When the Underdog does not Lead - Endogenous Prize and Leadership in Contests

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Abstract

This paper examines simultaneous versus sequential play in a simple two-player contest framework with a contest success function of the logit type. The timing of moves as well as the value of the prize is assumed to be endogenous. In a preplay stage to the basic contest subgame players decide whether they exert effort as soon, or as late as possible. No matter when exerted, the players' effort influences not only the size of the prize (negative) but also their win probability (positive) as well as the competitor's win probability (negative). It is found that in line with previous literature on contests with an exogenous prize, the subgame perfect equilibrium of the extended game is Pareto dominated by no other sequential or simultaneous play payoff and that, if sequential play emerges in equilibrium, the leader undercommits effort compared to the Nash equilibrium. But, contrary to previous findings, the win probability of the simultaneous move game is in no way crucial for the determination of an endogenous leadership. Moreover, the rent dissipation in the subgame perfect equilibrium of the extended game may not be minimized.

Keywords: Contests, Endogenous timing, First/second-mover advantage

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1 Introduction

This paper contributes to two strands of literature on contests. The first group of papers focusses on the distinction between Nash equilibrium (NE) and Stackelberg equilibria in models of contest with an exogenous prize. The second group of papers is broadly concerned with the impact of endogenous prizes on the NE in models of contest.

We propose a framework for the analysis of strategic behavior in a two-player contest with an endogenous prize based on the endogenous timing model of Hamilton and Slutsky (1990). In a preplay stage to the basic contest subgame players decide whether they exert effort as soon or as late as possible. No matter when exerted, the players' effort influences not only the size of the prize (negative) but also their win probability (positive) as well as the competitor's win probability (negative). Within this model we introduce a taxonomy of endogenous leadership, based on the properties of the players' best response functions as well as on the characteristics of the prize-production technology. We find that, unlike the literature on exogenous prize, the win probability of the simultaneous move game is in no way crucial for the determination of endogenous leadership. Thus, for example, it is possible that in the subgame perfect equilibrium (SPE) of the extended game it is the favourite who will act as a Stackelberg-leader and the underdog who will act as a Stackelberg-follower.[1] Moreover, simultaneous play may emerge in equilibrium. We prove that this finding holds for a general production technology of the prize and a logit contest success function (CSF). Moreover, we show that in line with previous literature on contests with an exogenous prize the subgame perfect equilibrium of the extended game is Pareto dominated by no other sequentiell or simultaneous play payoff.

Until Dixit's seminal paper on strategic behavior all of the literature on contests was primarily concerned with Nash equilibria. Dixit showed that if two unevenly

[1] According to Dixit (1987) the favorite (underdog) is the player whose odds of victory in a two-player contest exceed (fall below) one-half at the Nash equilibrium.

matched players compete over an exogenously given prize in a sequential manner, using a logit or probit form of the CSF, it is the favourite (underdog) who has an incentive to overcommit (undercommit) effort.[2] The reason for this is the underdog's (favourite's) effort being a strategic complement (substitute) for the favourite (underdog), i.e. the underdogs best response function is downward sloping in the NE of the game, the favourite's is upward sloping.[3] This implies directly that the SPE with the underdog being the leader minimizes the effort exerted in the game whereas in the reversed order this will be maximized.[4]

In line with model of endogenous commitment by Hamilton and Slutsky (1990), Baik and Shogren (1992) subsequently extended the basic contest by a preplay stage in which the two players determine their order of moves prior to the actual choice of effort. They showed that the favourite will never and the underdog will always move first. Hence, the unique SPE of the contest-subgame shows commitment by the weaker player, which means that the equilibrium order of moves minimizes the social cost. The reason for this SPE of the extended game is that by acting first the underdog reveals his relative weakness, thereby allowing the favourite to react efficiently.[5]

What has been neglected in the literature on endogenous timing in contests is the fact that prizes are more often endogenous than exogenous. In many cases the contestants themselves can influence the size of the prize either (1) directly or, in cases where the prize is contingent of the players effort, (2) indirectly. In the first case players choose their prize themselves knowing that whatever size of the prize they choose it will directly influence the incentive for effort-spending of the competitor.

[2] The logit form of the CSF gives the probability of winning as a function of the *relative* effort of players (see Tullock (1980), Grossman (2001), etc.). The probit form, on the other hand, determines the relative success by the *differences* among the efforts (see Hirshleifer (1991)).

[3] For the case of an oligopoly, this has been examined by Bulow et al. (1985) and Gal-or (1985).

[4] Similar results can be found in Linster (1993), who also allows for incomplete information with respect to the type of the second mover. Glazer and Hassin (2000) consider an n player sequential contest.

[5] The same result can be found in Leiniger (1993), who assumes asymmetric valuation of the prize.

An example is studied by Konrad (2002) where subsequently to the realization of a project an incumbent decides not only about his investment in the project but also about his effort-spending in a contest where he has to defend his project returns against a challenger. Compared to the case where the incumbent receives his return on investment with a probability smaller than one, in this example a higher investment in the project increases the effort-spending of the challenger, thus causing a moderation of project investment.[6]

An example for the indirect influence of effort-spending on the prize is when the contestants face a trade-off between production and appropriation if effective property rights are absent. In these models of conflict agents are endowed with an unappropriable resource which can be used as an input in a valuable prize (production) or for appropriation of the latter.[7] The strategic situation is that a further unit of effort not only decreases the winning probability of the competitor via the CSF but also decreases the prize available to the agents therefore making it less attractive for the competitor to win the prize.[8]

In order to unite these two strands of literature we provide a framework in which two competitors fight in a model of conflict over an endogenous prize. Each player is endowed with an inalienable resource which he can allocate between productive and appropriative effort. In the metaphor of conflict models the latter effort is

[6]See also Leidy (1994) who argues that a monopolist whose right is contested in a political market will spend lobbying effort and lower his price to defuse reformist opposition. Similar models are presented by Epstein and Nitzan (2003, 2004). Hoffmann (2009) shows in a two-player conflict model that the anticipation of potential appropriation forces agents to engage in trade, since this mutually reduces the gains from appropriation. In Hoffmann and Schmidt (2007) a monopolist invests in copy protection as well as lowers the price of his output (a digital product) in order to make piracy less attractive.

[7]See for example Hirshleifer (1991), Skaperdas (1992) and Skaperdas and Syropoulos (2001).

[8]It is worth noting that dependencies do not have to be negative. Chung (1996), for example, shows that promotional effort increases the market share of a firm as well as the size of the whole market. Thus, effort-spending does not have only a negative externality on the combatant. See also Baye and Hoppe (2003), where the research activity of a firm increases not only the probability of winning a contest but also the expected value of the contest prize. Another example with a positive side effect of contests is Morgan (2000). For the case of a lottery contest where the proceeds will be used to finance a public good, the size of the public good is increased compared to the case of voluntary contribution to the public good.

labeled as *guns*, the former as *butter*. The aggregate income produced by both competitors' butter is a common pool good, and the respective shares are contingent on the players' decision on guns. In this sense our paper is closely related to the work of Hirshleifer (1991), Skaperdas (1992), Grossman (2001), and Skaperdas and Syropoulos (2001). Moreover, we will use a CSF of the logit form and players are allowed to exert effort as soon as or as late as possible. Thus, sequential moves may arise in equilibrium. In this sense our paper is closely related to the work of Dixit (1987), Baik and Shogren (1992), Leiniger (1993), Baik et al. (1999) and Yildirim (2005). The timing game itself matches the *action commitment* model by Hamilton and Slutsky (1990) frequently used in games of endogenous timing.[9] We will examine how the endogeneity of the prize will influence the players' timing decision. In particular, we will provide a taxonomy of endogenous leadership. Hence, in a methodological sense, the paper is close to Kempf and Graziosi (2009) who develop an endogenous timing game where two countries provide public goods with spillovers. Here, a taxonomy is proposed depending on the sign of spillovers among countries and the nature of the interaction between various public goods.

The paper proceeds as follows. Section 2 presents the basic model and explores the nature of strategic substitutes vs. complements in our setting and its' influence on the players' first-mover and second-mover advantage. Furthermore, it describes the equilibrium concepts used in the paper. Section 3 provides the equilibria in the full game and shows when the findings of Baik and Shogren (1992) are no longer valid. Furthermore, we have added two simple examples presented in subchapter 3.3 of this paper.

[9] See for example van Damme and Hurkens (1999, 2004), and Amir and Stepanova (2006).

2 The model

Consider a situation in which each of two players possesses R_i unit of an inalienable primary resource, with $(i = 1, 2)$. This resource can be used to produce one-to-one two kinds of inputs, x_i and y_i, where the latter will be used in the joint production of a single consumption good representing the prize, and the former will be used as an input in the appropriative competition. The production technology $V(R_1 - x_1, R_2 - x_2)$ transforms the two productive inputs into a single consumption good (the prize). This endogenous prize is assumed to be quasi-concave and twice differentiable, and the competitors marginal product is positive and decreasing.

Assumption 1 (Endogenous rent)

$$V_i(\mathbf{x}) \equiv \frac{\partial V(\mathbf{x})}{\partial x_i} < 0, \tag{1a}$$

$$V_{ii}(\mathbf{x}) \equiv \frac{\partial^2 V(\mathbf{x})}{\partial x_i^2} < 0, \tag{1b}$$

$$V_{12}(\mathbf{x}) \equiv \frac{\partial^2 V(\mathbf{x})}{\partial x_1 \partial x_2} \gtreqless 0, \tag{1c}$$

$$V_{11}(\mathbf{x}) V_{22}(\mathbf{x}) \geqslant V_{12}^2(\mathbf{x}) \tag{1d}$$

for $\mathbf{x} = (x_1, x_2)$ and $i = 1, 2$ and $x_i \in [0, R_i]$. The first and second assumptions state that an increase in effort decreases the rent and that this negative effect increases in x_i. Note, that the marginal productivity with respect to x_i might differ for both players, i.e. $V_1(\mathbf{x}) \gtreqless V_2(\mathbf{x})$. The third assumption simply states that we allow for $q-$substitutes and $q-$complements, i.e., we do not restrict the sign of the cross derivatives.[10] The main role of the last assumption is in guaranteeing uniqueness of the equilibrium.

