Ethnic diversity, economic performance and civil wars.

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Abstract

We develop a behavioral model linking dissipation to the distribution of the population over an arbitrary number of groups. We extend the pure contest version of the model by Esteban and Ray (1999) to include a mixed public-private good. We analyze how the level of dissipation changes as the population distribution and the share of publicness of the prize change. The model is used to explain the sensitiveness of cross-country regressions associating ethnic diversity to economic performance and likelihood of civil wars to the index used to capture ethnic diversity. We find that we should use the fractionalization index when the prize at stake is purely private, the discrete polarization index when it is purely public, and a weighted average of the two when the prize is mixed. However, even in this case, empirical analysis would be still subject to measurement error. In particular, the fractionalization may still over-estimate the weight of bigger groups.

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1 Introduction

The empirical literature on ethnic diversity suggests two stylized relationships: ethnic diversity affects negatively steady-state GDP per capita (Easterly and Levine 1997) and affects positively the likelihood of experiencing a civil war (Montalvo and Reynal-Querol 2005b). The magnitude and significance of these reduced form relationships hinges on the measure used to capture ethnic diversity, either a fractionalization index either a polarization one. This sensitiveness may inform us as to the mechanisms through which they work. The research questions we tackle in this paper are: when should we use the fractionalization index and when the discrete polarization index? Above all, what drives the use of one index as opposed to the other? What does that imply about the underlying behavior of the economic agents? In order to answer these questions, we develop a behavioral model linking societal dissipation to the distribution of population across groups and we investigate how societal dissipation changes as the population distribution changes.

We conceive dissipation as a situation in which, in presence of weak institutions (absence of checks and balances, inefficiency of elections to discipline politicians or absence of elections overall) and in absence of a well-defined and agreed-upon collective decision rule, individuals incur costs to capture their most preferred outcome. The concept encompasses both inefficiency of economic policies and conflict. We choose such a broad concept because of two reasons. First, the paper is motivated by the empirical relationships between ethnic diversity on one side and (low) economic performance and likelihood of civil wars on the other. Second, our modelling strategy allows us to do so. We study a simple rent-seeking model with an arbitrary number of groups. The characteristic feature of this class of models is the diversion of resources from productive activities. This is why they are commonly used to explain rebel activities, open and latent conflict, lobbying and capture of the government and why it should capture not only the relationship between ethnic diversity and civil wars, but also the one between ethnic diversity and economic performance. The model borrows largely from the pure contest version of the model by Esteban and Ray (1999), who investigate the relationship between conflict and distribution. Since the properties of their model resemble closely those of the polarization index, one way to answer to our research questions is to extend it in a way that make the properties of the model resemble the fractionalization index for some parameter values, and those of the discrete polarization index for some others. By doing so, the model should suggest which features drive the change in the properties and which do not matter. The main novelty with respect to their model is the specification of the prize. Within the winning group, part of the outcome is public and is enjoyed in the same quantity by group members, no matter their number; another part is private, in the sense that it has to be shared among group members, which means that the per capita share shrinks with group size. If we were to consider both conflict and low economic performance as the outcome of the capture of government, then the public component could represent features like ideology and economic policies favoring members of the winning group, while the private component would represent not only any monetary component of the outcome, but also the capture of rent-seeking position whose value shrinks with the number of individuals having access to[1].

We find that the properties of the equilibrium are closest to those of the fractionalization index when the prize is a purely private good. We confirm that the properties of the equilibrium are closest to those of the discrete polarization index when the prize is a purely public good, as found by Esteban and Ray (1999). For all intermediate cases in which the prize is a mixed public-private good we show how some properties of the equilibrium vary as the degree of publicness of the prize varies. The relationship is very complex though. These results indicate that we should use the discrete polarization index when the prize individuals compete for is a pure public good, the fractionalization index when the prize is purely private and

[1] An example could be the often mentioned overvaluation of the exchange rate in African countries. The value of such access depends on the number of people having access to it.

a weighted average of the two when the prize is mixed.

The analysis of the underlying behavior of the economic agents suggests that even if we were to choose the combination of indexes according to the prize at stake, empirical analysis would be still subject to measurement error. We find that in case of pure private goods individuals belonging to bigger groups always contribute less. This suggests that the fractionalization index may systematically over-estimate the weight of bigger groups in the creation of dissipation even in case of purely private goods. In case of mixed prize, the pattern is more complex and the weighted average of the two indexes may just suffer an attenuation bias.

It should be stressed that the mixed public-private prize has introduced in a framework very similar to ours by Esteban and Ray (2009). We developed this modification independently though. Moreover, while both works investigate when to use a certain index, the way they answer is quite different: we develop analytically the properties of the equilibrium and compare them to those of the two indexes, while they run numerical simulations to test how accurate is the approximation done by using the indexes. The two works are also complementary: our analysis of the underlying behavior of the economic agents provide a theoretical ground to the systematic bias they find in correspondence of pure public goods and pure private goods.

The remainder of the paper is organized as follows. In section II we review the literature, present the empirical stylized facts and provide a short discussion of the fractionalization and polarization indexes. In section III we describe the model and derive some propositions. Section IV compare the predictions of the model with the properties of the two indexes. Section V concludes.

2 Literature review and stylized facts

More than ten years ago Easterly and Levine (1997) showed evidence suggesting that the degree of ethnolinguistic heterogeneity within a country affects negatively its growth prospects[2]. Later, Montalvo and Reynal-Querol (2005b) showed that ethnic diversity also affects the likelihood of civil war.

Notwithstanding the multitude of works following these papers[3], it is still not clear the way ethnic diversity is related to the economic performances and to social unrest. Easterly and Levine (1997) suggest three broad channels: political instability, rent-seeking policies and generalized corruption, low provision of public goods. However, they recognize that they cannot distinguish them in a useful way, since they are likely to be correlated. For example, they show that by including a measure of public infrastructures in the regression specification, the coefficient associated with ethnic diversity decreases greatly in magnitude and becomes insignificant. They interpret this result as evidence that an increase in ethnic diversity induces under-provision of pure public goods. This has been argued also by La Porta et al. (1999), Alesina et al. (2003) and Montalvo and Reynal-Querol (2005a). However, it seems rather difficult to infer whether ethnic diversity reduces the demand for pure public goods (as rationalized by Alesina, Baqir and Easterly 1999) or increases corruption and so its equilibrium provision (Mauro 1995)[4]. More in general, the proxy used by Easterly and Levine (1997) to capture the provision of public goods (logarithm of telephones per thousand workers) is not independent from the control variables used to capture political instability and rent-seeking policies[5]. Montalvo and Reynal-Querol (2005a) try to address the issue by including measures of ethnic diversity into first stage regressions explaining government consumption and investments, but they obtain

[2] Before them, the index of ethnic fractionalization had been used by Canning and Fay (1993) and Mauro (1995).

[3] There is a considerable number of parallel works in political science, mainly focussed in explaining the likelihood of civil wars. See the special issue in the Journal of Peace Research 2007.

[4] For example, Schleifer and Vishny (1993) suggest corruption motives may be so relevant that they may distort significantly the allocation of public spending away from schooling and health into infrastructures, given the lower level of transparency of the latter sector. This argument alone is sufficient to explain the previous empirical results. Notice also that inefficiency in the public sector (patronage and corruption) and targeted transfers could explain why there seems to be no effect of ethnic diversity on the size of the public sector notwithstanding the low provision of public goods (see Alesina et al. 2003 following the analysis pursued by La Porta et al. 1999).

[5] As the authors state by referring to the lack of independence of these variables in the sense of Levine and Renelt (1992). This is also the main critic suggested by Arcand et al. (2000).

mixed results[6].

