

Coasian Dynamics in Sequential Search*

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Abstract

Consumer-tracking technology offers new tools for price discrimination in digital markets. We examine the impact of sellers using this technology to adjust prices according to a buyer’s prior search length in a competitive search market where buyers differ in patience. We find “Coasian equilibria” wherein sellers reduce prices for buyers with longer search lengths which in turn requires them to reduce prices for buyers with shorter search lengths. In commonly studied environments, Coasian equilibria not only yield higher welfare for every buyer than all uniform-pricing equilibria, but are also the only equilibria when some mass of buyers are arbitrarily patient.

Keywords: sequential search, price discrimination, Coase conjecture

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1 Introduction

The shift toward a digital economy has given sellers a greater ability to target buyers with personalized prices and a richer set of consumer data for doing so. One important but largely unexplored dimension for price discrimination online is to adjust prices according to a buyer’s online behaviour, such as which website the buyer comes from or how long the buyer has been searching for the good.

Several aspects of online markets enable this type of price discrimination. For one, sellers can track a buyer’s past shopping experience more easily online than offline by using tools like cookies and software development kits (SDKs) (Zuiderveen Borgesius and Poort, 2017; Morrison, 2020).¹ At the same time, steady methodological and computational advances have expanded the capacity for sellers to analyze big datasets.² Moreover, data brokers collect data about consumers from a variety of sources and sell the data to interested firms (Ramirez et al., 2014).

In this paper, we ask whether sellers have an incentive to perform price discrimination based on the length of a buyer’s search history. And if so, does this form of price discrimination adversely affect the welfare of buyers? By investigating these questions, we contribute to the broader inquiry exploring online privacy and the effects of large-scale data collection in digital markets.³ We examine these questions in a competitive market in which buyers sequentially search sellers for a low price. Buyers are distinguished by their *search types* which measure their patience or willingness to continue searching; for example, reflecting their search costs or discount factors.

We find that there are equilibria in which sellers engage in price discrimination based on search duration and that this can actually improve the welfare of all buyers. Our results are driven by two countervailing effects. As the first effect, by cutting the price for buyers who have searched longer, sellers are able to split up buyer

¹Cookies are strips of code stored on a device’s browser that record information about a person’s interactions with a website (first-party cookies) or across websites (third-party cookies). SDKs are pieces of ready-made code that app developers can use to include some common functionalities to their apps without coding these themselves. An SDK’s provider can track people across apps.

²Bourreau and de Streel (2018), Ipsos et al. (2018), Ennis and Lam (2021).

³See Fuller (2019) for a recent summary of the literature.

types and target the least patient buyers with higher prices, potentially allowing sellers to extract more overall surplus. The second and countervailing effect comes from the fact that the promise of lower future prices makes continuing search more desirable. As a result, sellers must also charge lower prices to buyers with shorter search lengths so that they make a purchase and do not continue their search. When considering the net effect, the strategic complementarity inherent to the pricing decisions in many settings causes the second effect to dominate and each buyer's welfare increases.

Our result is reminiscent of the celebrated conjecture posed by Ronald Coase (Coase, 1972) and proven by Gul et al. (1986) regarding a durable good monopolist that sells over time to buyers who vary in their valuation for the good. The conjecture posits that the monopolist essentially competes with its future self: It is compelled to lower current prices because of the pressure it will face at later points in time to charge even lower prices when only the buyers who value the good the least remain in the market. Similarly, sellers in our equilibrium with price discrimination are compelled to lower current prices because of the pressure they face to charge even lower prices when only the buyers who are the least likely to stop searching remain in the market. In both settings, sellers would prefer commitment to uniform pricing, but the changing composition of buyers in the market leads to declining prices.

Drawing on this analogy, we call equilibria with a decreasing sequence of prices “Coasian equilibria”. In a Coasian equilibrium, buyers with the highest search types (i.e., most impatient) buy after searching once, buyers with somewhat lower search types buy after searching twice, and so on. We show that many commonly studied settings feature strategic complementarity, wherein reducing the price charged for one more search leads a seller to reduce its current price. When strategic complementarity holds, we show that only Coasian equilibria exist in addition to uniform-pricing equilibria (Propositions 2 and 3) and that all buyers benefit in a Coasian equilibrium as compared to all uniform-pricing equilibria (Proposition 6). Not only is a Coasian equilibrium beneficial to buyers, but only Coasian equilibria exists if the search type distribution includes *shoppers* (à la Stahl (1989)) who are buyers that enjoy the act of searching for prices (Proposition 5).