The conflict technology, here defined by a logit CSF, determines for any given value

[10]The terms $q-$substitutes and $q-$complements have been suggested by Hicks (1956). In the contest literature several specifications have been proposed with respect to the rent: Dixit (1987), Baik and Shogren (1992) and Grossman (2001) consider exogenous rents ($V(\mathbf{x}) = K$), Skaperdas and Syropoulos (1998) consider an endogenous rent, with $V_{12}(\mathbf{x}) = 0$, whereas Hirshleifer (1991) and Skaperdas (1992) assume $q-$complements ($V_{12}(\mathbf{x}) > 0$).

of the vector $\mathbf{x}$ each player's probability of winning the prize. For player i, this probability is represented by $p^i(\mathbf{x})$:

$$p^i(\mathbf{x}) = \begin{cases} \frac{f_i(x_i)}{f_i(x_i)+f_j(x_j)}, & \text{if } \mathbf{x} \neq \mathbf{0}, \\ \frac{1}{2}, & \text{if } \mathbf{x} = \mathbf{0}, \end{cases} \tag{2}$$

with $f_i(0) = 0$, $\frac{d\, f_i(x_i)}{d\, x_i} \equiv f_i'(x_i) > 0$ and $\frac{d^2\, f_i(x_i)}{d\, x_i^2} \equiv f_i''(\mathbf{x}) \leq 0$.[11] Thus, we assume that each player's effectivity function ($f_i(x_i)$) is a twice differentiable, increasing and concave function of the effort of player i. For brevity's sake, we introduce the notations $p^1(\mathbf{x}) \equiv p(\mathbf{x})$ so that $p^2(\mathbf{x}) = 1 - p(\mathbf{x})$. Note, that the logit specification involves

$$p_{12}p\,(1-p) - p_1 p_2\,(1-2p) = 0, \tag{3}$$

which corresponds to assumption $A3$ of **?**.

Given the above defined logit form we find that the cross derivative of the marginal win probability is given by

$$p_{12}(\mathbf{x}) \equiv \frac{\partial^2 p\,(\mathbf{x})}{\partial x_1 \partial x_2} = \frac{f_1'\,(x_1)\,f_2'\,(x_2)}{\left[f_1\,(x_1) + f_2\,(x_2)\right]^3}\,(f_1\,(x_1) - f_2\,(x_2))\,. \tag{4}$$

Since the CSF is symmetric we may thus use the following definition:

Definition 1 (Favourite and underdog) *Dixit (1987); Baik and Shogren (1992)* Agent 1 (2) is said to be the favorite (underdog) if $p_{12}(\mathbf{x}) > 0$ in the Nash equilibrium of the game; agent 1 (2) is the underdog (favourite) if $p_{12}(\mathbf{x}) < 0$ in the Nash equilbrium of the game.

The payoff function of agents 1 and 2 are given by $\Pi^1\,(\mathbf{x}) = p(\mathbf{x})\,V(\mathbf{x}) - C(x_1)$, and $\Pi^2\,(\mathbf{x}) = (1 - p(\mathbf{x}))\,V(\mathbf{x}) - C(x_2)$, with $C(x_i) = x_i$.[12] Each agent maximizes his expected payoff which equals the prize that goes to the sole winner, i.e. the

[11] To avoid repetition, we use $i, j = 1, 2$ and $i \neq j$ when it is obvious.

[12] Thus, the payoff function of player i is a strictly concave function of his effort (x_i) and strictly monotonic decreasing function of his opponent's effort (x_j).

aggregate income produced, weighted by the probability that he wins the conflict. Moreover, we have a game of *pure substitutes*, i.e. the sign of the cross derivatives of the payoff function is negative. Hence, the effort of agent i decreases the payoff of agent j, which means that we have negative spillovers with respect to the effort invested:

$$\Pi_2^1(\mathbf{x}) \equiv \frac{\partial \Pi^1(\mathbf{x})}{\partial x_2} = p_2(\mathbf{x})\, V(\mathbf{x}) + p(\mathbf{x})\, V_2(\mathbf{x}) < 0, \tag{5.1}$$

$$\Pi_1^2(\mathbf{x}) \equiv \frac{\partial \Pi_2(\mathbf{x})}{\partial x_1} = -p_1(\mathbf{x}) V(\mathbf{x}) + (1 - p(\mathbf{x})) V_2(\mathbf{x}) < 0. \tag{5.2}$$

2.1 Strategic complements vs. substitutes

Following Bulow et al. (1985), player i's effort is a strategic substitute (SS) to the competitor's effort if the marginal payoff of player j decreases in the effort of player i, and is a strategic complement (SC) if it increases.[13] Hence, given a two player game, either (1) both agents regard their effort as a SS, or (2) as a SC, or (3) effort of player i is a SS to player j, and the effort of player j is a SC to the effort of player i. We will now determine the condition that would allow these four different scenarios to emerge. Therefore, we will determine the change in the marginal payoffs caused by an increase in the strategy of the opponent:

$$\Pi_{12}^1(\mathbf{x}) = p_{12}(\mathbf{x})\, V(\mathbf{x}) + p_1(\mathbf{x}) V_2(\mathbf{x}) + p_2(\mathbf{x})\, V_1(\mathbf{x}) + p(\mathbf{x})\, V_{12}(\mathbf{x}), \tag{6.1}$$

$$\Pi_{12}^2(\mathbf{x}) = -p_{12}(\mathbf{x})\, V(\mathbf{x}) - p_1(\mathbf{x})\, V_2(\mathbf{x}) - p_2(\mathbf{x})\, V_1(\mathbf{x}) + (1 - p(\mathbf{x}))\, V_{12}(\mathbf{x}), \tag{6.2}$$

which involves that the sum of the cross effects on the marginal payoff function equals the cross derivatives of the production function, i.e. $\Pi_{12}^1(\mathbf{x}) + \Pi_{12}^2(\mathbf{x}) = V_{12}(\mathbf{x})$. Given that $V_{12} \gtreqless 0$, we therefore have three different cases. A game of strategic complements ($\Pi_{12}^i \in \mathbb{R}^+$) is only consistent with q-complements ($V_{12}(\mathbf{x}) > 0$) and

[13] Note that due to the properties of the CSF, the players' marginal payoff depend in a non-monotonic way on the competitors effort. Following Dixit (1987) we thus define SS and SC in the neighborhood of the Nash equilibrium (see Definition 2, page 9).

a game of strategic substitutes ($\Pi^i_{12} \in \mathbb{R}^-$) is only consistent with q-substitutes ($V_{12}(\mathbf{x}) < 0$). The mixed case ($\Pi^i_{12} > 0 > \Pi^j_{12}$) is consistent with either q-substitutes or q-complements and it is also the only case that emerges if $V_{12} = 0$.[14] Furthermore, we have to note that the assumption of an exogenous rent ($V(\mathbf{x}) = K$) would yield:

$$\Pi^1_{12}(\mathbf{x}) = -\Pi^2_{12} = p_{12}(\mathbf{x})\, K,$$

i.e., the favourite's (underdog's) effort is a SS (SC) to the underdog's (favourite's) effort, which is in line with the findings of Dixit (1987), Baik and Shogren (1992).

2.2 Guns in the three basic games

First, we consider the three basic games: the simultaneous move game (Γ^N) and the two Stackelberg-scenarios where agent 1 or agent 2 leads (Γ^{S_1} and Γ^{S_2}).[15] In each basic game the agents choose their optimal allocation of their initial resources subject to the conflict technology and the production technology.

The equilibrium of the game $\left(\Gamma^N\right)$ is defined by the following system of maximization programs

$$\begin{cases} x_i^N \equiv \arg\max_{x_i \in [0,R_i]} \Pi^i(\mathbf{x}), & x_j \text{ given} \\ x_j^N \equiv \arg\max_{x_j \in [0,R_j]} \Pi^j(\mathbf{x}), & x_j \text{ given.} \end{cases}$$

The FOCs for player 1 and 2 are therefore

$$p_1(\mathbf{x})V(\mathbf{x}) + p(\mathbf{x})V_1(\mathbf{x}) - 1 = 0, \tag{7.1}$$

$$-p_2(\mathbf{x})V(\mathbf{x}) + (1 - p(\mathbf{x}))\, V_1(\mathbf{x}) - 1 = 0. \tag{7.2}$$

[14]We do not examine the case where $\Pi^1_{12} = \Pi^2_{12} = 0$ in the NE as further restrictions on the third derivatives of the CSF would be required to find out whether there are local commitment incentives in this case, which is beyond the scope of this paper (see Dixit (1999) and Baye and Shin (1999)).

[15]Due to symmetry, we know that the studied games are neither supermodular nor submodular, i.e. $p^i_{ij}(\mathbf{x}) = -p^j_{ij}(\mathbf{x})$.

Applying the Envelop theorem to (7), it is easy to show that

$$\frac{dx_j}{dx_i} = -\frac{\Pi^j_{ij}(\mathbf{x})}{\Pi^j_{jj}(\mathbf{x})} \lesseqqgtr 0 \Leftrightarrow \Pi^i_{ij}(\mathbf{x}) \lesseqqgtr 0,$$

i.e., the slope of agent's i best response function at a point in the strategy space is solely determined by the cross effect on the marginal payoff function which - as was said before - may vary. Therefore, we will use the following definition:

Definition 2 (Strategic substitutes and complements)

If player i's effort is a SS (SC) to his competitor's effort then the best response function of player j is downward (upward) sloping **in the Nash equilibrium** of the game.