The strand of literature on ethnic diversity and likelihood of civil wars could be thought as concerning one of the above-mentioned channels (political instability). In this respect, Fearon and Laitin (2003) and Caselli and Coleman (2006) suggest that ethnicity may be a strategic way of grouping to avoid sharing the benefits from winning with non-members. More recently, Esteban and Ray (2008a) suggest a theory according to which conflicts along ethnic lines are more likely than conflicts along class lines.

Finally, there is an emerging strand of literature investigating whether ethnic diversity can really be taken as exogenous and fixed over time. The relevant level of diversity may be subject to manipulation by politicians in the short term (Posner 2000), evolve over time (Michalopulous 2008, Campos and Kuzenyev 2007) and may be the result of long-run human history (Ahlerup and Olsson 2008, Spolaore and Wacziarg 2006).

2.1 Sources of data

Easterly and Levine (1997) use the index of ethnolinguistic fragmentation computed by Taylor and Hudson (1972) with data from Atlas Norodov Mira (1964)[7]. The latter is the result of a twenty years research programme in the Soviet Union.

Alesina et al. (2003) consider language, religion and ethnic diversity separately. For the ethnic index, which involves both racial and linguistic characteristics[8], they

[6] Their work seems not to be robust to a number of points. Their measures of ethnic diversity do not affect significantly government consumption when entererd one at the time (and there is no clear reason to include them both if the aim is predicting the dependent variable instead of assessing the relative explanatory power, as we do here). They do not distinguish private from public investment, while the common idea is that the two respond to very different mechanisms: public investment may suffer low provision of public goods as well as corruption; private investment suffers the low provision of compelmentary infrastructures and high political instability, which depress the marginal return of capital. Finally, their measure of ethnic diversity remains significant even after including consumption and investments, which means that they do not identify all the channels the relationship works through.

[7] The same data were also used by Canning and Fay (1993) and Mauro (1995).

[8] For example, they find that the source refers mainly of race for south american countries; language for european ones (ex. Belgium, Switzerland); mixed for sub-saharan countries.

use the Encyclopedia Britannica (124 on 190 countries; henceforth EB), CIA World Factbook (25 countries), Levinson (1998, 23 countries), Minority Rights Group International (1997, 13 countries). When more than one source provided information over the same country, they first computed the fractionalization index according to each source and then they chose among them according to the following rule: "if two or more sources for the index of ethnic fractionalization were identical to the third decimal point, we used these sources (..). If sources diverged in such a way that the index of fractionalization differed to the second decimal point, we used the source where ethnic groups covered the greatest share of the total population. If this was 100 percent in more than one source, we used the source with the most disaggregated data (i.e. the greatest number of reported groups)" (p.160).

Montalvo and Reynal-Querol (2002, 2005a, 2005b) use the World Christian Encyclopedia (WCE), which provides more details on the way used to discriminate groups. First, it considers six different characteristics: race and color, culture and language, ethnic origin, nationality. Then, it combines them to provide a classification of ethnolinguistic diversity with several levels: 7 major races, 7 colors, 13 geographical races, 4 sub-race, 71 ethnolinguistic families, 432 major peoples, 7010 distinct languages, 8990 sub-peoples, 17000 dialects. Following Vanhanen (1999), they consider an intermediate level of disaggregation, ethnolinguistic families[9]. However, since the identification strategy used in the WCE is more flexible[10], this choice led the two researchers to aggregate proportions or groups of peoples or sub-peoples, when these are identified as the relevant cleavages.

Simple comparison of the two data collection processes then suggests that, if there is a bias in the data used by Alesina et al. (2003), it is in favor of disaggregation; on the contrary, if there is a bias in the data used by Montalvo and Reynal-Querol,

[9] Fearon (2003) also considers an intermediate level of disaggregation: he considers only groups whose population share is greater then one percent. His sources are the EB, the CIA World Factbook and Library of Congress Country Study.

[10] Since the WCE relies on survey questions like the following "What is the first, or main, or primary ethnic or ethnolinguistic term by which persons identify themselves, or are identified by people around them?"

it is in favor of aggregation.

2.2 Measures of ethnic diversity: fractionalization vs polarization

The measures commonly used to capture ethnic diversity are the fractionalization index, constructed by Taylor and Hudson (1972), and the discrete polarization index, adapted by Montalvo and Reynal-Querol (2002) from Esteban and Ray (1994). Both these measures aggregate data on group shares into an index ranging along the unit interval[11].

The fractionalization index is the probability that any two randomly chosen individuals belong to different ethnic groups. Let the size of a generic group be denoted by n_i and the entire population be normalized to unity $\left(\sum_{i=1}^{G} n_i = 1\right)$, then the fractionalization index is[12]:

$$F = \sum_{i=1}^{G} n_i \left(1 - n_i\right) = 1 - \sum_{i=1}^{G} n_i^2 .$$

It has the following properties[13]:

1. for a given number of groups G, F is maximized at the uniform population distribution over these groups;
2. over the set of uniform distributions, F increases with the number of groups;
3. any transfer of population to a smaller group increases F;
4. the split of any group into two new groups increases F.

[11] Ideally, one would like to count for inter-group distances as well. However, to the best of my knowledge there are no convincing measures of ethnic hate. Indeed, this constitutes one of the main reason of interest for the recent research on genetic distances, which draws largely upon the seminal work by Cavalli-Sforza et al. (1994).

[12] The index has two theoretical backgrounds: one is the Gini coefficient (the fractionalization index can be seen as its semplification); the other is Herfindal index (the fractionalization index is its complement).

[13] See Esteban and Ray (2008b).

The discrete polarization index is a simplified version of the polarization index introduced by Esteban and Ray (1994)[14]. The expression for its discrete version (Q) is:

$$Q = 4\sum_{i=1}^{G} n_i^2 \left(1 - n_i\right)$$

where n_i denotes the population share for group i and the population is normalized to unity: $\sum_{i=1}^{G} n_i = 1$.

It has the following properties[15]:

1. for a given number of groups G, Q is maximized when the population is concentrated on two equally sized groups only (bimodal symmetric distribution);

2. over the set of uniform distributions, Q decreases with the number of groups, provided there are at least two groups to begin with;

3. a transfer of population to a smaller group increases Q if both groups are larger than 1/3. If both groups are smaller than 1/3, the transfer decreases Q;

4. the split of a group in two increases Q if and only if the initial group size was at least 2/3.

Notice that in case $G = 2$, both measures reach their maximum in correspondence of the uniform distribution $(n_1 = n_2 = 1/2)$ and transfers from big to small groups increase both indexes[16]. The two indexes diverge more and more as the number of groups with positive population shares increase $(G \geq 3)$, since Q maintains its maximum in correspondence of the bimodal distribution (population concentrated

[14] Essentially, Montalvo and Reynal-Querol (2002, 2005a, 2005b) simplified the expression for the general index to exclude the use of ethnic distances, normalized the index to unity to make it easier to be interpreted, and chose a particular value of a polarization sensitiveness (see one of the paper for details). Notice that the main purpose of the latter was to provide an alternative to the Gini coefficient in the field of inequality measurement and that the fractionalization index constitutes a semplification of the Gini coefficient itself.

[15] See Esteban and Ray (2008b).

[16] Indeed, Montalvo and Reynal-Querol (2002) show that, within the two-group case, even when group sizes diverge, the two indexes continue to be proportional to each other.

in any two groups with equal population shares $n_i = n_j = 1/2$), while the maximum for F becomes the uniform distribution over all groups.

2.3 Ethnic diversity, economic performance and civil wars: empirical evidence

What we want to investigate in this sub-section is whether the fractionalization and the discrete polarization indexes differ also in their explanatory power. In the following regression analysis the two indexes will be the explanatory variables of interest, while we will consider two types of dependent variables: growth rates of real GDP per capita and likelihood of civil wars. In order to make the comparison as rigorous as possible, we choose the same time period (1960-1989) and the same level of observation (country data averaged over decades: one observation per decade). The econometric technique, the treatment of standard errors and the set of control variables are those of our reference papers: Easterly and Levine (1997) for economic performance; Montalvo and Reynal-Querol (2005b) for civil wars. With respect to the former, we include both indexes at the same time in the specification. With respect to the latter, we only change the time period. We also run separate regressions corresponding to different data sources: Montalvo and Reynal-Querol (2005b), who use the World Christian Encyclopedia, and Alesina et al. (2003), who use the Encyclopedia Britannica.