We motivate the model with a consumer-search example, but our setup is a

general sequential search setting. In particular, the search type is a general ordering of buyers such that continuing to search is less costly for a buyer with a lower search type. The search type embeds the most commonly used ways of measuring cost of search: additive search costs and discounting. Also the instantaneous utility that a buyer gets from transacting immediately is more general than the usual quasilinear utility assumed in consumer search. Our setup also accommodates, for example, log and other concave utility functions. Thus, the model's applications include not only consumer search, but also searching for jobs and financial assets. In these settings, the search length can be inferred from the CV of a job-seeker if the search is for a job and from past transaction records if the search is for a financial asset.

A rich literature has emerged identifying settings where the Coase conjecture holds and where it fails.⁴ In this vein, our finding sharply contrasts the result of [Board and Pycia \(2014\)](#) which demonstrates the failure of the Coase conjecture in a search model where buyers' tastes vary independently across sellers. The reason behind this discrepancy is that we allow buyers to differ in their propensity to search, i.e., search types, and so equilibria in our model are able to witness deteriorating market conditions from one period to the next. In contrast, by maintaining both homogeneity in the level of patience across buyers and independence in match values across sellers, the setup in [Board and Pycia \(2014\)](#) introduces stability in sellers' pricing decisions. Our baseline model captures the novel effect of buyer heterogeneity on price dynamics in the simplest possible, yet general, framework, leading to Coasian dynamics in a competitive environment.

Our paper joins an active literature that investigates the various ways price discrimination can emerge in a market with search frictions.⁵ In an early contribution, [Salop \(1977\)](#) shows how a monopolist can perform second-degree price discrimination by setting distinct price distributions at different store fronts, inducing the highest search cost buyers to visit the store with a higher expected price, but less price dispersion. This literature has so far mostly ignored price discrimination based on search duration. An exception is by [Preuss \(2023\)](#) which demonstrates that prices

⁴See [Nava and Schiraldi \(2019\)](#) for a discussion of the literature and a general framework for understanding Coasian dynamics for a durable good monopolist.

⁵See, for example, [Fabra and Reguant \(2020\)](#), [Ronayne and Taylor \(2022\)](#), and [Mauring \(2023\)](#).

increase in search length if some buyers learn, in addition to prices, about whether they have a match with the sellers' products and that learning is the more likely the longer a buyer searches. In a sequential search setting, [Bergemann et al. \(2021\)](#) develop a tight upper bound on the equilibrium price distribution when sellers use information about a buyer's search cost to perform price discrimination. In our approach, sellers draw inferences about a buyer's search type by observing their prior search length, but do not have direct information about their search types. A literature also considers price discrimination based on whether a visitor is a first-time or returning customer.⁶

We also relate to the broader recent literature exploring the effect of personalized pricing that comes from greater access to consumer data in digital markets without search frictions. [Rhodes and Zhou \(2022\)](#) study personalized pricing in a general oligopoly framework with differentiated products. They generalize the classic results in [Thisse and Vives \(1988\)](#) and find that when there is full market coverage (all buyers make a purchase under uniform pricing), then personalized pricing harms sellers and benefits buyers, but the reverse conclusion can hold when market coverage is low. [Albrecht \(2020\)](#) demonstrates that, under Bertrand competition, when consumer data in the form of buyer preferences is public information, price discrimination can increase competition and benefit buyers. However, if buyer preferences are only privately known by a single seller, then price discrimination harms buyers. [Elliott et al. \(2021\)](#) takes an information design approach to characterize the optimal information structure for competing sellers and also shows that sufficient information can benefit buyers through increased competition.

Our results also contribute to the study of ordered search. Because sellers can adjust prices according a buyer's search length, our setup resembles a model where search proceeds according to an exogenously given order as in [Arbatskaya \(2007\)](#). Our paper shows how consumer-tracking technology can effectively produce the exogenous search order, expanding the applicability of the results of this literature. More importantly, we provide broader equilibrium existence results and characterize the conditions under which exogenously ordered search does and does not improve

⁶See, for example, [Chen \(1997\)](#), [Villas-Boas \(1999\)](#), [Fudenberg and Tirole \(2000\)](#), [Taylor \(2003\)](#), [Armstrong and Zhou \(2016\)](#), and [Groh and Preuss \(2022\)](#).

each buyer’s payoff relative to random search (without price discrimination).

Our model of price discrimination based on search duration can alternatively be interpreted as introducing dynamic pricing into a sequential search model. From this perspective, we also contribute to the relatively small literature studying truly dynamic search markets, i.e., search markets where market conditions change over time.⁷

We describe the model in Section 2. We tackle equilibrium existence in our model and describe the equilibria in Section 3. The welfare analysis of discriminatory pricing and Coasian dynamics is in Section 4. Section 5 concludes. The proofs missing from the main text are in the appendix.