The Stackelberg equilibrium is determined by applying backward induction. Thus, in the game where agent i leads (Γ^{S_i}), we first focus on the follower's (F) maximization program which is $x_j^F(x_i) \equiv \arg\max_{x_j \in [0,R_j]} \Pi^j(\mathbf{x})$. This yields

$$\Pi^j_j\left(x_j^F(x_i), x_i\right) = 0. \tag{8}$$

The leader's (L) program is now $x_i^L \equiv \arg\max_{x_i \in [0,R_i]} \Pi^i\left(x_i, x_j^F(x_i)\right)$, which involves[16]

$$\Pi^i_i\left(x_i^L, x_j^F(x_i^L)\right) + \Pi^i_j\left(x_i^L, x_j^F(x_i^L)\right)\frac{dx_j^F(x_i)}{dx_i} = 0. \tag{9}$$

Given the optimizing behavior in the basic games, we are now in the position to rank the level of guns:

Lemma 1

The levels of guns for the Nash and Stackelberg games are such that

(1) if both players regard their effort as a SC ($\Pi^i_{ij}(\mathbf{x}) > 0$): $x_i^N > \max\left\{x_i^F, x_i^L\right\}$ and $x_j^N > x_j^F > x_j^L$,

[16] For the rest of the paper, we pose $x_j^F \equiv x_j^F(x_i^L)$.

(2) if both players regard their effort as a SS ($\Pi^i_{ij}(\mathbf{x}) < 0$): $x_i^L > x_i^N > x_i^F$,

(3) if $\Pi^i_{ij}(\mathbf{x}) > 0 > \Pi^j_{ij}(\mathbf{x})$, player j regards the effort of player i as a SS and player i regards the effort of player j as a SC. In this case $x_i^L > x_i^N > x_i^F$ and $x_j^N > \max\left\{x_j^L, x_j^F\right\}$.

Proof. See Appendix A.2. ■

In our general framework with an endogenous rent, a variation of the level of appropriation activities has two effects: one on the contest success function and one on the rent. In the first case ($\Pi^i_{ij}(\mathbf{x}) > 0$) both players regard their effort as a SC and this is only consistent with $q-$complements (cf. equations 6). Thus, the higher the effort of player j, the lower the (negative) marginal effect of an increase of player i's effort on the endogenous prize. Therefore, both best response functions are *increasing* in the NE of the contest subgame. Consistent with games of plain substitutes and strategic complements the equilibrium levels of guns in both Stackelberg games are lower than the one obtained at the NE ($x_1^N > x_1^L$ and $x_2^N > x_2^L$). Here, the leader, say agent 1, undercommits his effort relative to the one in the NE, which induces the follower, agent 2, to decrease his own effort because of the SC property. In turn, this increases the leader's payoff because of the negative externality induced by effort ($\Pi^i_j(\mathbf{x}) < 0$). However, it may happen that the follower's effort is higher than the leader effort of the same player ($x_i^F > x_i^L$). This comes from the differences in the payoff functions. For a sufficient degree of asymmetry among agents, the interaction effects are much stronger from player 2 to player 1, than vice versa. Then it might be the case that x_1^L is very close to x_1^N and x_2^L is very far from x_2^N as well as x_1^F from x_1^N. This explains the obtained possible rankings.

In the second case both players regard their effort as a SS ($\Pi^i_{ij}(\mathbf{x}) < 0$) which is only consistent with $q-$substitutes (cf. equations 6). Thus, the higher the effort of player j, the higher the (negative) marginal effect of an increase of player i's effort on the endogenous prize and therefore both best response functions are *decreasing* in the NE of the contest subgame. Consistent with games of plain substitutes and strategic substitutes the equilibrium levels of guns in both Stackelberg games are higher than the one obtained at the NE ($x_1^N < x_1^L$ and $x_2^N < x_2^L$). Here, the leader, say agent 2, overcommits effort compared to the NE, which induces the follower, agent 2, to decrease its own effort because of the SS property.

In the third case ($\Pi_{ij}^i(\mathbf{x}) > 0 > \Pi_{ij}^j(\mathbf{x})$) player j, whose effort is regarded by player i as a SC overcommits effort compared to the NE ($x_i^L > x_i^N$), whereas player i, whose effort is regarded by player j as a SS undercommits effort ($x_j^L < x_j^N$). Moreover, both players' effort as a follower falls short compared to the NE ($x_i^F < x_i^N$ and $x_j^F < x_j^N$).

2.3 First-mover and second-mover advantages.

Given these rankings, we can now compare the payoffs in the two Stackelberg games (Γ^{S_1} and Γ^{S_2}), which will give us the opportunity of detecting first-mover and second-mover advantages. Given the findings of lemma 1, we can state that:

Lemma 2

(1) If both players regard their effort as a SC $\left(\Pi_{ij}^i(\mathbf{x}) > 0\right)$, at least one player has a second-mover advantage,

(2) if both players regard their effort as a SS $\left(\Pi_{ij}^i(\mathbf{x}) < 0\right)$, each player has a first-mover advantage,

(3) if $\Pi_{ij}^i(\mathbf{x}) > 0 > \Pi_{ij}^j(\mathbf{x})$, player j who regards the effort of player i as a SS, has a first-mover advantage. Player i who regards the effort of player j as a SC, has a first-mover (resp. second-mover) advantage if $x_j^L > x_j^F$ (resp. $x_j^L < x_j^F$).

Proof. see Appendix A.3. ∎

In the first case ($\left(\Pi_{ij}^i(\mathbf{x}) > 0\right)$) we may have $x_i^N > x_i^F > x_i^L$ for both players, so that there is a second-mover advantage for both players since both prefer to follow. If the leader undercommits effort compared to the NE, both players benefit indirectly from the reduced cutback of the prize. But, additionally the follower benefits from a higher effort compared to the leader, which increases his win probability, compared to the other Stackelberg game. When $x_i^N > x_i^L > x_i^F$ and $x_j^N > x_j^F > x_j^L$, only agent i prefers to follow.

In the second case $\left(\left(\Pi_{ij}^i(\mathbf{x}) < 0\right)\right)$ we have $x_i^L > x_i^N > x_i^F$ for both players. Hence, each agent prefers to lead. If agent i increases his level of guns, the other player has to decrease it since guns are strategic substitutes. This improves the expected payoff of player i twofold by increasing his probability of winning the prize and by increasing the value of the prize itself. The same arguments can be now be replicated for the third case $(\Pi_{ij}^i(\mathbf{x}) > 0 > \Pi_{ij}^j(\mathbf{x}))$.

3 Selecting a leader through a timing game

The issue of endogenous timing is examined according to the concept proposed by Hamilton and Slutsky (1990) in their *extended game with observable delay*. This extended game $\tilde{\Gamma}$ allows players to choose non-cooperatively and simultaneously their effort in a preplay stage either as soon as (*early*) or as late as possible (*late*). Their decision is announced by the players subsequently. In the consecutive *basic game* (Γ^k, with $k = \{N, S_1, S_2\}$) the players choose their effort according to their timing decision to which they are committed. Thus, the *basic game* consists of three different games: Γ^N if both players decide to exert effort at the same time (whether early or late), Γ^{S_1} if player 1 chooses to move early and player 2 chooses to move late, and Γ^{S_2} in the permutation of the players. Thus, if players decide to choose effort at different times, the player who chooses to move late observes the effort exerted by the player who chose to move *early* and acts accordingly.[17] It is worth

[17] Following Hamilton and Slutsky (1990) and Amir and Stepanova (2006), we restrict our attention to the SPE of $\tilde{\Gamma}$.

noting that the order of moves does not affect the payoffs which are conditional only on the players' strategies. Thus, the payoff of player i when $\mathbf{x} = (\bar{x}_i, \bar{x}_j)$ is the same whether player i's effort is taken before player j's effort, after or if taken simultaneously.

The normal form representation of the preplay stage is shown in table 1.[18]

		Player 2	
		early	***late***
Player 1	***early***	$\Pi^1(x_1^N, x_2^N), \Pi^2(x_1^N, x_2^N)$	$\Pi^1(x_1^L, x_2^F), \Pi^2(x_1^L, x_2^F)$
	late	$\Pi^1(x_1^F, x_2^L), \Pi^2(x_1^F, x_2^L)$	$\Pi^1(x_1^N, x_2^N), \Pi^2(x_1^N, x_2^N)$

Table 1: Normal form representation of $\tilde{\Gamma}$

3.1 Solutions to the leadership problem

The solution to this reduced form game is equivalent to characterizing the solution to the leadership problem. There is no leader if both players choose the same action; a leader emerges when they choose complementary roles. We obtain the following proposition:

Proposition 3

(1) If both players regard their effort as a SC $\left(\Pi^i_{ij}(\mathbf{x}) > 0\right)$, the subgame perfect equilibria are the two Stackelberg situations,

(2) if both players regard their effort as a SS $\left(\Pi^i_{ij}(\mathbf{x}) < 0\right)$, the subgame perfect equilibrium is the simultaneous moves game,

(3) if $\Pi^i_{ij}(\mathbf{x}) > 0 > \Pi^j_{ij}(\mathbf{x})$, player j who regards the effort of player j as a SS will act as a Stackelberg-leader. Player i who regards the effort of player i as a SC

[18] We remark that the literature on endogenous timing remains divided about how to qualify the situation where both players choose to lead. Indeed, Dowrick (1986) and more recently van Damme and Hurkens (1999) consider a Stackelberg warfare where both countries apply their action as a leader. In contrast, Hamilton and Slutsky (1990) or Amir and Stepanova (2006) apprehend this situation as the static Nash game. They emphasize that Stackelberg warfare can occur only through error, since the underlying strategy of one player is not consistent with the other player's strategy (Hamilton and Slutsky, 1990, p. 42).

will act as a Stackelberg-follower.