In table 1 (Appendix) we show the results for economic performance. They suggest that the fractionalization index explains growth rates better than the discrete polarization index, no matter the source of data.

In table 2 and 3 (see Appendix) we present parallel evidence on the relationship between ethnic diversity and civil wars. There is no clear cut index measuring the likelihood of civil wars. There are several measures used extensively in the literature: broadly speaking, they are based on the number of deaths due to battles between the government and an opposing faction within a given year and they differ mainly for the threshold number of deaths above which a country is defined as going through a civil war. Here we present results for five measures: at least 25 deaths (PRIO25)[17], at least 25 deaths plus at least 1000 death over the course of the war (PRIOcw), at least 100 deaths on both sides plus 1000 deaths over the course of the war (FLcw)[18], at least 1000 deaths (PRIO1000), at least 1000 deaths plus it challenged the sovereignty of the State (SDcw)[19]. Table 2 reports the results associated with the use of data from Montalvo and Reynal-Querol (2005b). Ethnic polarization outperforms ethnic fractionalization in all but one measure.

In table 3 we run the same regression specifications using data from Alesina et al. (2003). Here the results are less clear-cut: the ethnic polarization index outperforms the fractionalization one only when using the most sensitive index (at least 25 deaths in a given year). Thus, the evidence on this relationship is ambiguous. However, given that the data used by Montalvo and Reynal-Querol (2005b) seem more accurate (the WCE provides the criteria used for the classification) and that they find the same results when considering five year averages, we rely on the findings in table 2 for the rest of the analysis.

With the caveat that sensitiveness to the indexes may be due also to the data source, we draw the following stylized facts from this literature:

- ethnic fractionalization explains (low) economic performance better than ethnic polarization;
- ethnic polarization explains likelihood of civil wars better than ethnic fractionalization.

[17] Prio-Uppsala database.
[18] based on Fearon and Laitin (2003).
[19] based on Doyle and Sambanis (2000).

3 The model

We provide a behavioral model linking dissipation to the distribution of the population over a set of groups.

We consider the pure contest version of the model by Esteban and Ray (1999)[20]. Individuals belonging to different groups compete for the capture of a prize. We extend their model by specifying a mixed public-private prize. This feature introduces an additional channel through which group size determines the incentives of economic agents to contribute. Group size determines the per capita share of the private component: the bigger the group, the smaller the per capita share. Whether this means introducing the Pareto-Olson argument into the model will be discussed later in the section. It should be stressed that the mixed public-private prize has been used in a different framework by Esteban and Ray (2001) and only recently it has been introduced in a framework very similar to ours by Esteban and Ray (2009). We developed this modification independently though. Their research question concerns which index is more appropriate to use and whether existing indexes are accurate approximations of the model. They investigate this issue by running numerical simulations. We answer to the same question by deriving analytically some properties on the level of dissipation and on the pattern of dissipation. In a sense, the two works are complementary.

The model has some limits too. First, we neglect the productive side of the economy. In this sense the relationship between dissipation and distribution is a very reduced form. Although the marginal cost of contributing is increasing and captures the rising opportunity cost of devoting resources to a non-productive activity, the prize is exogenous and independent from the level of dissipation in the society. Sec-

[20] We neglect the notion of distance between groups. In this respect, our model is less ambitious, but this is a precise choice. The full version of their model includes the distance between groups. Our paper is empirically motivated though, and there is no reliable measure of distance between groups in the literature on ethnic diversity. This is also why in our model we neglect the provision of public goods, which is the third channel suggested by Easterly and Levine (1997) to explain the relationship between ethnic diversity and economic performance.

ond, the utility function considered is linear in income and we assume that members of the same group enjoy the same utility and so behave identically. Both modelling choices are driven by reasons of tractability: allowing an arbitrary number of groups in the society complicates the analysis considerably and we had to simplify other aspects of the economy.

In section 3.1 we describe the model and how it differs from the literature. In section 3.2 we settle the existence and uniqueness of the equilibrium. In section 3.3 we analyze the relationship between equilibrium dissipation and population distribution.

3.1 Description of the model

Agents. There is a unit mass of individuals distributed over the unit interval, where i indicates the group and k indicates the individual. Individuals are distributed across G groups, each with population n_i, so that $n_i \in (0,1]$ and $\sum_{i=1}^{G} n_i = 1$.

Actions. Society must choose the allocation of a prize. We model this prize directly in terms of the utility individuals receive from it (w_{ik}). We assume that individuals can influence the allocation of the prize by devoting resources into a non-productive activity. The decision process can be interpreted as a lottery, where the probability of receiving the prize is distributed over the population according to a vector of resources. Let $a_{ik} \in \mathbb{R}^{+}$ denote the resources devoted by individual k in group i. The aggregate amount of resources devoted by the entire population is $A \equiv \sum_{i=1}^{G} \sum_{h \in i} a_{ih}$, (where h indicates the generic individual in group i), where $A \in \mathbb{R}^{+}$. We will use A as a measure of societal dissipation in the non-productive activity.

Timing. The timing is the following: i) all individuals of all groups choose simultaneously their contributions; ii) nature chooses the winning group with probabilities π_i; iii) the prize is distributed across members of the winning group.

Information. The payoff structure of all individuals is common knowledge.

Payoffs. Let $c(a)$ denote the utility cost of a generic amount of resources. The cost function $c : \mathbb{R}^+ \to \mathbb{R}^+$ is homogeneous across all groups. Assume that

Assumption 1. c is continuous, increasing and twice differentiable with $c(0) = 0$, $c' > 0$, $c'' > 0$ for all $a > 0$, and $\lim_{a \longrightarrow 0^+} c'(a) = c'(0) = 0$.

Define the winning probability of individual k in group i (π_{ik}) as the share of resources devoted by members (indexed by h) of group i:

$$\pi_{ik}(a_{ik}) = \frac{\sum_{h \in i} a_{ih}}{A}, \tag{1}$$

provided $A > 0$. By definition (1) individuals belonging to the same group have the same winning probability: $\pi_{ik} = \pi_{il} = \pi_i \ \forall (k,l) \in i, \forall i = 1, .., G$.

Let w_{ik} be the individual benefit from winning the prize. We specify the prize as a mixed private-public good[21]. Let $\lambda \in [0,1]$ denote the share of publicness of the prize:

$$w_{ik} = w(\lambda, n_i) = \lambda + \frac{1-\lambda}{n_i}. \tag{2}$$

It is important to specify exactly the nature of the prize. Both the public component (λ) and the private component $(1-\lambda)$ are enjoyed *exclusively by members of the winning group.* The difference between the two is that the per capita benefit associated with the public component is constant, while the one associated with the private component shrinks with group size.

For simplicity, we assume that the share of publicness of the prize (λ) is the same across groups. By definition (2), individuals of the same group receive the same benefit in case of capture of the prize: $w_{ik} = w_{il} = w_i \ \forall (k,l) \in i, \forall i$.

We assume a utility function for individual k in group i linear in the expected benefit from winning the prize net of the cost of contributions:

[21] Esteban and Ray (2001) suggested this specification in a different framework.

$$u_{ik}\left(a_{ik}\right)=\pi_i\left(a_{ik}\right)w_i-c\left(a_{ik}\right).$$

We assume that individual k in group i chooses his contribution so as to maximize his extended utility function (v_{ik}), which includes the ones of his fellow members:

$$v_{ik}\left(a_{ik}\right)=\sum_{l\in i}u_{il}\left(a_{il}\right)=u_{ik}\left(a_{ik}\right)+\sum_{l\in i,l\neq k}u_{il}\left(a_{il}\right) \tag{3}$$

This is what Esteban and Ray (1999, 2008) and Montalvo and Reynal-Querol (2002, 2005a) named absence of free-riding.This assumption can be founded on either one of two theoretical background: either individual contributions are really determined by a group leader, like in Esteban and Ray (2008a), because of coercion or group ideology, either individuals maximize an extended utility, which includes the utility of fellow members.