2 Model

A unit mass of sellers each produce an identical good at a constant marginal cost (normalized to zero) and a unit mass of buyers each wish to purchase at most one unit of the good. Buyers search sequentially and randomly for a low price and have free recall.⁸ When a buyer visits a seller, the seller first detects how long the buyer has been engaged in search and then offers a price. That is, rather than committing to charge all buyers a single uniform price, a seller is able to adjust its price based on how long the buyer has been searching. Our main interpretation is that sellers have access to a tracking technology (e.g., via cookies or SDKs) or buy information from a data intermediary and can thus infer a buyer’s shopping history. Let p_n^i denote the price that seller i charges a buyer when it observes that the buyer has visited $n - 1$ previous sellers.⁹

Each buyer receives her first price quote without delay and the payoff for each additional price quote depends on her *search type*, capturing her degree of impatience. Search types t are independently and identically distributed according to a

⁷Some papers in this literature focus on cyclical equilibria (Janssen and Karamychev, 2002; Muring, 2020) while others study the sale of goods with an expiration date (Garcia, 2022; Garcia et al., 2022).

⁸All results are maintained under costly recall.

⁹Alternatively, in a model where all buyers enter the market in the same period, we could adopt a “temporal” interpretation so that p_n^i is the price seller i charges in time period n .

distribution F with support $[0, \bar{t}]$ (possibly unbounded) and with a density f that is differentiable on $(0, \bar{t})$. The utility a buyer of type t obtains from buying a good at price p after searching n sellers is given by the continuously differentiable utility function $U_n(p, t)$ which is strictly decreasing in the price.¹⁰ The utility from purchasing the good from the first seller that is searched is the same for all search types, $u(p) = U_1(p, t)$ and satisfies $u(\bar{p}) = 0$ for some $\bar{p} > 0$. Buyers are free to exit the market at any time and receive an outside option of zero.

We model impatience by supposing that utility satisfies the following *single crossing condition* for all $n, k \in \mathbb{N}$ and prices in $[0, \bar{p}]$:

$$U_n(p, t) - U_{n+k}(p', t) \geq 0 \implies U_n(p, t') - U_{n+k}(p', t') > 0, \text{ for all } t < t'. \quad (1)$$

The single crossing condition orders search types so that buyers with type t' are less patient than buyers with type t as long as $t < t'$. We refer to buyers with the smallest search type $t = 0$ as *perfectly patient* and assume that their utility function $U_n(p, 0)$ is constant in the search length n .

As is standard in the search literature, we solve for symmetric perfect Bayesian equilibria where buyers hold passive beliefs. Passive beliefs mean that if buyers observe a price deviation by a seller, they assume all other sellers continue to follow the equilibrium strategy thereafter.

2.1 Examples

The following are the two forms of search types that are most commonly used in the sequential search literature.

Example 1 (Additively separable search costs). Within the consumer search literature, the classic approach to capture the idea that search requires costly time and effort is to assume that a buyer incurs search costs $C(t, n)$ to obtain n price quotes, which is increasing in both arguments, satisfies $C(0, n) = C(t, 1) = 0$ for all t and

¹⁰Note that buyers exhibit dynamic consistency. At each point along the search path, a buyer of type t evaluates $U_n(p, t)$ to be the utility from buying after n searches at price p . In Appendix B, we show that all of our main results continue to hold in the presence of dynamic inconsistency.

n , and with utility expressed as

$$U_n(p, t) = u(p) - C(t, n).$$

Single crossing is satisfied whenever the difference $C(t, n) - C(t, n+k)$ is strictly decreasing in the search type. For example, the condition holds if $C(t, n) = c((n-1)t)$ and c is a strictly increasing and convex function. Usually, the literature assumes search costs to be linear in the search length, i.e., $C(t, n) = (n-1) \cdot t$.

Example 2 (Discounting). Within the labor literature, the most common way to model costly search is through discounting. In this setup, there is a discounting function $D(t, n)$ that is strictly decreasing in both arguments when $t > 0$, satisfies $D(0, n) = D(t, 1) = 1$ for all t and n , and utility is written as

$$U_n(p, t) = D(t, n) \cdot u(p).$$

Single crossing is satisfied whenever the ratio $D(t, n)/D(t, n+k)$ is strictly increasing in the search type for all $n, k \in \mathbb{N}$. For example, the discounting function can take an exponential form $D(t, n) = \delta(t)^{n-1}$ with $\delta(t)$ strictly decreasing and $\delta(0) = 1$. The ratio $D(t, n)/D(t, n+k) = \delta(t)^{-k}$ is increasing in t for all $k \in \mathbb{N}$, and so the single crossing condition is satisfied. Similarly, single crossing is satisfied for other discount functions such as $D(t, n) = \frac{1}{1+t \cdot (n-1)}$ as the ratio $D(t, n)/D(t, n+k) = \frac{1+t \cdot (n-1+k)}{1+t \cdot (n-1)}$ is also increasing in the search type for all $n, k \in \mathbb{N}$.