Proof. See APPENDIX A.4. ■

In the first case $\left(\Pi^i_{ij}(\mathbf{x}) > 0\right)$ at least one player has a second-mover advantage (cf. lemma 2.1) and both players prefer their Stackelberg-follower payoff over their simultaneous play payoff. Given the fact that the leader's payoff is always higher than the payoff in the Nash equilibrium, a coordination game results with two pure strategy Nash equilibria, Γ^{S_1} and Γ^{S_2}. In the second case $\left(\Pi^i_{ij}(\mathbf{x}) < 0\right)$ both players have a first-mover advantage (cf. lemma 2.2) and prefer their simultaneous play payoff over their Stackelberg-follower payoff. Thus, both players have the dominant strategy *early* which leads to a simultaneous move game (Γ^N). In the third case ($\Pi^i_{ij}(\mathbf{x}) > 0 > \Pi^j_{ij}(\mathbf{x})$) player j, who has a first-mover advantage (cf. lemma 2.3), prefers his NE payoff over his follower payoff and has therefore a dominant strategy (*early*). Player i, on the other hand, always prefers his follower payoff over his NE payoff, whether or not he has a first-mover advantage. Therefore, given the dominant strategy of his opponent, a rational player i will always choose *late*, so that the unique solution to the timing game is Γ^{S_j}.

From PROPOSITION (3), we may deduce the following COROLLARY.

Corollary 4 *Hamilton and Slutsky (1990)* and rent dissipation

Every SPE of the extended game $\widetilde{\Gamma}$ is Pareto undominated although rent dissipation might be higher than in non-SPE. More precisely

(1) If both players regard their effort as a SC $\left(\Pi^i_{ij}(\mathbf{x}) > 0\right)$, both subgame perfect equilibria (Γ^{S_1} and Γ^{S_2}) Pareto-dominate the NE. Moreover, the levels of guns for the Nash and Stackelberg games are such that $x_i^N + x_j^N > x_i^L + x_j^F$.

(2) If both players regard their effort as a SS $\left(\Pi^i_{ij}(\mathbf{x}) < 0\right)$, the payoffs in the simultaneous move game and the two sequentiell payoffs are not Pareto-rankable. Moreover, the levels of guns for the Nash and Stackelberg games are such that $x_i^N + x_j^N \lesseqgtr x_i^L + x_j^F$.

(3) If $\Pi_{ij}^{i}(\mathbf{x}) > 0 > \Pi_{ij}^{j}(\mathbf{x})$, the subgame perfect equilibrium (Γ^{S_j}) Pareto-dominates Γ^{S_i} as well as Γ^{N}. Moreover, the levels of guns for the Nash and Stackelberg games are such that $x_i^N + x_j^N > x_j^L + x_i^F$.

Proof. Immediate. ■

These findings are based on the following facts: If we observe sequential play in equilibrium, the leader always *undercommits* effort compared to the NE. If we observe simultaneous play in equilibrium, both players' effort is - ceteris paribus - lower than as a Stackelberg leader.

In the first case $\left(\Pi_{ij}^{i}(\mathbf{x}) > 0\right)$ both players' best response functions enter the Pareto superior set if we specify it at the simultaneous NE. Moreover, the leader, in this case, undercommits effort compared to the NE (cf. lemma 1.1) and therefore both Stackelberg scenarios Pareto dominate the NE and the rent dissipation in Γ^{S_1} and Γ^{S_2} fall short compared to Γ^{N}. In the second case $\left(\Pi_{ij}^{i}(\mathbf{x}) < 0\right)$ both players prefer their NE payoff over their follower payoff (cf. proposition 3.2). Thus, neither of the best response functions enter the Pareto-superior set and that is why the SPE of $\widetilde{\Gamma}$ (Γ^{N}) Pareto dominates Γ^{S_1} as well as Γ^{S_2}. Note that the difference in the rent dissipation between the simultaneous move game and the two sequentiell move games is indeterminate in this case. Thus, choosing effort sequentially might lead to social improvement in terms of resources spent in the contest compared to the SPE of the game. In the third case ($\Pi_{ij}^{i}(\mathbf{x}) > 0 > \Pi_{ij}^{j}(\mathbf{x})$) only player i prefers his NE payoff over his follower payoff. Thus, only player i's best response function enters the Pareto superior set. Moreover, player j undercommits effort and since in this case the SPE of $\widetilde{\Gamma}$ is the Stackelberg scenario with player j being the leader, Γ^{S_j} Pareto dominates Γ^{N} as well as Γ^{S_i}.

3.2 When the underdog leads or follows

With Proposition (3) we will now establish when the underdog leads or follows. If we consider an exogenous prize we observe that the favorite regards the effort of the underdog as a SC, while the underdog considers the effort of the favourite as a SS. Thus, applying Proposition (3), we conclude unambiguously that at the SPE the underdog leads while the favorite follows. This result does no longer hold if the prize is endogenous, i.e. if it depends on the appropriative activities of each player. *The underdog does not lead* means that we find in equilibrium

- that the underdog may choose effort simultaneously with his competitor,
- that the favourite may lead.

Thus, using the results of Proposition (3) we get the following taxonomy of endogenous leadership based on the slope of the players' best response functions in the NE of the contest subgame ($\Pi^i_{ij}(\mathbf{x})$) as well as on the characteristics of the prize-production technology ($V_{12}(\mathbf{x})$).

$\Pi^1_{12}(\mathbf{x})$	$V_{12}(\mathbf{x}) > 0$	$V_{12}(\mathbf{x}) < 0$	$V_{12}(\mathbf{x}) = 0$
> 0	Both players may lead if $V_{12}(\mathbf{x}) < \Pi^1_{12}(\mathbf{x})$ 1 follows 2 leads if $V_{12}(\mathbf{x}) > \Pi^1_{12}(\mathbf{x})$	1 follows, 2 leads	1 follows 2 leads
< 0	1 leads, 2 follows	no country leads if $V_{12}(\mathbf{x}) > \Pi^1_{12}(\mathbf{x})$ 1 leads, 2 follows if $V_{12}(\mathbf{x}) < \Pi^1_{12}(\mathbf{x})$	1 leads 2 follows

Table 2: Leadership in the extended game contingent on $sign\left(V_{12}(\mathbf{x})\right)$.

Let us first consider the case where guns are $q-$complements, i.e. $V_{12}(\mathbf{x}) > 0$ and where player 1 regards the effort of player j as a SS ($\Pi^1_{12}(\mathbf{x}) < 0$). Here, we unambiguously have leadership of the player 1. This, as we already established, is independent of the players' win probability. Thus, the leader in the SPE of the extended game might very well be the favourite. In a similar way, we can assume that guns are $q-$substitutes ($V_{12}(\mathbf{x}) < 0$). Let again, $\Pi^1_{12}(\mathbf{x}) < 0$ and $V_{12}(\mathbf{x}) > \Pi^1_{12}(\mathbf{x})$ (from which follows that $\Pi^2_{12}(\mathbf{x}) < 0$). In this case both players want to move early, since becoming a follower leads to the lowest possible payoff. Hence, the SPE of $\tilde{\Gamma}$ is the NE in the contest subgame. This means that both players choose to exert effort at the same time as their opponent, independently of the players' win probability.

3.3 Two examples

In order to give the reader a better understanding of our findings we now introduce two simple examples.

3.3.1 Example 1

Let $V(\mathbf{x}) = ((R_1 - x_1) + (R_2 - 2\,x_2))^{\alpha}$, with $\alpha = \frac{3}{4}$ so that $V_{12}(\mathbf{x}) < 0$. Let $R_1 = R_2 = 10$ and $f_1(x_1) = x_1$ and $f_2(x_2) = 2\,x_2$. In the NE of the contest subgame it is $x_2^N \approx 1.67368$ and $x_1^N \approx 4.73387$, so that player 2 is the underdog ($p_{12} > 0$). Given the leadership of player 1, it is $x_1^L \approx 5.48093$ and $x_2^F \approx 1.64894$, so that the favourite overcommits effort, compared to the NE as explained by Dixit (1987). In

Player 1	Player 2
$\Pi^1\left(x_1^N, x_2^N\right) \approx 2.93447$	$\Pi^2(x_1^N, x_2^N) \approx 2.07498$
$\Pi^1(x_1^L, x_2^F) \approx 2.94849$	$\Pi^2(x_1^L, x_2^F) \approx 1.77411$
$\Pi^1(x_1^F, x_2^L) \approx 2.49145$	$\Pi^2(x_1^F, x_2^L) \approx 2.08981$

Table 3: Payoffs in the three basic games for the 1st example.

the other Stackelberg scenario where player 2 is the leader we get $x_2^L \approx 1.94087$ and $x_1^F \approx 4.62778$, so that the underdog also overcommits effort compared to the NE.

As explained above, this results from the fact that both players regard their effort as a SS. Therefore, no matter whether $p_{12}(\mathbf{x}) \gtreqless 0$ both players increase their effort as a Stackelberg-leader compared to the NE:

$$x_i^L > x_i^N > x_i^F.$$

Given this, it is easy to compute the payoffs in all three games, which are given in table 3, where we can see that both players rakn their payoffs according to

$$\Pi^i(x_i^L, x_j^F) > \Pi^i(x_i^N, x_j^N) > \Pi^i(x_i^F, x_j^L).$$

Given these payoffs it is obvious that both player prefer their leader-payoff over their follower-payoff, i.e. both player have a first-mover advantage. Comparing the payoff in the NE with those in both Stackelberg games leads to the conclusion that being a Stackelberg follower is the worst case scenario for both players. Hence, both players will choose to move *early*, or to put it differently, favourite and underdog will play simultaneously. This can also be seen from figure 1. Here, the thick concave (convex) function represents the best response function of player 2 (player 1), and the concave (convex) black dashed line represents the iso-payoff-curve of player 1 (player 2) in the NE of the contest subgame (Γ^N). The grey surface represents the set of strategy combinations that improve upon the NE of the contest subgame, in the Pareto sense. The dotted line represents the set of strategy combinations that cause $p(\mathbf{x}) = \frac{1}{2}$. Thus, since the NE of the contest subgame lies to the south-east of the dotted line, player 1 is, according to Dixit (1987) the favourite. However, as mentioned earlier, both players regard their effort as a SS so that both have an incentive to increase their effort compared to the level in the NE if they are supposed to make a strategic precommitment. Hence, the subgame perfect equilibrium of the contest subgame with player 1 (player 2) being the leader lies to the east (north) of the NE and outside the Pareto-set.