To complete the specification of the model, we describe the outcome when $A=0$. We take this to be an arbitrary vector $\bar{\pi}=(\bar{\pi}_1,..,\bar{\pi}_G)$ [22].

The following table summarizes all variables and functions included in the model.

[22] Esteban and Ray (1999) provide a similar assumption to complete the specification of their model.

Table 4 - List of the variables in the model.		
a_{ik}	individual contribution of member of individual k in group i	choice variable
n_i	size of group i	exogenous
w_i	utility for any member of group i for outcome $i : \lambda + \frac{1-\lambda}{n_i}$	exogenous
λ	share of publicness of the prize: $\lambda \in [0,1]$	exogenous
π_i	winning probability for any member of group $i : \sum_{i=1}^{G} \pi_i = 1$	endogenous
A	dissipation: $A = \sum_{i=1}^{G} \sum_{h \in i} a_{ih}$	endogenous
$c\,()$	cost of effort $c : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ and $c\,(.) : c'\,(.) > 0, c''\,(.) > 0$	
a^*	vector of individual contributions $a^* \equiv \left(a^*_{11}, .., a^*_{1n_1}, .., a^*_{G1}, .., a^*_{Gn_G}\right)$	equilibrium
π	vector of winning probabilities $\pi \equiv (\pi_1, .., \pi_G) : \sum_{i=1}^{G} \pi_i = 1$	
N	vector of group sizes $N \equiv \left(n_1, .., n_G\right) : \sum_{i=1}^{G} n_i = 1$	

3.2 Agents' Behavior and Equilibrium

All the proofs to the propositions henceforth are relegated in the Appendix.

Notice that individuals belonging to the same group have exactly the same payoff structure. Therefore, they will devote the same per capita contributions in equilibrium: $a_{ik} = a_{il} = a_i \ \forall (k,l) \in i, \ \forall i = 1, .., G$, where a_i denotes the per capita contribution of members of group i.

Proposition 1 *Suppose that assumption 1 holds. Provided $a_j > 0$ for some $j \neq i$, the amount of resources devoted members of groups i are strictly positive and completely described by the first-order condition (FOC)*

$$\frac{n_i}{A}\left(1 - \pi_i\right) w_i = c'\left(a_i\right) \tag{4}$$

Since $\pi_i = \frac{n_i a_i}{A}$, we can write (4) as follows:

$$\pi_i (1 - \pi_i) w_i (\lambda, n_i) = c' (a_i) a_i \tag{5}$$

Equation (5) shows that, besides the share of publicness of the prize (λ) and group size (n_i), individual contributions depend on the group's winning probability (π_i). With the caveat that the endogeneity of the latter requires additional analysis for a full discussion of the implications of the model, we consider the main forces at work. Notice that a larger group means a larger number of contributions and so a higher winning probability (π_i) for any given level of individual contribution within the group. As Esteban and Ray (1999) put it: "members of a larger group generate larger externalities for one another, and as such may be cajoled, inspired, or compelled by group leaders to put in more resources per capita" (p.385). A larger group also means smaller population to face, and so weaker competition over the prize (smaller probability of "losing" the prize $(1 - \pi_i)$). Finally, a larger group also means greater dissipation of the private component of the prize, and so reduced incentives to contribute (smaller benefit w_i). This latter force is more relevant the greater the share of the private component within the prize. Therefore, we may expect both level and pattern of dissipation to vary with the level of this parameter.

Define the equilibrium as a vector of resources $a^* \equiv (a_1^*, .., a_G^*)$ such that a_{ik}^* solves the maximization of (3) subject to (1); equilibrium dissipation is given by $A^* = \sum_{i=1}^{G} n_i a_i^*$; equilibrium resource shares are $\pi_i^* = \frac{n_i a_i^*}{A^*}$.

Proposition 1 states that the solution to the individual's maximization problem is always interior. Thus, any equilibrium must involve positive contributions by all individuals. The following proposition characterizes the equilibrium:

Proposition 2 *Suppose assumption 1 holds. Then an equilibrium exists and it is* *unique.*

3.3 Dissipation and distribution: levels and patterns

We analyze how equilibrium dissipation (A) varies as we let population distribution (N) for each degree of publicness of the prize (λ). By doing so, we compare dissipation across equilibria. Later in the paper (section 4) we will compare the properties of this relationship with those of the indexes used in the empirical literature (section 2.3). In order to understand the behavior of the economic agents implied by the model, we shift the analysis to a more disaggregated level: how per capita contributions (a_i) vary across groups. In order to do so, we compare contributions within a given equilibrium (A fixed).

Recall that our model is an extension of the pure contest version of Esteban and Ray (1999) to mixed public-private goods. With respect to our model, their results cover the case of pure public goods $(\lambda = 1)$. Throughout the analysis, we refer to their results as a benchmark against which evaluate ours $(\lambda \in [0, 1))$.

3.3.1 Dissipation and distribution: levels

We start our analysis with the case of two groups. Following Tullock (1980), we would expect the uniform distribution to be the global maximum. This is how both the frationalization (F) and discrete polarization index (Q) behave and what Esteban and Ray (1999) find for pure public goods. The next proposition extends this result to all other cases.

Proposition 3 *Suppose that assumption 1 holds. Then in the two groups case the uniform distribution is the strict global maximum.*

The proposition implies that any departure from the uniform distribution, which corresponds to increased population inequality, lowers the level of dissipatio. We investigate whether this result extends to an arbitrary number of groups $G \geq 3$.Propositions 4 and 5 summarize what we know about uniform distributions. Over this class of distributions, Esteban and Ray (1999) find that equilibrium dissipation decreases with the number of groups, provided there at least two groups to begin with. This is exactly in line with the second property of the discrete polarization index (see section 2). We investigate whether this continues to be true for all types of goods.

Proposition 4 *Suppose that assumptions 1 holds. Then over the set of uniform distributions, equilibrium dissipation increases with the number of groups up to a threshold* $G(\lambda)$*, and decreases thereafter. The number of groups maximizing dissipation increases as the prize becomes more private* $\left(\frac{\partial G(\lambda)}{\partial \lambda} < 0\right)$*, and approaches infinity as the prize is half public half private* ($\lambda = 1/2$)*.*

So the Esteban and Ray's finding is not robust over all types of goods. Most importantly, the dissipation-distribution relationship does not resemble the property of the discrete polarization index anymore. On the contrary, for a large set of goods $\left(\lambda \in \left[0, \frac{1}{2}\right]\right)$, dissipation increases with the number of groups, thus resembling the second property of the fractionalization index[23].