2.2 Logconcavity and Strategic Complementarity

The structure of equilibria in the economy turns out to depend on a buyer's incentive to search one seller more at each point along the search path. To identify the marginal buyer type that is indifferent between buying after n searches and continuing once more, let $\mathbf{P}_n \subset \mathbb{R}_+^2$ be the set of price pairs (p, p') that satisfy $U_n(p, t) = U_{n+1}(p', t)$ for some type $t \in [0, \bar{t}]$ and let $\tau_n : \mathbf{P}_n \rightarrow [0, \bar{t}]$ denote the marginal search type, satisfying $U_n(p, \tau_n(p, p')) = U_{n+1}(p', \tau_n(p, p'))$. Given that utility is constant in the search length for perfectly patient types and smoothly varies in the price and search type, \mathbf{P}_n has a nonempty interior. The marginal type $\tau_n(p, p')$ varies smoothly in the prices, is strictly increasing in p , and strictly decreasing in p' (see Lemma

A.1).

Our analysis focuses on settings that satisfy natural regularity conditions. Our first condition provides that sellers' profit is strictly concave in its price when considering the buyer's utility from searching one step more.¹¹

Definition 1. The game features **logconcavity** if the function $\frac{\partial}{\partial p} \log(1 - F(\tau_n(p, p')))$ is strictly decreasing in p on \mathbf{P}_n for all $n \in \mathbb{N}$.

Logconcavity serves the usual purpose of ensuring that a seller's first order conditions are sufficient to characterize its best replies.

Our second condition provides that increasing the price charged when searching one step more yields an increase in the marginal "local" profit from increasing the current price.

Definition 2. The game features **strategic complements** if the function $\frac{\partial}{\partial p} \log(1 - F(\tau_n(p, p')))$ is strictly increasing in p' on \mathbf{P}_n for all $n \in \mathbb{N}$.

Roughly speaking, strategic complementarity provides that the prices that a seller optimally sets move in the same direction: if the price p_{n+1} increases, so does the optimal price p_n .

There are simple sufficient conditions which ensure that the game is logconcave and features strategic complements. By differentiating $\log(1 - F(\tau_n(p, p')))$ twice in p and taking the cross partial derivative in p and p' , we obtain, respectively:¹²

$$\frac{d^2}{dt^2} \log(1 - F(\tau_n)) \cdot \left(\frac{\partial \tau_n}{\partial p} \right)^2 + \frac{d}{dt} \log(1 - F(\tau_n)) \cdot \frac{\partial^2 \tau_n}{\partial p^2} \quad (2)$$

$$\frac{d^2}{dt^2} \log(1 - F(\tau_n)) \cdot \left(\frac{\partial \tau_n}{\partial p} \right) \cdot \left(\frac{\partial \tau_n}{\partial p'} \right) + \frac{d}{dt} \log(1 - F(\tau_n)) \cdot \frac{\partial^2 \tau_n}{\partial p' \partial p} \quad (3)$$

Logconcavity requires (2) to be negative and strategic complements requires (3) to be positive. By inspecting the above expressions, we see that for both conditions to be satisfied, it is sufficient for one minus the distribution $(1 - F(t))$

¹¹For our main equilibrium existence result in Proposition 3, logconcavity can be weakened to only requiring $1 - F(\tau_n(p, p'))$ to be strictly -1 -concave in p . See Zhou (2020) for a discussion.

¹²Note, we write $\frac{d}{dt} \log(1 - F(\tau_n)) = \frac{d}{dt} \log(1 - F(t))|_{t=\tau_n}$ and suppress the arguments for $\tau_n(p, p')$.

to be strictly logconcave, $\tau_n(p, p')$ convex in p , and the cross partial derivative of τ_n nonpositive. Using these conditions, we can identify when logconcavity and strategic complementarity are satisfied in our running examples.

Example 1–Continued. Consider the environment with additively separable search costs. When utility is quasilinear ($u(p) = \bar{p} - p$), then $\frac{\partial^2 \tau_n}{\partial p^2} = \frac{\partial^2 \tau_n}{\partial p \partial p'} = 0$ and so both conditions are satisfied as long as $1 - F(t)$ is strictly logconcave. If search costs are linear, then the marginal buyer type is $\tau_n(p, p') = u(p') - u(p)$. The marginal type $\tau_n(p, p')$ is convex in p if and only if u is concave in p . The cross partial derivative is zero. Thus, both conditions are satisfied in the linear search cost environment when $1 - F(t)$ is strictly logconcave and $u(p)$ is concave.

