ADVANCED MATHEMATICS IN ECONOMICS
PROBLEM SET 6

Due by Oct 12 (before the tutorial)

Problem 1 - Convexity, Concavity and Similar Concepts (7 Points)

Prove the following statements:

(a) If $f : \mathbb{R}^n_{\geq 0} \to \mathbb{R}$ is concave then f is quasiconcave.
NOTE: The same claim holds true for f convex and quasiconvex.

(b) If $f : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is strictly monotone increasing then f is strictly quasiconvex and strictly quasiconcave.

(c) If $f : \mathbb{R}^n_{\geq 0} \to \mathbb{R}$ is quasiconcave and $g : \mathbb{R} \to \mathbb{R}$ is monotone increasing then $g \circ f$ is quasiconcave.

(d) Show that there are quasiconcave functions that are not continuous.

Problem 2 - Cobb-Douglas Utilities and Quasiconvexity and -concavity (8 Points)

Let $u : \mathbb{R}^2_{>0} \to \mathbb{R}$ with $u(x_1, x_2) := kx_1^{\alpha} x_2^{1-\alpha}$, $\alpha \in (0;1)$. Then u is called a Cobb-Douglas utility function.

(a) Show that u is quasiconcave.
HINT: Show that $\ln(u)$ is concave on $\mathbb{R}^2$ by showing that the Hessian of u is negative semidefinite. Then apply results from Problem 1 to argue that u must be quasiconcave.

(b) Solve the following utiliy maximization problem of a consumer household:

$$\max_{x_1, x_2} u(x_1, x_2) \text{ s.t. } p_1 x_1 + p_2 x_2 = w$$

(c) What is the optimal utility level in Problem 2 (b) with respect to p and w? Let us denote this optimal utility level as a function $v : \mathbb{R}^3_{>0} \to \mathbb{R}$, $(p, w) \to$

$v(p, w)$.

Show that $v(p, w)$ is quasiconvex.

HINT: You may use the fact that the if the lower contour set $\{(p, w) : v(p, w) \leq \overline{v}\}$ is convex, this already implies that v is quasiconvex.