Next, it would be interesting to know whether there are other similarities between the properties of dissipation-distribution relationship and those of the fractionalization index. In section 2 we have seen that population transfers to smaller groups

[23] Even the Esteban and Ray's finding that the symmetric bimodal distribution is the global maximum is not robust to our extension. In fact, although we do not identify the global maximum for each possible degree of publicness, we can rule the symmetric bimodal distribution out of the potential candidates for a large set of goods. In order to establish this, it is enough to notice that ER's global maximum is a uniform distribution. Since over the set of uniform distributions dissipation is greatest in correspondence of the three-point uniform distribution for $\lambda = \frac{3}{4}$, then the two-points uniform distribution can be ruled out for that and for smaller values: $\lambda \in \left[0, \frac{3}{4}\right]$.

we find that the uniform distribution over two groups is associated with a level of dissipation strictly lower than that associated with the three-point uniform distribution for $\lambda \in$
ter values such that dissipation is higher in correspondence of a uniform distribution over a different number of groups. Since dissipation is maximized over three groups for $\lambda = 3/4$ (see proof in the Appendix), the uniform distribution over two groups cannot be the global maximum for $\lambda \leq 3/4$.

affect the index in the same direction no matter the distribution of the groups not involved in the transfer. This is what Esteban and Ray call "monotonicity" of the index. Such property distinguishes the fractionalization index from the discrete polarization one and contributes to the major complexity of the latter. They are able to rule out monotocity from the dissipation-distribution relationship by referring to transfers leading from the uniform distribution over G groups to that over $G-1$ groups. This is how they prove it: they know that dissipation corresponding to the former is lower than that corresponding to the latter; they argue convincingly that a consistent series of population transfer aiming to reach the uniform distribution over $G-1$ groups would be to move population out of one group towards all others. In other words, one group will tend to disappear while the others grow uniformly. The distribution they have just before moving the last bit of population from it is closest to the uniform distribution over $G-1$ groups and so, by continuity, must have a level of dissipation close to it. This implies that the uniform distribution over G groups is a local maximum without being a global maximum over the set of $G-$point distributions. This, joint to the uni-directional series of transfers, let them state that the dissipation-distribution relationship is non-monotonic. Would it be possible to adopt the same strategy in out case? Notice that the previous proposition about dissipation over the set of uniform distributions is exactly opposite to that found by Esteban and Ray. This is relevant. Consider the set of goods for which $\lambda \in \left[0, \frac{1}{2}\right]$. Dissipation associated with the G-point uniform distribution is greater with that associated with the $G-1$-point uniform distribution. In order to pass from the latter to the former with a series of transfers "moving in the same direction", we would have to split one of the groups first, thus switching to the class of $G-$point distributions, and then transfer population from all groups to the smaller one. This requires

The next proposition concerns local maximality with respect for the special case of pure private goods. Let α denote the elasticity of marginal cost of the contribution $c'(a)$ with respect to the contribution itself $a : \alpha(a) = \frac{c''(a)a}{c'(a)}$, $\forall a > 0$. We make the following regularity assumption on such elasticity

assumption 2. c is three times differentiable and $\frac{\alpha'(a)a}{\alpha(a)} > -\frac{(2-\pi)\alpha(a)+\pi}{\pi}$.

Essentially we restrict the class of cost functions. The intuition behind this assumption is that we want the cost function is be "convex enough". It is not very restrictive though. For example the entire set of iso-elastic cost functions $c(a) = \beta a^{\alpha}$, satisfying assumption 1 is included. Just observe that the latter requires $\alpha(a) > 0$, which makes the RHS always negative. By definition, the derivative of the elasticity of an iso-elastic function is zero, which makes the LHS equal to zero as well and the assumption hold.

Proposition 5 *Suppose that assumptions 1 and 2 hold. Then in case of purely private goods, any uniform distribution is always a strict local maximum.*

We find that uniform distributions are all local maximum in case of purely private goods. We also find that the level of dissipation in correspondence of $G-1$ groups is always lower than in correspondence of G groups. These two results, combined together, imply that the relationship between dissipation and distribution is non-monotonic. Suppose you were to decompose the change from a uniform distribution over $G-1$ groups to the one over G groups in a sequence of population shifts. From proposition 4 we know that the initial level of dissipation is strictly lower than its final level. However, a generic sequence of transfers from the initial to the final situation will not necessarily be all dissipation-increasing. Proposition 5 states exactly that any local departure from the uniform distribution over $G-1$ groups will be dissipation-decreasing, since the latter is a local maximum. So, we have established that there are at least some distributional changes that cannot be broken down in a series of smaller changes going in the same direction. This is at odds with the monotonicity of the fractionalization index (section 2, property 3).

Although we do not expect a generic sequence of changes to affect the level of dissipation uni-directionally in case of pure private goods and we may not expect it even for the other cases, we may ask ourselves whether there exists a sequence of changes providing uni-directional impacts on dissipation. Following Esteban and Ray (1999), we explore the possibility of groups merging together. They find that the repeated merge of the two smallest groups or the one-step merge of the smallest $G-1$ groups unambiguously increase dissipation. The next proposition summarizes our finding for all other cases.

Proposition 6 *In case of pure private goods any merge must lower equilibrium dissipation.*

Esteban and Ray (1999) comment on their result as having "implications for the phenomenon of *divide and conquer*" (p.397). In case of pure public goods, starting from a uniform distribution over two groups, a split of one of them always causes dissipation to fall. Proposition 6 indicates that in case of pure private goods, we expect exactly the opposite.

3.3.2 Dissipation and distribution: patterns

We have already mentioned the possibility that individuals belonging to groups with different sizes may devote different contributions. In order to investigate this point in more detail, we describe individuals' contribution choices (a_i) with respect to the average contribution across the entire population (A). Define the ratio between the two $(\frac{a_i}{A})$ as intensity of lobbying. Define activism any equilibrium such that at least two groups differ in their intensity of lobbying: $a_i \neq a_j$ for some i, j.

The following proposition complements proposition 5.2 in Esteban and Ray (1999). They find that "contests with two groups can never involve activism. On the other hand, contests with more than two groups display activism whenever all groups are not equal sized, and larger groups always lobby more than smaller groups" (p.398). This is how results change once we allow the prize to be not purely public.

Proposition 7 *Contests between two groups involve activism whenever the prize is not purely public and the two groups are not equal sized. In this case, the larger group always lobbies less intensively than the smaller one. Contests with more than two groups display activism whenever all groups are not equal sized. In case of purely private goods, larger groups always lobby less intensively than smaller ones.*

The case of two groups illustrates clearly the forces at work described in section 3.2: a larger group means a greater number of contributions (greater incentive to contribute), but also a smaller opponent (lower incentive to contribute) and lower per capita benefit from the private component of the prize. In case of pure public goods, the latter component does not exist, the first two forces exactly cancel each other out and individuals contribute the same no matter the population distribution. For all intermediate cases though, the additional incentive created by the private component of the prize plays a role and individuals belonging to the smaller group contribute more than the opponents. Notice that this does not mean that the share of resources devoted by the larger group is smaller than the share of resources devoted by the smaller group. Indeed, the larger group continues to have a greater winning probability (see Lemma 8.1), but not as much as it would have had in case of pure public goods. Therefore, whether we may say that the Pareto-Olson argument plays a role in the model depends on the definition of the latter. According to Esteban and Ray (2001), the Pareto-Olson argument dominates when larger groups have smaller winning probability than smaller groups, which is not the case here[24]. The case of an arbitrary number of groups $G \geq 3$ is more complex. Essentially, the second force we listed becomes weaker. Esteban and Ray (1999) find that the first force always dominates and larger groups lobby more intensively than smaller ones; on the other hand, we find that for pure private goods the third force dominates and individuals

[24]Indeed, it is quite surprising that this does not happen in our framework and that the value of the elasticity of the cost function, which is crucial in Esteban and Ray (2001), here does not play any role. Since the difference between their framework and the current one is the assumption of free-riding, this difference indirectly suggests the value of that assumption.

belonging to larger groups always contribute less than those belonging to smaller groups.

4 Conclusions

In this paper we asked when to use the fractionalization index as opposed to the discrete polarization index, and what this implies about the underlying behavior of the economic agents. In order to answer these questions we developed a behavioral model linking dissipation to the distribution of the population across an arbitrary number of groups and we studied the properties of the relationship.