Example 2–Continued. We can characterize the setting with discounting by examining the ratio $r_n(t) \equiv \frac{D(t, n+1)}{D(t, n)}$. Logconcavity and strategic complementarity both hold whenever the utility index $u(p)$ is logconcave, $r_n(t)$ is concave, and $1 - F(t)$ is strictly logconcave. Instances where $r_n(t)$ is concave include when buyers perform exponential discounting with $\delta(t) = \frac{1}{1+t}$ and when they use the discount function $D(t, n) = \frac{1}{1+t \cdot (n-1)}$.

2.3 Responsive Demand

The structure of equilibria also depends on demand being sufficiently responsive to prices in a neighborhood in which the most patient types make a purchase. Formally, define $\theta_n(p) \equiv \lim_{\tilde{p} \rightarrow p^+} \frac{\partial}{\partial \tilde{p}} \tau_n(\tilde{p}, p)$ for $p \in [0, \bar{p})$ to capture the marginal change in the buyer type willing to make a purchase when increasing the current price at a point where both the current and subsequent price are the same.

Definition 3. The game features **responsive demand** if $\theta_n(p)$ is positive and finite for all $p \in [0, \bar{p})$ and $\lim_{p \rightarrow \bar{p}} [\theta_n(p) \cdot p \cdot f(0)] > 1$ for all $n \in \mathbb{N}$.

Intuitively, demand responsiveness is related to the price elasticity of demand at a point where prices are identical from one search length to the next. When demand is responsive, it means that there is a high enough price p such that, if all sellers charge it for a search length of $n+1$, then demand is sufficiently elastic so that a seller i does not want to charge a price higher than p for a search length of n , and thus sets

$p_n^i = p$ as well. Technically, having responsive demand provides that equilibrium prices are interior maximizers to sellers' objectives and, importantly, allows for the existence of nontrivial uniform pricing equilibria to serve as a benchmark for comparing our discriminatory pricing equilibria. In the additively separable search cost framework, we have $\theta_n(p) = -\frac{u'(p)}{c'(0)}$. Instances in which demand is responsive in this setting include when search costs are linear and the utility is quasilinear ($u(p) = \bar{p} - p$) or logarithmic ($u(p) = \log(\bar{p} - p + 1)$) with $\bar{p} \cdot f(0) > 1$. In the discounting framework, we have $\theta_n(p) = \frac{u'(p)}{u(p)r'_n(0)}$. When utility is quasilinear or logarithmic, $\theta_n(p)$ diverges to infinity as $p \rightarrow \bar{p}$ and so the responsive demand condition is satisfied if $f(0) > 0$ and $r'_n(0) < 0$.

3 Equilibrium Characterization

3.1 Uniform Pricing Benchmark

To understand the effect of discriminatory pricing, it is useful to first revisit the classic uniform-pricing benchmark and recall how search frictions shift market power toward sellers in a competitive market (Diamond, 1971; Stahl, 1996). Uniform pricing can be embedded into our model by requiring each seller i 's prices to be constant in the search duration: $p_n^i = p_{n+k}^i$ for all $n, k \in \mathbb{N}$.

In this uniform-pricing setup, consider a seller's problem when it must select a constant price \tilde{p} when all other sellers charge the constant price $p \leq \tilde{p}$. In a symmetric uniform pricing equilibrium, it is clear that buyers will only ever receive their first price quotation, buy the good, and conclude their search. By charging \tilde{p} , a seller makes a sale to a buyer with type t as long as buying the good is preferable to extending the search to another seller, $u(\tilde{p}) \geq U_2(p, t)$ and to exiting the market $u(\tilde{p}) \geq 0$. Notice that it can never be optimal to charge a strictly lower price than the other sellers since a marginal decrease in the price results in the same number of sales but at a lower revenue per unit. So we can consider prices $\tilde{p} \geq p$. Supposing that the price \tilde{p} is close enough to p so that some type prefers to buy from the seller than to search further, the single crossing condition provides that all buyers with search types $t \geq \tau_1(\tilde{p}, p)$ will buy from the seller. Therefore, we can compute

the probability of a sale and write the seller's objective as: select \tilde{p} to maximize $\tilde{p}(1 - F(\tau_1(\tilde{p}, p))) \cdot \mathbf{1}_{\{\bar{p} \geq \tilde{p}\}}(\tilde{p})$.