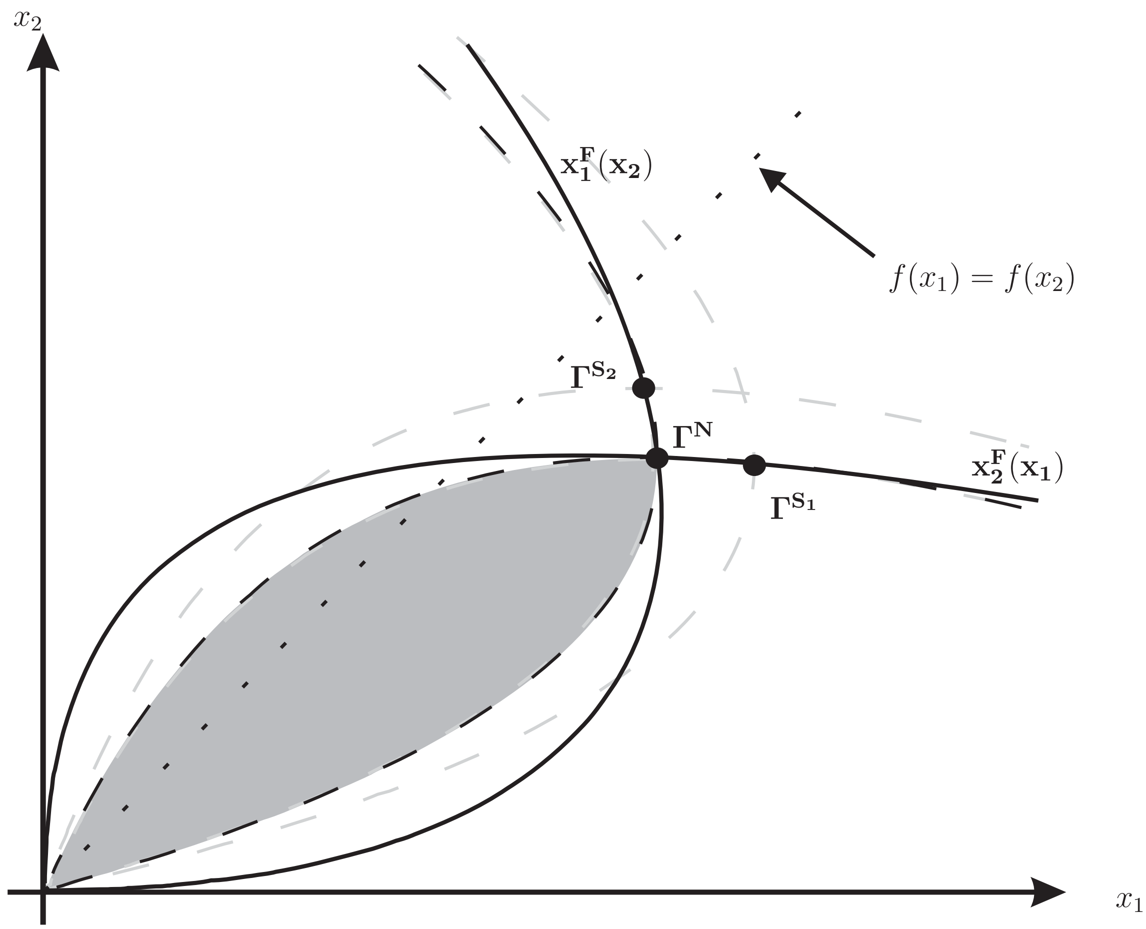

Figure 1: Strategy space in the 1st example

3.3.2 Example 2

Let $V(\mathbf{x}) = (R_1 - x_1)(R_2 - 2x_2)$ so that $V_{12}(\mathbf{x}) > 0$. Again, let $R_1 = R_2 = 10$ and $f_1(x_1) = x_1$ and $f_2(x_2) = 2x_2$. In the NE of the contest subgame it is $x_1^N \approx 2.84146$ and $x_2^N \approx 1.4993$, so that player 1 is the underdog ($p_{12} < 0$). Given the leadership

Player 1	Player 2
$\Pi^1\left(x_1^N, x_2^N\right) \approx 21.5442$	$\Pi^2(x_1^N, x_2^N) \approx 24.2349$
$\Pi^1(x_1^L, x_2^F) \approx 22.4007$	$\Pi^2(x_1^L, x_2^F) \approx 32.7104$
$\Pi^1(x_1^F, x_2^L) \approx 27.6314$	$\Pi^2(x_1^F, x_2^L) \approx 24.6733$

Table 4: Payoffs in the three basic games for the 2nd example.

of player 2, it is $x_2^L \approx 1.1511$ and $x_1^F \approx 2.70439$, so that the underdog undercommits his effort, compared to his effort in the NE of contest subgame as explained by Dixit (1987). In the other Stackelberg scenario, where player 1 is the leader we get $x_1^L \approx 1.9555$ and $x_2^F \approx 1.36804$, so that the favourite also undercommits effort compared to the NE. As explained above this results from the fact that both players regard their effort as a SC. Therefore, no matter whether p_{12} is greater or smaller than zero, both players decrease their effort compared to the NE:

$$x_i^N > x_i^F > x_i^L.$$

Given this, it is easy to compute the payoffs in all three games, given in table 4, where we can see that both players rank their payoffs according to

$$\Pi^i(x_i^F, x_j^L) > \Pi^i(x_i^L, x_j^F) > \Pi^i(x_i^N, x_j^N).$$

Given these payoffs it is obvious that the NE of the contest subgame is Pareto-dominated by both Stackelberg scenarios, where the latter can not be Pareto ranked. Hence, we have a coordination issue, since both Stackelberg scenarios are a SPE of the extended game. This can also be seen from figure 2. Here, the thick black lines represent the best-response-functions of player 1 and 2 respectively, intersecting at Γ^N, where both curves have a positive slope, so that $\Pi_{ij}(\mathbf{x})^i > 0$, for $i, j = 1, 2$. Furthermore, the NE lies to the north-west of the dotted black line, where $f_1(x_1) = f_2(x_2)$, so that player 2 is the favourite. The dashed black curves, again, represent the iso-payoff curves of both players given the NE of the contest subgame. Due to the fact that both players regard their effort as a SC, they both will decrease their effort as a leader, compared to the simultaneous move game. And since both best-response functions are upward sloping, this will also decrease the best response of the competitor. Thus, both Stackelberg-equilibria (Γ^{S_1} and Γ^{S_2}) lie inside the Pareto-set (grey lense), proving that indeed they both Pareto-dominate the NE of the contest subgame, whereas the SPE cannot be Pareto-ranked among each other.