We find that the properties of the equilibrium are closest to those of the fractionalization index when the prize is a purely private good. We confirm that the properties of the equilibrium are closest to those of the discrete polarization index when the prize is a purely public good, as found by Esteban and Ray (1999). For all intermediate cases in which the prize is a mixed public-private good we show how some properties of the equilibrium vary as the degree of publicness of the prize varies. The relationship is very complex though. These results indicate that we should use the discrete polarization index when the prize individuals compete for is a pure public good, the fractionalization index when the prize is purely private and a weighted average of the two when the prize is mixed. These results may explain why cross-country regressions associating ethnic diversity to economic performance and likelihood of civil wars are sensitive to the index used to capture the former. To the extent both reflect competition for the capture of the State, our results suggest that the latter is perceived as a public good in case of open conflict, while it is perceived as a private good in case of lobbying and generalized corruption.

The analysis of the underlying behavior of the economic agents suggests that even if we were to choose the combination of indexes according to the prize at stake, empirical analysis would be still subject to measurement error. We find that in case of pure private goods individuals belonging to bigger groups always contribute less. This suggests that the fractionalization index may systematically over-estimate the weight of bigger groups in the creation of dissipation even in case of purely private goods. In case of mixed prize, the pattern is more complex and the weighted average of the two indexes may just suffer an attenuation bias.

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Appendix

Proof. Proposition 1.

Given that individuals within the same group, then maximizing (3) subject to (1) becomes maximizing

$$\frac{n_i a_i}{\sum_{j=1}^{G} n_j a_j} w_i - c(a_i) \tag{6}$$

Equation (9) is well-defined for every a_i since we have assumed that $a_j > 0$ for some $j \neq i$. The end-point restriction on c in assumption 1 and the observation that the existence of a positive lower bound on the benefit from winning the prize $(w_i \geq \lambda > 0)$ ensure that the solution to the maximization problem is interior (the FOC must hold with equality). Differentiation of (9) with respect to a_i provides exactly (4). Since differentiation of the expected benefit $(\pi_i w_i)$ with respect to a_i shows its strict concavity and assumption 1 ensures the strict convexity of the cost function, then the individual utility function is strictly convex, which means that equation (4) is also sufficient to define the solution. ■

Proof. Proposition 2.

Define a function $\phi : [0,1]^2 \times \mathbb{R}^+ \rightarrow \mathbb{R}$ such that the single element $\phi(\pi_i, A, n_i)$ is defined by the first order derivative of the maximization problem in terms of winning probability, dissipation and group size $\left(a_i = \frac{\pi_i A}{n_i}\right)$:

$$\begin{aligned} & \frac{n_i}{A}(1-\pi_i) w_i - c'\left(\frac{\pi_i A}{n_i}\right) \\ = \quad & \phi(\pi_i, A, n_i) \end{aligned}$$

Re-define the equilibrium as any combination of winning probabilities $\pi^* = (\pi_1^*, .., \pi_G^*)$ and total effort A^*, such that $\phi(\pi_i^*, A^*, n_i) = 0 \ \forall i$, and $\sum_{i=1}^{G} \pi_i^* = 1$.

The determination of the equilibrium can be shown in two step: first, by making reference to the individual optimality condition (FOC); second, by making reference to the population consistency condition (the sum of winning probability must equal one).

Suppose A (and N) fixed, and consider the behavior of the first derivative $\phi(\pi_i, A, n_i)$ as the winning probability (π_i) varies along its domain $[0,1]$:

- $\frac{\partial\phi(\pi_i,A,N_i)}{\partial\pi_i} = -\frac{n_i}{A}w_i - \frac{A}{N_i}c''\left(\frac{\pi_i A}{n_i}\right) < 0$ (strictly decreasing);
- $\lim\limits_{\pi\longrightarrow 0^+} \phi(\pi_i, A, n_i) = \frac{n_i}{A}w_i > 0$;
- $\lim\limits_{\pi\longrightarrow 1^-} \phi(\pi_i, A, n_i) = -c'\left(\frac{A}{n_i}\right) < 0$;

The intermediate value theorem ensures the existence and uniqueness of a winning probability satisfying the equilibrium condition: $\exists!\pi_i^* : \phi(\pi_i^*, A, n_i) = 0$. This value can be thought of as a function depending on the remaining variables: $\pi_i^* = \pi(A, n_i)$.

Aggregate consistency requires the sum of these winning probabilities to equal unity: $\sum\limits_{i=1}^{G} \pi(A, n_i) = 1$. Suppose N fixed and consider the behavior of the sum of winning probabilities $\left(\sum\limits_{i=1}^{G} \pi(A, n_i)\right)$ as total dissipation (A) varies along its domain $[0, +\infty)$. Since we have not derived an explicit expression for the equilibrium winning probability, we refer to the implicit function theorem to study it. Re-write the FOC function $\phi_i \equiv \phi(\pi_i, A, n_i)$, then we know:

$$\frac{\partial\phi_i}{\partial\pi_i}\frac{d\pi(A, n_i)}{dA} + \frac{\partial\phi_i}{\partial A} = 0$$

which means

$$\frac{d\pi(A, n_i)}{dA} = -\frac{\frac{\partial\phi_i}{\partial A}}{\frac{\partial\phi_i}{\partial\pi_i}}$$

since $\frac{\partial\phi_i}{\partial A} = -\frac{n_i}{A^2}(1-\pi_i)w_i - \frac{\pi_i}{n_i}c''\left(\frac{\pi_i A}{n_i}\right) < 0$, and $\frac{\partial\phi_i}{\partial\pi_i} = -\frac{n_i}{A}u - \frac{A}{N_i}c''\left(\frac{\pi_i A}{n_i}\right) < 0$, then

$$\frac{d\pi(A, n_i)}{dA} < 0 \;\forall i.$$

which implies

$$\sum_{i=1}^{G}\frac{d\pi\left(A,n_i\right)}{dA}<0\Longrightarrow\frac{d\left[\sum\limits_{i=1}^{G}\pi\left(A,n_i\right)\right]}{dA}<0.$$

Again, we derive the behavior of this function as total dissipation approaches the limits of its domain. In order to do so, we focus on the single winning probability $\pi\left(A,n_i\right)$. In order to determine the behavior of the winning probability for any member of group i as total dissipation shrinks to zero, fix such winning probability and consider the behavior of the first derivative ϕ as total dissipation shrinks to zero:

$$\begin{aligned}\lim_{A\longrightarrow 0^+}\phi\left(\pi_i,A,n_i\right) &= \lim_{A\longrightarrow 0^+}\frac{n_i}{A}\left(1-\pi_i\right)w_i-\lim_{A\longrightarrow 0^+}c'\left(\frac{\pi_i A}{n_i}\right)\\ &= \infty-c'(0)=\infty.\end{aligned}$$

If the first order condition ($\phi_i=0$) is to continue to hold, the winning probability must approach unity as total dissipation $\left(\lim\limits_{A\longrightarrow 0^+}\pi\left(A,n_i\right)=1\right)$. This implies that the sum of winning probabilities will exceed unity: $\lim\limits_{A\longrightarrow 0^+}\left[\sum\limits_{i=1}^{G}\pi\left(A,n_i\right)\right]=G(>1)$. In order to determine the behavior of the winning probability for any member of group i as total dissipation increases to infinity, fix such winning probability and consider the behavior of the first derivative ϕ as total dissipation increases to infinity:

$$\begin{aligned}\lim_{A\longrightarrow +\infty}\phi\left(\pi_i,A,n_i\right) &= \lim_{A\longrightarrow +\infty}\frac{n_i}{A}\left(1-\pi_i\right)w_i-\lim_{A\longrightarrow +\infty}c'\left(\frac{\pi_i A}{n_i}\right)\\ &= 0-\infty=-\infty.\end{aligned}$$

If the first order condition ($\phi_i=0$) is to continue to hold, the winning probability must shrink to zero $\left(\lim\limits_{A\longrightarrow +\infty}\pi\left(A,n_i\right)=0\right)$. This implies that the sum of winning probabilities will shrink to zero as well: $\lim\limits_{A\longrightarrow +\infty}\left[\sum\limits_{i=1}^{G}\pi\left(A,n_i\right)\right]=0$.