If the other sellers charge the monopoly price $p = \bar{p}$, then a seller's best reply is to do the same due the fact that charging a higher price leads almost all visitors to exit the market. Therefore, a monopoly pricing equilibrium always exists.

To identify any other symmetric equilibria, suppose that other sellers charge less than the monopoly price $p < \bar{p}$. When the game satisfies the logconcavity condition, then the seller's profit is strictly concave in its price \tilde{p} between p and \bar{p} . Differentiating the seller's profit, the seller's optimum is the smallest price in $[p, \bar{p}]$ that satisfies

$$1 - F(\tau_1(\tilde{p}, p)) - \tilde{p} \cdot f(\tau_1(\tilde{p}, p)) \cdot \frac{\partial \tau_1(\tilde{p}, p)}{\partial \tilde{p}} \leq 0. \quad (4)$$

For it to be unprofitable to charge a higher price than the others it must be that the above first order condition is nonpositive when $\tilde{p} = p$, which occurs if and only if $p \cdot f(0) \cdot \theta_1(p) \geq 1$. Intuitively, there needs to be enough buyers who are very patient ($f(0)$ must be high enough) and demand must be sufficiently responsive to prices around the equilibrium price ($\theta_1(p)$ must be high enough). Therefore, for all prices p satisfying $\frac{1}{f(0)\theta_1(p)} \leq p \leq \bar{p}$, there exists a uniform pricing equilibrium. We summarize the benchmark with the following proposition.

Proposition 1 (Stahl (1996)). *If the game is logconcave, then a strategy profile is a symmetric equilibrium under uniform pricing if and only if all sellers charge a price p satisfying either $\frac{1}{f(0)\theta_1(p)} \leq p < \bar{p}$ or $p = \bar{p}$.*

Under uniform pricing, search frictions grant considerable market power to sellers, even permitting equilibria in which they extract the entire market surplus. Notice that when the responsive demand condition is satisfied, then there are also nontrivial uniform pricing equilibria featuring prices below \bar{p} so that buyers are able to receive a positive surplus.

3.2 Discriminatory Pricing

We turn to the discriminatory pricing problem in which sellers have the ability to adjust the price they charge based on the prior length of a buyer's search. As our

first point, observe that the uniform pricing strategies described in Proposition 1 remain equilibria when sellers have the ability to perform price discrimination. To see why, notice that if buyers expect all sellers to play one of these uniform pricing strategies and if all other sellers do so, then a seller only receives buyers who visit it for their first price quote. But then it remains a best reply for the seller to play the same uniform pricing strategy. Note that this conclusion will no longer hold when we introduce shoppers into the market (see Section 3.3).

We now demonstrate how allowing sellers to engage in discriminatory pricing broadens the set of equilibria to include those in which all buyers obtain higher surplus. We make the following definitions.

Definition 4. We say that an equilibrium is a **discriminatory equilibrium** if it is not a uniform-pricing equilibrium. That is, along the equilibrium path in a discriminatory equilibrium, a positive fraction of buyers pay a different price than others.

Definition 5. We say that an equilibrium is a **Coasian equilibrium** if it is a discriminatory equilibrium where the prices charged along the equilibrium path strictly decrease in the buyers' active search length.

Proposition 2. *The only discriminatory equilibria that exist are Coasian.*

Proof. Toward a contradiction, suppose that an equilibrium exists in which prices weakly increase in the search length for some search lengths, i.e., that $p_m \geq p_l$ for some $m > l$ and a positive fraction of buyers search m sellers or beyond. We argue that a seller wants to deviate from these prices. Continuing searching exactly until the m th seller cannot be optimal for any imperfectly patient buyer because the buyer could have paid a lower price by stopping earlier and also engaged in less of the costly search. Thus, it must be that almost all buyers that search at least m sellers continue beyond the m th seller. In particular, there must be a $k > m$ such that $p_k < p_m$ and a positive fraction of buyers make a purchase at the k th search. It must be that $p_k > 0$; otherwise, a seller would profitably deviate by increasing the k th price to a small positive amount $p'_k > 0$ which secures positive demand and therefore positive profit. But this also implies that a seller has a profitable deviation by reducing the m th price to $p'_m = p_k$ as this secures positive profit. \square

Note that this result does not depend on the assumptions that we introduced in Sections 2.2 and 2.3.

Now we turn to the question of whether discriminatory and therefore Coasian equilibria exist. To this end, let us construct a symmetric equilibrium in which each seller i charges $p^i = (p_1, p_2, \dots)$ and a positive fraction of buyers purchase the good on the n th search for at least two values of $n \in \mathbb{N}$. Let $\bar{n} \in \mathbb{N} \cup \{\infty\}$ denote the supremum of the equilibrium search lengths.