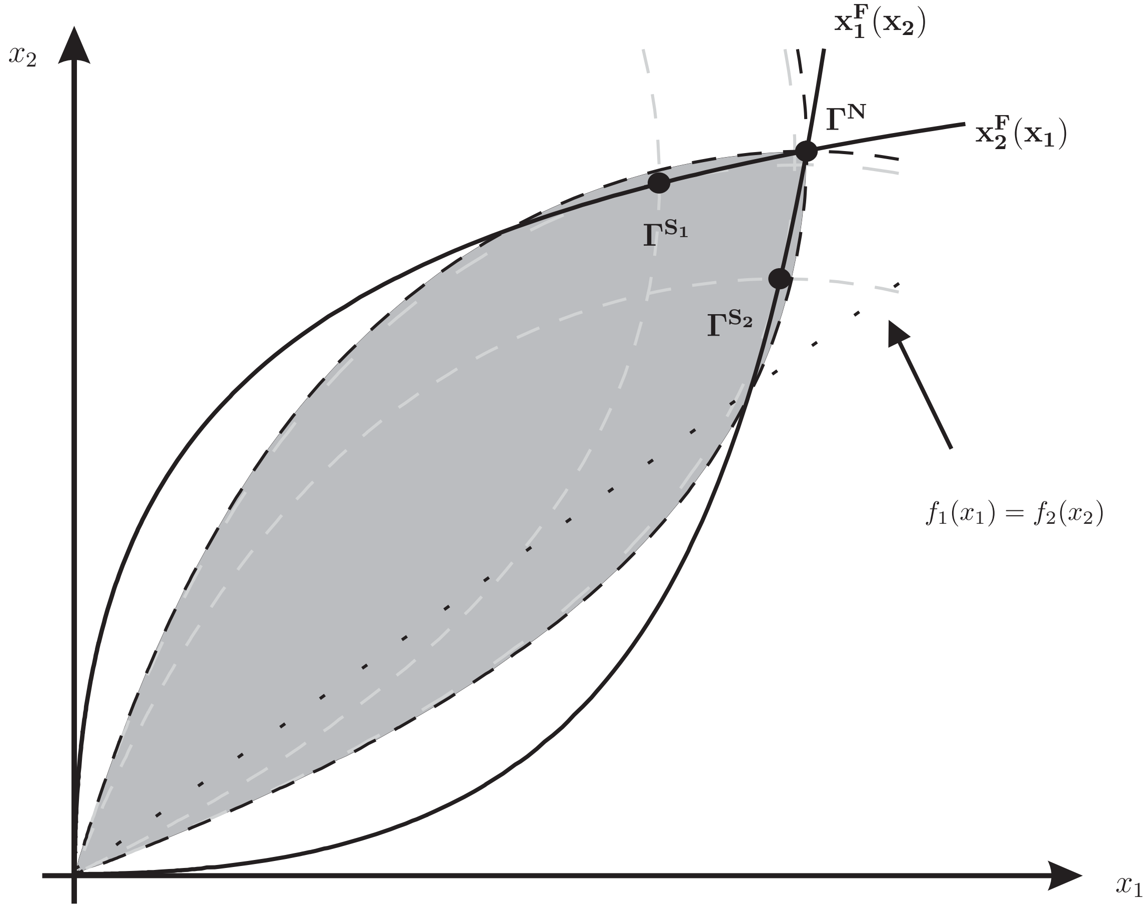

Figure 2: Strategy space in the 2nd example

4 Conclusion

Based on the endogenous timing game by Hamilton and Slutsky (1990) we have provided a framework for the analysis of endogenous leadership in models of contest for the case of an endogenous prize, given a CSF of the logit type. In a preplay stage to a basic contest subgame players decided whether they exert effort as soon or as late as possible and their decision, to which the players are commited, is announced by the players subsequently. Within this model we introduced a taxonomy of endogenous leadership, based on the properties of the players' best response functions as well as on the characteristics of the prize-production technology.

We found that if sequential play emerges in the SPE of the extended game, than the leader always undercommits effort compared to the NE of the contests subgame, which is in line with the findings of Baik and Shogren (1992).

But, we also found (1) that even though the leader undercommits effort, he does not have to be the underdog of the contest subgame, i.e. the one player whose win probability in the NE of the contest subgame falls short compared to the win probability of his competitor, which is contrary to the findings of Dixit (1987) and Baik and Shogren (1992). *Thus, the Stackelberg-leader in the SPE of the extended game may very well be favourite.*

Moreover, (2) we found that sequential play is in no way the only solution to the endogenous leadership game. As in the case of q-substitutes *it might be that favourite and underdog decide to move simultaneously* (when $\Pi^i_{ij} < 0$).

Furthermore, we found that every SPE of the extended game is Pareto-undominated, which is in line with the findings of Baik and Shogren (1992). These findings are based on the following facts: If we observe sequential play in equilibrium, the leader always *undercommits* effort compared to the NE. If we observe simultaneous play in equilibrium, both players' effort is - ceteris paribus - lower than as a Stackelberg leader.

But, we also found that for the case when both players regard their effort as a SS the rent dissipation in a non-SPE (any sequentiell move game) might be lower than in the SPE of the extended game (the simultaneous move game), which is contrary to Baik and Shogren (1992).

A Appendix

A.1 Existence and uniqueness of Nash equilibrium

From the First Order Conditions, we have at the Nash equilibrium:

$$V_1\left(\mathbf{x}^N\right) = \frac{1 - p_1 V\left(\mathbf{x}^N\right)}{p}, \tag{10}$$

and

$$V_2\left(\mathbf{x}^N\right) = \frac{1 + p_2 V\left(\mathbf{x}^N\right)}{1-p}. \tag{11}$$

The Second Order Conditions are given by

$$\Pi^1_{11}(.) = p_{11}V(\mathbf{x}) + 2p_1 V(\mathbf{x}) + pV_{11}(\mathbf{x}), \tag{12}$$

and

$$\Pi^2_{22}(.) = -p_{22}V(\mathbf{x}) - 2p_2 V(\mathbf{x}) + (1-p)V_{22}(\mathbf{x}). \tag{13}$$

From Assumptions ($V_{ii}(.) < 0$) and ($f_i''(.) < 0$), we have $p_{ii} < 0$ and then

$$\Pi^1_{11}(.) < 0 \quad \text{and} \quad \Pi^1_{22}(.) < 0.$$

The concavity of the objective functions insure the existence of a Nash equilibriun. To establish the uniqueness we consider the cross derivatives of the objective functions given in equations 6. Let define $\Omega = p_{12}V(\mathbf{x}) + p_1 V_2(\mathbf{x}) + p_2 V_1(\mathbf{x})$, we have

$$\Pi^1_{12}(.) = \Omega + pV_{12}(\mathbf{x}), \tag{14}$$

and

$$\Pi^2_{12}(.) = -\Omega + (1-p)V_{12}(\mathbf{x}). \tag{15}$$

We consider several cases depending on the sign of $V_{12}(\mathbf{x})$, $\Pi^1_{12}(.)$ and $\Pi^2_{12}(.)$. Two situations are not relevant:

- If $V_{12}(\mathbf{x}) > 0$, $\Pi^1_{12}(.) < 0$ and $\Pi^2_{12}(.) < 0$, the production function still highlights q-complements and efforts are strategic substitutes. It is easy to compute that this case is not possible. Indeed, we have a contradiction:

$$\begin{cases} \Pi^1_{12}(.) < 0 \\ \Pi^2_{12}(.) < 0 \end{cases} \Leftrightarrow \begin{cases} \Omega < -pV_{12}(\mathbf{x}) < 0 \\ \Omega > (1-p)V_{12}(\mathbf{x}) > 0 \end{cases}.$$

- In a similar way, we can eliminate the situation where the production function has q-substitutes and efforts are strategic complements, that is when $V_{12}(\mathbf{x}) < 0$, $\Pi^1_{12}(.) > 0$ and $\Pi^2_{12}(.) > 0$. We obtain

$$\begin{cases} \Pi^1_{12}(.) > 0 \\ \Pi^2_{12}(.) > 0 \end{cases} \Leftrightarrow \begin{cases} \Omega > -pV_{12}(\mathbf{x}) > 0 \\ \Omega < (1-p)V_{12}(\mathbf{x}) < 0 \end{cases}.$$

- We now consider the mixed cases, that is when efforts are strategic complements for one player, while they are strategic substitutes for the other. For instance, if $V_{12}(\mathbf{x}) > 0$, $\Pi^1_{12}(.) > 0 > \Pi^2_{12}(.)$, we have

$$\begin{cases} \Pi^1_{12}(.) > 0 \\ \Pi^2_{12}(.) < 0 \end{cases} \Leftrightarrow \begin{cases} \Omega > -pV_{12}(\mathbf{x}) \\ \Omega > (1-p)V_{12}(\mathbf{x}) \end{cases} \Rightarrow \Omega > 0. \qquad (16)$$

  The reaction function of player 1 is increasing in the effort of her opponent while the reaction function of player 2 is decreasing. To establish the uniqueness, we will assume that there are at least two Nash equilibriums denoted by $\mathbf{x}^N \equiv \left(x^N_1, x^N_2\right)$ and $\mathbf{y}^N \equiv \left(y^N_1, y^N_2\right)$. If the slopes of the reaction functions at the Nash equilibrium (x_1^*, x_2^*) are given by (16), that is

$$R_1'\left(x_2^N\right) = -\frac{\Pi_{12}^1(.)}{\Pi_{11}^1(.)} > 0 \quad \text{and} \quad R_2'\left(x_1^N\right) = -\frac{\Pi_{12}^2(.)}{\Pi_{11}^2(.)} < 0,$$

these at the following Nash equilibrium $\mathbf{y}^N$ cannot be identical (since the objective function are continuous) and then we have

$$R_1'\left(y_2^N\right) = -\frac{\Pi_{12}^1(.)}{\Pi_{11}^1(.)} < 0 \quad \text{and} \quad R_2'\left(y_1^N\right) = -\frac{\Pi_{12}^2(.)}{\Pi_{11}^2(.)} > 0.$$

Implementing (10) and (11) into the expressions of $\Pi_{12}^1(.)$ and $\Pi_{12}^2(.)$ yields at $\mathbf{x}^N$

$$\begin{aligned}\Pi_{12}^1\left(\mathbf{x}^N\right) &= \frac{1}{p(1-p)}\left[p_{12}(1-p)p - p_2p_1(1-2p)\right]V\left(\mathbf{x}^N\right) + \frac{p_1}{1-p} + \frac{p_2}{p} + pV_{12}\left(\mathbf{x}^N\right) \\ &= \frac{p_1}{1-p} + \frac{p_2}{p} + pV_{12}\left(\mathbf{x}^N\right),\end{aligned}$$

since the logit form of the CSF involves (3). Similarly, we deduce that

$$\Pi_{12}^2\left(\mathbf{x}^N\right) = -\frac{p_1}{1-p} - \frac{p_2}{p} + (1-p)V_{12}\left(\mathbf{x}^N\right).$$

From (16) we can establish that

$$-\frac{p_1}{1-p} - \frac{p_2}{p} + (1-p)V_{12}\left(\mathbf{x}^N\right) < \frac{p_1}{1-p} + \frac{p_2}{p} + pV_{12}\left(\mathbf{x}^N\right),$$

or equivalently

$$V_{12}\left(\mathbf{x}^N\right) < \frac{2}{1-2p}\frac{pp_1 + (1-p)p_2}{(1-p)p}, \tag{17}$$

and

$$V_{12}\left(\mathbf{y}^N\right) > \frac{2}{1-2p}\frac{pp_1 + (1-p)p_2}{(1-p)p}. \tag{18}$$

From $p_{12}p(1-p) - p_1p_2(1-2p) = 0$, we deduce that

$$\frac{2}{1-2p}\frac{pp_1 + (1-p)p_2}{(1-p)p} = \frac{2}{(1-2p)^2}p_{12}\frac{p}{p_2},$$

which is negative if player 1 is the favorite ($p_{12} > 0$) or positive otherwise. Thus the inequalities (17) and (18) involve a contradiction.