Given the last three results $\left(\frac{d\left[\sum_{i=1}^{G}\pi(A,n_i)\right]}{dA}<0,\ \lim\limits_{A\longrightarrow 0^+}\left[\sum\limits_{i=1}^{G}\pi\left(A,n_i\right)\right]=G,\ \lim\limits_{A\longrightarrow +\infty}\left[\sum\limits_{i=1}^{G}\pi\left(A,n_i\right)\right]=0\right)$, the intermediate value theorem ensures the existence and uniqueness of a value of total dissipation satisfying the equilibrium condition: $\exists! A^* : \sum_{i=1}^{G} \pi(A^*, n_i) = 1$. Such value can be thought as depending on the vector of group sizes $N = (n_1, .., n_G)$: $A^* = A(N)$.

In summary, for any vector of group sizes N there is one and only one level of total effort and vector of winning probabilities satisfying the equilibrium conditions. ■

Proof. Proposition 3.

Proposition states that the uniform distribution $(\bar{N} = (\frac{1}{2}, \frac{1}{2}))$ is the strict global maximum for $G = 2$. Since there are only two groups (1,2), and their sizes (n_1, n_2) must add to unity, we can just re-define their sizes as $n_1 = n$ and $n_2 = 1 - n$. The dissipation function $A(N)$ can be re-defined accordingly $A(n)$. Re-define the group's winning probability $\Pi_i(n) \equiv \pi_i(A(n), n)$. Since winning probabilities also to unity, then $\Pi_1 = \Pi$ and $\Pi_2 = 1 - \Pi$. Re-define the first-order derivative accordingly: $\phi(\pi, A, n) = \Phi(\Pi(n), A(n), n) \equiv \Phi_1$, where . The first-order derivative of this function with respect to n is:

$$\frac{d\Phi(\Pi, A, n)}{dn} = \frac{\partial \Phi_1}{\partial \Pi}\frac{d\Pi}{dn} + \frac{\partial \Phi_1}{\partial A}\frac{dA}{dn} + \frac{\partial \Phi_1}{\partial n} = 0 \tag{7}$$

Explicit the derivative of the winning probability with respect to n :

$$\frac{d\Pi}{dn} = -\frac{\frac{\partial \Phi_1}{\partial A}\frac{dA}{dn} + \frac{\partial \Phi_1}{\partial n}}{\frac{\partial \Phi_1}{\partial \Pi}}.$$

Since population is normalized to unity an infinitesimal change in the size of group 1 (n) directly affects also the size of group 2 $(1 - n)$. Let the first-order derivative for the generic member of group 2 be: $\Phi_2 \equiv \Phi(1 - \Pi, A, 1 - n)$. There will be another direct and indirect effect to count for. However, we know that the sum of winning probabilities must be equal unity before and after the shift. Therefore, the two aggregate changes in winning probabilities must compensate each other: $\sum_{i=1}^{2} \frac{d\Pi_i}{dn} = 0$. Then we can explicit the total derivative of dissipation A with respect to the population parameter $\left(\frac{dA}{dn}\right)$:

$$\frac{dA}{dn} = -\frac{\sum_{i=1}^{2}\left[\frac{\partial\Phi_i/\partial n}{\partial\Phi_i/\partial\Pi}\right]}{\sum_{i=1}^{2}\left[\frac{\partial\Phi_i/\partial A}{\partial\Phi_i/\partial\Pi}\right]}.$$

The two initial first-order derivatives Φ_i are: $\Phi_1 = \frac{n}{A}(1-\Pi)\,w(n) - c'\left(\frac{\Pi A}{n}\right)$ and $\Phi_2 = \frac{1-n}{A}\Pi w(1-n) - c'\left(\frac{(1-\Pi)A}{(1-n)}\right)$. Differentiation of these two expressions and some manipulation provides the following expression, where $\alpha_1 = \alpha\left(\frac{\Pi A}{n}\right)$ and $\alpha_2 = \alpha\left(\frac{(1-\Pi)A}{(1-n)}\right)$:

$$\frac{dA}{dn} = \frac{A}{n(1-n)} \frac{(\alpha_1+\theta_1)(1-n)\left[\Pi\alpha_2 + (1-\Pi)\right] + (\alpha_2+\theta_2)\,n\left[(1-\Pi)\,\alpha_1 + \Pi\right]}{(1-2n)(\alpha_1\alpha_2 - 1)}.$$

It follows that:

$$sign\left\{\frac{dA}{dn}\right\} = sign\left\{(1-2n)\right\},$$

which means that $A(n)$ is increasing in n for $n \in \left(0, \frac{1}{2}\right]$ and decreasing afterwards. Therefore, $A(n)$ attains its maximum at $n = \frac{1}{2}$, which corresponds to the uniform distribution over the two groups. ■

Proof. Proposition 4.

As we restrict our attention to uniform distributions $(n_i = n\ \forall i)$, the maximization problem becomes identical for individuals across all groups. Per capita contributions are identical $(a_i = a_j = a\ \forall i, j)$ and so are winning probabilities $(\pi_i = \pi = n\ \forall i)$. Therefore, equilibrium contributions across individuals. Given the normalization of total population to unity, equilibrium contributions will also equal total dissipation $(a = A)$. Equation (6) reduces to:

$$n(1-n)\,w(n) = c'(A)\,A.$$

Define a new function $f : \mathbb{R}_+ \to \mathbb{R}_+$ such that $f(a) \equiv c'(a)a$. This let us re-write the previous equality as:

$$n(1-n)\,w(n) = f(A).$$

Assumption 1 ensures that the $f(.)$ is strictly increasing: $f'(A) = c''(A)A + c'(A) > 0$. This means that f is invertible and the dissipation-maximizing problem reduces to maximizing the LHS:

$$\begin{aligned} & \max_n \{n(1-n)w(n)\} \\ = & \max_n \left\{n(1-n)\left(\lambda + \frac{1-\lambda}{n}\right)\right\} \\ = & \max_n \{n(1-n)\lambda + (1-n)(1-\lambda)\}, \end{aligned}$$

$$FOC : (1-2n)\lambda - (1-\lambda) \leq 0 \ (= 0 \ if \ n > 0).$$

If the share of publicness of the prize (λ) is equal or smaller than $\frac{1}{2}$, the solution is corner $(n = 0)$. Otherwise the solution is interior and equal to:

$$n = 1 - \frac{1}{2\lambda} \equiv n(\lambda).$$

The number of groups corresponding to these solutions is $G(\lambda) = \frac{1}{n(\lambda)}$, which means $G(\lambda) = +\infty \ \forall \lambda \in \left[0, \frac{1}{2}\right]$ and $G(\lambda) = \frac{2\lambda}{2\lambda - 1}$. In particular, notice that

$$\frac{\partial G(\lambda)}{\partial \lambda} < 0 \quad \forall \lambda \in \left(\frac{1}{2}, 1\right],$$

and $G(1) = 2$. ■

Proof. Proposition 5.

In order to clarify the exposition, we drop the subscripts. The following definition will be used frequently throughout the proof. Define the subjective degree of publicness of the prize (θ) as the ratio between the degree of publicness of the prize (λ) and the benefit from winning the prize (w):

$$\theta = \frac{\lambda}{w} = \frac{\lambda}{\lambda + (1-\lambda)/n}. \quad (8)$$

The following lemma describes properties that will be needed in the proof of proposition 5, 6 and 7. ■

Lemma 8 *Suppose assumption 1 holds. The the function $\pi(.)$ is strictly increasing and twice continuously differentiable.*

Proof. Recall that $\pi(.)$ is implicitly defined by equation (4), which we can re-write in terms of (π, A, n):

$$\frac{n}{A}(1-\pi)\,w(n) = c'\left(\frac{\pi A}{n}\right). \tag{9}$$

Let $\alpha(a)$ denote the the elasticity of the marginal cost of effort $c'(a)$ with respect to effort $a : \alpha(a) = \frac{ac''(a)}{c'(a)}$. Set A fixed and differentiate equation (12) with respect to n to obtain:

$$\pi'(n) = \frac{\pi}{n}\frac{(1-\pi)\left[\alpha(a)+\theta\right]}{(1-\pi)\,\alpha(a)+\pi}. \tag{10}$$

Assumption 1 ensures $\alpha(a) > 0\ \forall a > 0$.Therefore $\pi'(.) > 0\ \forall n > 0$. So part 1 is established. ■

Proof. We return to the main proof.