In a Coasian equilibrium, if a buyer of type t prefers to buy the good on the n th search rather than to continue and receive any additional price quotes, then it must be that $U_n(p_n, t) \geq U_{n+k}(p_{n+k}, t)$ for all $k \in \mathbb{N}$. From the single crossing condition, it follows that a buyer who is more impatient $t' > t$ must share the same preference for halting search no later than the n th step. By the argument for Proposition 2, we also know that in any symmetric Coasian equilibrium, sellers obtain strictly positive demand from buyers of each search length $n \leq \bar{n}$. We can therefore partition the set $(0, \bar{t}) = \cup_{n=1}^{\bar{n}} [t_n, t_{n-1})$ so that a buyer with search type $t \in [t_n, t_{n-1})$ buys after n searches. Buyers with search type $t = t_n$ are indifferent between buying on the n th search and continuing once more, implying $U_n(p_n, t_n) = U_{n+1}(p_{n+1}, t_n)$.

When determining the price to charge in the n th search, \tilde{p}_n , a seller knows that the most impatient buyers have already made a purchase and exited the market. The distribution of search types among buyers who remain in the market is $F(t)/F(t_{n-1})$ for all $t \leq t_{n-1}$ and zero otherwise. Raising the price \tilde{p}_n has the effect of raising the marginal buyer type who is willing to make a purchase. In a neighborhood of the equilibrium price, the marginal type's outside option is to continue searching once more and make a purchase. Thus, near the equilibrium price, the seller makes a sale to all visitors for whom $U_n(\tilde{p}_n, t) \geq U_{n+1}(p_{n+1}, t)$.¹³ From the single crossing condition, a sale is therefore made to buyers with search types satisfying $\tau_n(\tilde{p}_n, p_{n+1}) \leq t < t_{n-1}$, yielding profit

$$\tilde{p}_n (F(t_{n-1}) - F(\tau_n(\tilde{p}_n, p_{n+1}))).$$

From the same reasons given in the case with uniform pricing, it is never optimal to

¹³Buyers' passive beliefs mean that, even after observing a deviating price \tilde{p}_n after n searches, the price that a buyer expects to be offered at the next search is the equilibrium price p_{n+1} .

set the price above \bar{p} or below p_{n+1} . Differentiating profit, we obtain the first order condition for an optimum

$$F(t_{n-1}) - F(\tau_n(\tilde{p}_n, p_{n+1})) - \tilde{p}_n \cdot f(\tau_n(\tilde{p}_n, p_{n+1})) \cdot \frac{\partial(\tau_n(\tilde{p}_n, p_{n+1}))}{\partial \tilde{p}_n} \leq 0, \quad (5)$$

binding with equality whenever the optimum is strictly larger than p_{n+1} .

In a symmetric equilibrium, $\tilde{p}_n = p_n$. Therefore, setting $t_0 = \bar{t}$, for there to exist a Coasian equilibrium of length \bar{n} , it is necessary and sufficient for there to be a pair of sequences, $\{p_n\}_{n \in \mathbb{N}}$ and $\{t_n\}_{n \in \mathbb{N}}$, such that $\tau_n(p_n, p_{n+1}) = t_n$ is satisfied for all $n \leq \bar{n}$, condition (5) holds for all $n \leq \bar{n}$ and strictly for $n < \bar{n}$, $p_{\bar{n}+1} = p_{\bar{n}}$ and $p_n \geq p_{\bar{n}}$ for all $n \geq \bar{n} + 2$.

Proposition 3. *If the game is logconcave, features strategic complements, and demand is responsive, then a Coasian equilibrium exists.*

Proof. We construct a Coasian equilibrium featuring $\bar{n} = 2$ so that a positive fraction of buyers purchase from the second seller that they search and the remaining fraction purchase from the first seller that they search.

Let x denote the first price in the sequence. Rearranging the first order conditions (5), we need to identify the set of x values for which there is a value $p' \in [0, x]$ satisfying

$$\begin{aligned} \lambda(x, p') &\equiv x - \frac{1 - F(\tau_1(x, p'))}{f(\tau_1(x, p')) \cdot \frac{d\tau_1(x, p')}{dx}} \\ &= x + \left(\frac{\partial}{\partial x} \log(1 - F(\tau_1(x, p'))) \right)^{-1} = 0. \end{aligned} \quad (6)$$

Given that the game is logconcave and features strategic complements, $\lambda(x, p')$ is strictly increasing in x on $[p', \bar{p}]$ and strictly decreasing in p' on $[0, x]$. This means that if an equilibrium involves $p_1 = x$, then p_2 must equal the unique value $\hat{p}_2(x)$ satisfying $\lambda(x, \hat{p}_2(x)) = 0$.