- A similar reasoning applies if we are in presence of q-substitutes ($V_{12}(\mathbf{x}) < 0$).

- We consider the two last situations where the production function has q-complements (q-substitutes) and efforts are strategic complements (substitutes). We adopt the contraction aproach. For $V_{12}(\mathbf{x}) > 0$, $\Pi^1_{12}(.) > 0$ and $\Pi^2_{12}(.) > 0$, we have

$$\begin{cases} \Pi^1_{12}(.) > 0 \\ \Pi^2_{12}(.) > 0 \end{cases} \Leftrightarrow \begin{cases} \Omega > -pV_{12}(\mathbf{x}) \\ \Omega < (1-p)V_{12}(\mathbf{x}) \end{cases}.$$

We have

$$\begin{aligned} \left|\Pi^1_{12}(.)\right|\left|\Pi^2_{12}(.)\right| &= \Pi^1_{12}(.)\,\Pi^2_{12}(.) \\ &= (\Omega + pV_{12}(\mathbf{x}))(-\Omega + (1-p)V_{12}(\mathbf{x})) \\ &< (\Omega + V_{12}(\mathbf{x}))(-\Omega + V_{12}(\mathbf{x})) = (V_{12}(\mathbf{x}))^2 - \Omega^2 \\ &< (V_{12}(\mathbf{x}))^2. \end{aligned}$$

From (12) and (13), we know that under assumption ($f''_i(.) < 0$)

$$\Pi^1_{11}(.) < pV_{11}(\mathbf{x}) < V_{11}(\mathbf{x}),$$

and

$$\Pi^2_{22}(.) < (1-p)V_{22}(\mathbf{x}) < V_{22}(\mathbf{x}).$$

Thus, assumption (1d) yields

$$\left|\frac{\Pi_{12}^{1}\left(.\right)}{\Pi_{11}^{1}\left(.\right)}\frac{\Pi_{12}^{2}\left(.\right)}{\Pi_{22}^{1}\left(.\right)}\right| < \frac{V_{12}^{2}}{V_{11}V_{22}} \leqslant 1.$$

A.2 Proof of Lemma 1 (Comparison of the levels of guns).

By definitions of the Stackelberg and the Nash equilibria, we have

$$\begin{aligned} &\Pi^{i}\left(x_{i}^{L}, x_{j}^{F}\left(x_{i}^{L}\right)\right) \geqslant \Pi^{i}\left(x_{i}, x_{j}^{F}\left(x_{i}\right)\right) \\ &\geqslant \Pi^{i}\left(x_{i}^{N}, x_{j}^{F}\left(x_{i}^{N}\right)\right) \geqslant \Pi^{i}\left(x_{i}^{N}, x_{j}^{N}\right) \end{aligned} \tag{19}$$

The leader of the Stackelberg game always has a utility level superior or equal to the utility level obtained at the Nash equilibrium.

Moreover, we may establish from the FOCs at the Nash and Stackelberg equilibria that:

$$\Pi_{i}^{i}\left(x_{i}^{N}, x_{j}^{N}\right) = \Pi_{i}^{i}\left(x_{i}^{F}, x_{j}^{L}\right) = 0 \tag{20}$$

We distinguish several cases depending on the signs of Π_{12}^{i} for each player. Remark that $\Pi_{ii}^{i} < 0$ in order that the SOCs are respected.

If $\Pi_{ij}^{i}(\mathbf{x}) > 0$, expression (20) then yields:

$$\begin{aligned} &x_{i}^{N} < x_{i}^{F} \Leftrightarrow x_{j}^{N} < x_{j}^{L} \\ &x_{i}^{N} > x_{i}^{F} \Leftrightarrow x_{j}^{N} > x_{j}^{L} \end{aligned} \tag{21}$$

If $\Pi_{ij}^{i}(\mathbf{x}) < 0$, we have

$$\begin{aligned} &x_{i}^{N} < x_{i}^{F} \Leftrightarrow x_{j}^{N} > x_{j}^{L} \\ &x_{i}^{N} > x_{i}^{F} \Leftrightarrow x_{j}^{N} < x_{j}^{L} \end{aligned} \tag{22}$$

Since $\Pi^i_j(\mathbf{x}) < 0$ the definition of the Nash equilibrium induces

$$\Pi^i\left(x_i^N, x_j^N\right) \geqslant \Pi^i\left(x_i, x_j^N\right) \geqslant \Pi^i\left(x_i^L, x_j^N\right)$$

The inequality $x_j^N < x_j^F$ then involves

$$\Pi^i\left(x_i^N, x_j^N\right) \geqslant \Pi^i\left(x_i^L, x_j^N\right) > \Pi^i\left(x_i^L, x_j^F\right),$$

which contradicts the relation (**??**). We deduce that

$$\Pi^i_j(\mathbf{x}) < 0 \Leftrightarrow x_j^N > x_j^F. \tag{23}$$

Combining our preceding results (21), (22) and (23), we obtain

$$\Pi^i_{ij}(\mathbf{x}) > 0 \Leftrightarrow \begin{cases} x_i^N > x_i^F \\ x_i^N > x_i^L \end{cases} \tag{24}$$

and

$$\Pi^i_{ij}(\mathbf{x}) < 0 \Leftrightarrow x_i^L > x_i^N > x_i^F \tag{25}$$

We now consider the different cases.

1. $\forall i \in \{1,2\}, \quad \Pi^i_{ij}(\mathbf{x}) > 0$: the guns' levels are strategic complements for both players. Since by definition $p(.) > 0$, $V(.) > 0$, $p_2(.) < 0$, $V_2(.) < 0$ and $sign\left\{\frac{dx_j^F(x_i)}{dx_i}\right\} = sign\left\{\Pi^i_{ij}(\mathbf{x})\right\}$, we have:

$$\left[p_2\left(x_i, x_j^F(x_i)\right) V\left(x_i, x_j^F(x_i)\right) + p\left(x_i, x_j^F(x_i)\right) V_2\left(x_i, x_j^F(x_i)\right)\right] \frac{dx_j^F(x_i)}{dx_i} < 0 \tag{26}$$

   Combining the FOCs for the three basic games and the sign of (26) yields

$$\forall i \in \{1,2\}, \quad \Pi^i_i\left(x_i^L, x_j^F\right) > \Pi^i_i\left(x_i^F, x_j^L\right) = \Pi^i_i\left(x_i^N, x_j^N\right) \tag{27}$$

From (27), we have

$$x_i^L > x_i^F \Longrightarrow x_j^F > x_j^L \tag{28}$$

From (24) and (28), we have two following rankings:

$$\begin{cases} x_i^N > x_i^F > x_i^L \\ x_j^N > x_j^F > x_j^L \end{cases} \quad \text{or} \quad \begin{cases} x_i^N > x_i^L > x_i^F \\ x_j^N > x_j^F > x_j^L \end{cases} \tag{29}$$

2. $\forall i \in \{1,2\}$ and $i \neq j$, $\quad \Pi_{ij}^i(\mathbf{x}) < 0$: the guns' levels are strategic substitutes for both players. From (24), we immediately have the following ranking:

$$\begin{cases} x_i^L > x_i^N > x_i^F \\ x_j^L > x_j^N > x_j^F \end{cases} \tag{30}$$

3. $\forall i \in \{1,2\}$, $\quad \Pi_{ij}^i(\mathbf{x}) > 0 > \Pi_{ij}^j(\mathbf{x})$: the guns' levels are strategic complements for country i and strategic substitutes in country j. We then have

$$\begin{aligned} \left[p_2\left(x_i, x_j^F(x_i)\right)V\left(x_i, x_j^F(x_i)\right) + p\left(x_i, x_j^F(x_i)\right)V_2\left(x_i, x_j^F(x_i)\right)\right]\frac{dx_j^F(x_i)}{dx_i} &> 0 \\ \left[-p_2\left(x_i^F(x_j), x_j\right)V\left(x_i^F(x_j), x_j\right) + \left(1 - p\left(x_i^F(x_j), x_j,\right)\right)V_2\left(x_i^F(x_j), x_j\right)\right]\frac{dx_i^F(x_j)}{dx_j} &< 0 \end{aligned}$$

which induce

$$\begin{aligned} \Pi_i^i\left(x_i^L, x_j^F\right) &< \Pi_i^i\left(x_i^F, x_j^L\right) = \Pi_i^i\left(x_i^N, x_j^N\right) \\ \Pi_j^j\left(x_j^L, x_i^F\right) &> \Pi_j^j\left(x_j^F, x_i^L\right) = \Pi_j^j\left(x_j^N, x_i^N\right) \end{aligned} \tag{31}$$

From (24) and (25), we always have $x_{i,j}^N > x_{i,j}^F$. From the equalities in (31), we deduce that:

$$x_i^F < x_i^N \Leftrightarrow x_j^L < x_j^N$$
$$x_j^F < x_j^N \Leftrightarrow x_i^L > x_i^N$$

and from the inequalities in (31)

$$x_j^F < x_j^L \Longrightarrow x_i^L > x_i^F$$
$$x_j^F > x_j^L \Longrightarrow x_i^L > x_i^F$$

There are two possible rankings:

$$\left\{ \begin{array}{l} x_i^L > x_i^N > x_i^F \\ x_j^N > x_j^L > x_j^F \end{array} \right. \quad \text{or} \quad \left\{ \begin{array}{l} x_i^L > x_i^N > x_i^F \\ x_j^N > x_j^F > x_j^L \end{array} \right. . \tag{32}$$

□

A.3 Proof of Lemma 2 (First-mover and second-mover advantages).