Consider the G-point uniform distribution $\bar{N}_G = (n, n, ..)$. Call the corresponding level of dissipation $\bar{A}_G$. Set $\bar{A}_G$ fixed and differentiate (13) with respect to n. After some manipulation, obtain the following expression:

$$\pi''(n) = \frac{[\pi'(n)]^2}{\pi(1-\pi)\left[\alpha(a)+\theta\right]\left[(1-\pi)\,\alpha(a)+\pi\right]} \left\{ \begin{array}{c} \left[\theta-(\theta+1)\,\pi\right]\left[(1-\pi)\,\alpha(a)+\pi\right] + \frac{\theta(1-\theta)}{\alpha(a)+\theta}\left[(1-\pi)\,\alpha(a)+\pi\right]^2 + \\ -\left[\alpha(a)+\theta\right]\pi - \frac{\alpha'(a)a}{\alpha(a)+\theta}\left[\theta-(\theta+1)\,\pi\right]^2 \end{array} \right\},$$

where $\theta = \theta(\lambda, n)$.

Define the expression in curly brackets as $\varphi(\lambda, n)$. Clearly, $sign\{\pi''(n)\} = sign\{\varphi(\lambda, n)\}$.

The case of purely private goods corresponds to setting $\lambda = 0$, which means $\theta(0, n) =$

0. By substitution we find:

$$\varphi(0,n)=\left\{-\pi\left[(1-\pi)\alpha(a)+\pi\right]-\alpha(a)\pi-\frac{\alpha'(a)a}{\alpha(a)}\pi^2\right\}. \tag{11}$$

By assumption 2 the expression in brackets is always negative and so $\varphi(0,n)<0$ $\forall n>0$. The winning probability $\pi(.)$ is locally strictly concave in an open neighborhood around the point combination $(\bar{A}_G(\lambda),n)$. Pick any G-point non-uniform distribution $\tilde{N}_G=(\tilde{n}_{G1},..,\tilde{n}_{GG})$ such that the combination $(\bar{A}_G(\lambda),\tilde{n}_{Gi})$ lies in the open neighborhood of $(\bar{A}_G(\lambda),n)$ for every i. By local strict concavity and the equilibrium condition $\sum\limits_{i=1}^{G}\pi(A,n_i)=1$,

$$1=G\pi\left(\bar{A}_G(\lambda),n\right)>\sum_{i=1}^{G}\pi\left(\bar{A}_G(\lambda),\tilde{n}_{Gi}\right).$$

Let $\tilde{A}_G(\lambda)$ be the equilibrium dissipation associated with $\tilde{N}_G$. Recall that $\pi(.)$ is strictly decreasing in A : $\frac{d\pi(A_G(\lambda),\tilde{n}_{Gi})}{dA}<0$ $\left(\text{as well as } \frac{d\left[\sum\limits_{i=1}^{G}\pi(A_G(\lambda),\tilde{n}_{Gi})\right]}{dA}<0\right)$ $\forall i$. This, joint to the previous inequality, implies $\tilde{A}_G(\lambda)<\bar{A}_G(\lambda)$. ■

Lemma 9 *Suppose assumption 1 holds and $\lambda=0$. Then:*

1. $\left(\frac{\pi(0,n)}{n}\right)$ *is strictly decreasing;*
2. *If* $(a,b)>>0$, *then* $\pi(a+b)>\pi(a)+\pi(b)$.

Proof. Using (13) we can derive the derivative of the ratio between winning probability and group size $\left(\frac{\pi}{n}\right)$ with respect to size (n):

$$\frac{\partial\left(\frac{\pi}{n}\right)}{\partial n}=\frac{\pi'(n)}{n}\frac{\theta-(\theta+1)\pi}{(1-\pi)\left[\alpha(a)+\theta\right]}. \tag{12}$$

Equation (14) shows that

$$sign\left\{\frac{\partial\left(\frac{\pi}{n}\right)}{\partial n}\right\}=sign\left\{\theta-(\theta+1)\pi\right\}.$$

In case of pure private goods $\theta = 0$, so $\frac{\partial\left(\frac{\pi}{n}\right)}{\partial n} < 0 \ \forall n, \forall \lambda$. So part 1 is established.

Consider $(a, b) >> 0$. From part 1 we know that $\frac{\pi(a+b)}{a+b} < \frac{\pi(a)}{a}$ and $\frac{\pi(a+b)}{a+b} < \frac{\pi(b)}{b}$. It follows that:

$$\begin{aligned} \pi(a+b) &= \frac{a+b}{a+b}\pi(a+b) = a\frac{\pi(a+b)}{a+b} + b\frac{\pi(a+b)}{a+b} \\ &< a\frac{\pi(a)}{a} + b\frac{\pi(a)}{a} = \pi(a) + \pi(a). \end{aligned}$$

So part 2 is established. ■

Proof. Proposition 6.

Sort groups according the their winning probabilities (π_i). Consider any sub-set M of the G groups. From Lemma 9.2 we know that

$$\pi\left(A, \sum_{i\in M} n_i\right) < \sum_{i\in M} \pi(A, n_i).$$

Add the winning probabilities of all remaining groups $(j \neq M)$, evaluated at the initial level of dissipation A :

$$\pi\left(A, \sum_{i\in M} n_i\right) + \sum_{j\neq M} \pi(A, n_j) < \sum_{i\in M} \pi(A, n_i) + \sum_{j\neq M} \pi(A, n_j) = 1.$$

For the sum of winning probabilities to equal unity also in the final distribution, the level of dissipation must decrease: $A' < A$. Therefore any merge must decrease the level of dissipation (and any split must increase it). ■

Proof. Proposition 7.

Notice that the ratio between group's per capita contribution and average contribution $(\frac{a_i}{A})$ is exactly equal to the ratio between winning probability and group size $\left(\frac{\pi_i}{n_i}\right)$.

Consider the case $G = 2$. Let n be the size of group 1 and $(1-n)$ the size of group 2. Let π be the winning probability of group 1 and $(1-\pi)$ the winning probability of group 2. Consider the ratio between the FOC of two individuals belonging to different groups:

$$\begin{aligned}\frac{c'(a_1)\,a_1}{c'(a_2)\,a_2} &= \frac{w(\lambda,n)}{w(\lambda,1-n)}\\ &= \frac{1-n}{n}\frac{\lambda n+1-\lambda}{\lambda(1-n)+1-\lambda}.\end{aligned}$$

If the RHS is greater then unity, group 1 lobbies more intensively than group 2. If the two groups have equal size $(n=\frac{1}{2})$, the RHS is equal to unity, which means absence of activism. Consider the general case:

$$\frac{1-n}{n}\frac{\lambda n+1-\lambda}{\lambda(1-n)+1-\lambda}>1.$$

After some manipulations, we find that

$$(1-2n)(1-\lambda)>0.$$

The inequality is satisfied for any value of n if the good is purely public $(\lambda=1)$, while it is satisfied for $n<\frac{1}{2}$ for any other type of good $(\lambda<1)$. This means exactly the the bigger group lobbies less intensively than the smaller one.

Consider the case $G\geq 3$ and the special case $\lambda=0$. Sort groups with respect to their size. Recall from Lemma 9.1 that the ratio $(\frac{\pi}{n})$ is decreasing in n, which means that bigger groups lobby less intensively than smaller ones. ■