We consider the three situations:

1. For $\forall i \in \{1,2\}$, $\Pi_{ij}^i(\mathbf{x}) > 0$. The rankings of guns' levels are given by (29). Using the definition of the first and second-mover advantage, we deduce that when $x_j^F > x_j^L$,

$$\Pi^i\left(x_i^F, x_j^L\right) \geqslant \Pi^i\left(x_i^L, x_j^L\right) > \Pi^i\left(x_i^L, x_j^F\right),$$

   where the first inequality results from the definition of the follower's maximization program and the second from the fact that $x_j^L < x_j^F$ and $\Pi_j^i(\mathbf{x}) < 0$. Since at least one country experiments $x_i^F > x_i^L$ in the rankings given in (29), at least one country has a second-mover advantage.

2. For $\forall i \in \{1,2\}$, $\Pi_{ij}^i(\mathbf{x}) < 0$. The rankings are given by (30). In a similar way as in the preceding case, we establish that

$$\begin{aligned} \Pi^i\left(x_i^L, x_j^F\right) &\geqslant \Pi^i\left(x_i^N, x_j^N\right) \\ &> \Pi^i\left(x_i^F, x_j^N\right) \\ &> \Pi^i\left(x_i^F, x_j^L\right) \end{aligned}$$

where the first inequality results from (??), the second from the definition of the Nash maximization program, and the third from the fact that $x_j^N < x_j^L$ and $\Pi_j^i(\mathbf{x}) < 0$. Each player then has a first-mover advantage.

3. $\Pi_{ij}^i(\mathbf{x}) > 0 > \Pi_{ij}^j(\mathbf{x})$. We have the rankings (32). Since $x_i^L > x_i^N$ always holds, we establish that

$$\begin{aligned} \Pi^j\left(x_j^L, x_i^F\right) &\geqslant \Pi^j\left(x_j^N, x_i^N\right) \\ &> \Pi^j\left(x_j^F, x_i^N\right) \\ &> \Pi^j\left(x_j^F, x_i^L\right), \end{aligned}$$

which means that player j has a first-mover advantage. It may happen that country i also benefits from a first-mover advantage too, that is when the ranking $x_j^L > x_j^F$ obtains. If $x_j^L < x_j^F$, then it benefits from a second-mover advantage. □

A.4 Proof of Proposition 3 (Subgame Perfect Equilibria).

From (??) we always have: $\Pi^i\left(x_i^L, x_j^F\right) > \Pi^i\left(x_i^N, x_j^N\right)$, $\forall i \in \{1,2\}$. In order to determine the SPE, we only have to compare the utility levels when the country follows and when it plays simultaneously ($\Pi^i\left(x_i^F, x_j^L\right) \lesseqgtr \Pi^i\left(x_i^N, x_j^N\right)$). We consider the preceding situations:

1. $\forall i \in \{1,2\}, \quad \Pi_{ij}^i(\mathbf{x}) > 0$: the rankings (29) yield

$$\begin{aligned} \Pi^i\left(x_i^F, x_j^L\right) &\geqslant \Pi^i\left(x_i^N, x_j^L\right) \\ &> \Pi^i\left(x_i^N, x_j^N\right), \end{aligned}$$

where the first inequality results from the definition of the Follower's maximization program, and the second from the fact that $x_j^L < x_j^N$ and $\Pi_j^i(\mathbf{x}) < 0$. The SPEs are then the two Stackelberg situations.

2. $\forall i \in \{1,2\}, \Pi^i_{ij}(\mathbf{x}) < 0$: the ranking (30) yields

$$\Pi^i\left(x_i^N, x_j^N\right) \geqslant \Pi^i\left(x_i^F, x_j^N\right) > \Pi^i\left(x_i^F, x_j^L\right),$$

since $x_j^N < x_j^L$ and $\Pi^i_{ij}(\mathbf{x}) < 0$. We deduce that the SPE corresponds to the static Nash situation.

3. If $\Pi^i_{ij}(\mathbf{x}) > 0 > \Pi^j_{ij}(\mathbf{x})$: the rankings (32) yield

$$\Pi^i\left(x_i^F, x_j^L\right) \geqslant \Pi^i\left(x_i^N, x_j^L\right) > \Pi^i\left(x_i^N, x_j^N\right),$$

since $x_j^L < x_j^N$ and $\Pi^i_j(\mathbf{x}) < 0$. We have also

$$\Pi_j\left(x_j^N, x_i^N\right) \geqslant \Pi_j\left(x_j^F, x_i^N\right) > \Pi_j\left(x_j^F, x_i^L\right),$$

since $x_i^L > x_i^N$ and $\Pi^i_j(\mathbf{x}) < 0$. Player j, for who guns are strategic substitutes, has a strict dominant strategy (*Leads*). Player i always prefers to follow than to play the simultaneous game ($\Pi^i\left(x_i^F, x_j^L\right) > \Pi^i\left(x_i^N, x_j^N\right)$). The SPE corresponds to the situation where player j leads and player i follows.$\square$

References

Amir, Rabah and Anna Stepanova, "Second-mover Advantage and Price Leadership in Bertrand Duopoly," *Games and Economic Behavior*, 2006, *55*, 1–20.

Baik, Kyung Hwan and Jason F. Shogren, "Strategic behavior in Contests: Comment," *The American Economic Review*, March 1992, *82* (1), 359–362.

_, Todd L. Cherry, Stephan Kroll, and Jason F. Shogren, "Endogenous Timing in a Gaming Tournament," *Theory and Decision*, 1999, *47* (1-21).

Baye, Michael R. and Heidrun Hoppe, "The Strategic Equivalence of Rent-seeking, Innovation, and patent-race games," *Games and Economic Behavior*, 2003, *44* (2), 217–226.

Baye, Michale R. and Onsong Shin, "Strategic Behavior in Contests: Comment," *American Economic Review*, 1999, *89* (3), 691–93.

Bulow, J. I., J. D. Geanakoplos, and P. D. Klemperer, "Multimarket Oligopoly: Strategic Substitutes and Complements," *Journal of Political Economy*, 1985, *93* (3), 488–511.

Chung, Tai-Yeon, "Rent-Seeking Contest when the prize increases with aggregate efforts," *Public Choice*, 1996, *87*, 55–66.

Dixit, Avinash, "Strategic Behaviour in Contests," *The American Economic Review*, December 1987, *77* (5), 891–98.

_ , "Strategic Behavior in Contests: Reply," *American Economic Review*, 1999, *89*, 694.

Dowrick, Steve, "von Stackelberg and Cournot Duopoly: Choosing Roles," *The RAND Journal of Economics*, 1986, *17* (2), 251–60.

Epstein, Gil S. and Shmuel Nitzan, "Political culture and monopoly price determination," *Social Choice and Welfare*, 2003, *21*, 1–19.

— and — , "Strategic restraint in contests," *European Economic Review*, 2004, *48*, 201–210.

Gal-or, Esther, "First Mover and Second Mover Advantage," *International Economic Review*, 1985, *26* (3), 649–53.

Glazer, Amihai and Refael Hassin, "Sequential Rent Seeking," *Public Choice*, 2000, *102*, 219–28.

Grossman, Hershel I., "The Creation of Effective Property Rights," *American Economic Review*, 2001, *91* (2), 347–352.

Hamilton, Jonathan H. and Steven M. Slutsky, "Endogenous Timing in Oligoply Games: Stackelberg or Cournot Equilibria," *Games and Economic Behavior*, 1990, *2*, 29–46.

Hicks, John R., *A Revision of Demand Theory*, Oxford University Press, 1956.

Hirshleifer, Jack, "The Technology of Conflict as an Economic Activity," *The American Economic Review, Papers and Proceedings of the Hundred and Third Annual Meeting of the American Economic Association*, May 1991, *81* (2), 130–4.

Hoffmann, Magnus, "Enforcement of property rights in a barter economy," *Social Choice and Welfare*, 2009. forthcoming.

— **and Fredrik Schmidt**, "Piracy of Digital Products: A Contest Theoretical Approach," MPRA Working Paper 3289, 2007.

Kempf, Hubert and Gregoire Rota Graziosi, "Leadership in Public Good Provision: A Timing Game Perspective," *Journal of Public Economic Theory*, 2009. forthcoming.

Konrad, Kai A., "Investment in the Absence of Property Rights - the Role of incumbency advantages," *European Economic Review*, 2002, *46* (8), 1521–1537.

Leidy, Michael P., "Rent dissipation through self-regulation: The social cost of monopoly under threat of reform," *Public Choice*, July 1994, *80*, 105–28.

Leiniger, Wolfgang, "More efficient rent-seeking - A Muenchhausen solution," *Public Choice*, 1993, *75*, 43–62.

Linster, Bruce G., "Stackelberg Rent-Seeking," *Public Choice*, 1993, *77*, 307–21.

Morgan, John, "Financing Public Goods by Means of Lotteries," *Review of Economic Studies*, 2000, *67* (4), 761–784.

Skaperdas, Stergios, "Cooperation, Conflict, and the Power in the Absence of Property Rights," *The American Economic Review*, September 1992, *82* (4), 720–39.

_ and Constantinos Syropoulos, "Complementarity in contests," *European Journal of Political Economy*, November 1998, *14* (4), 667–684.

_ and _ , "Guns, Butter, and Openness: On the Relationship Between Security and Trade," *American Economic Review, Papers and Proceedings*, 2001, *91* (2), 353–7.

Tullock, Gordon, "Efficient Rent Seeking," in J. Buchanan, R. Tollison, and G. Tullock, eds., *Towards a theory of the Rent–Seeking Society*, A and M University Press, 1980, pp. 97–112.

van Damme, Eric and Sjaak Hurkens, "Endogenous Stackelberg Leadership," *Games and Economic Behavior*, 1999, *28*, 105–29.

_ and _ , "Endogenous Price Leadership," *Games and Economic Behavior*, 2004, *47*, 404–20.

Yildirim, Huseyin, "Contests with Multiple Rounds," *Games and Economic Behavior*, 2005, *51*, 213–27.