If we consider $p' = x$, then the expression for (6) simplifies to $\lambda(x, x) = x - \frac{1}{f(0)\theta_1(x)}$. Observe that $\lambda(0, 0) = -\frac{1}{f(0)\theta_1(0)} < 0$ and also, because the responsive demand condition is satisfied, $\lim_{x \rightarrow \bar{p}} \lambda(x, x) > 0$.¹⁴ Therefore, we can say that

¹⁴Note that $\lambda(x, x) > 0$ if and only if $x \cdot f(0) \cdot \theta_1(x) > 1$.

$\lambda(x, x)$ has a root in $(0, \bar{p})$ and denote its smallest root by R . Because $\lambda(x, p')$ is strictly decreasing in its second argument, we have that $0 < \lambda(R, 0)$. As a result of both this observation and the fact that $\lambda(x, p')$ is strictly increasing in its first argument, there is a unique value $L \in (0, R)$ satisfying $\lambda(L, 0) = 0$. By this construction, for all values $x \in [L, R]$ we have, $\lambda(x, 0) \geq 0$ and $\lambda(x, x) \leq 0$ and thus there is a value $p' \in [0, x]$ that satisfies $\lambda(x, p') = 0$. Therefore, there is a strictly increasing function $\hat{p}_2 : [L, R] \rightarrow [0, \bar{p}]$ that is implicitly defined by $\lambda(x, \hat{p}_2(x)) = 0$.

Now consider, the seller's first order condition (5) when determining the price \tilde{p}_2 to charge a visiting buyer who has previously visited one other seller. We are interested in identifying the prices at which all buyers who visit two sellers purchase from the second seller. To this end, substituting $p_1 = x$ and $\tilde{p}_2 = p_2 = p_3 = \hat{p}_2(x)$, the first order condition becomes

$$F(\tau_1(x, \hat{p}_2(x))) - \hat{p}_2(x) \cdot f(0) \cdot \theta_2(\hat{p}_2(x)) \leq 0. \quad (7)$$

When $x = L$, we have $\hat{p}_2(L) = 0$ which makes the left side of (7) equal $F(\tau_1(L, 0)) > 0$. At $x = R$, $\tau_1(R, R) = 0$, implying that the left side of (7) is $-R \cdot f(0) \cdot \theta_2(R) < 0$. Thus, there is a nonempty subset $S \subset [L, R]^\circ$ such that, if $x \in S$, then (7) holds. \square

In the setting with additively separable search costs and quasilinear utility, the logconcavity condition in fact implies that the game features strategic complements. In this setting, we can develop a more comprehensive equilibrium characterization.

Proposition 4. *Consider the additively separable search cost environment with quasilinear utility. If the game is logconcave and demand is responsive, then a Coasian equilibrium of every length $\bar{n} \in \{2, 3, \dots\} \cup \{\infty\}$ exists.*

The proof is found in the appendix and builds on the construction developed in the proof of Proposition 3. In the setting with additively separable search costs, as we increase p_{n+1} and at the same time increase p_n so that the seller's n th step first order condition is satisfied, then the marginal type τ_n which buys at the n th step decreases. The conclusion of Proposition 4 applies to any setting that has this property and satisfies the conditions of Proposition 3.

Thus we see that allowing for price discrimination on a buyer’s search length expands the set of equilibria. Specifically, there are now Coasian equilibria in which prices decline over the search length. Now we show that if we introduce shoppers into the model, then only Coasian equilibria can exist.

3.3 All Equilibria Are Coasian when There Are Shoppers

In the previous sections, we showed that both Coasian and uniform-pricing equilibria coexist when sellers have the ability to engage in price discrimination. The key structural difference between the two is that a Coasian equilibrium involves a fraction of buyers obtaining multiple price quotes while a uniform pricing equilibrium requires buyers to almost surely accept the first price quote they see.

However, the uniform pricing equilibrium necessarily excludes a mass of buyers who derive enjoyment from shopping and become fully informed about prices before deciding which good to buy. [Stahl \(1989\)](#) and [Stahl \(1996\)](#) show that when a mass of buyers have zero search cost and always break indifference by continuing to search (called “shoppers”), then when there are finitely many sellers, the equilibrium involves price dispersion. As the number of sellers grows large, the equilibrium price distribution converges to a unit mass on the largest price in the interval described in [Proposition 1](#). In other words, under uniform pricing, introducing shoppers leads prices to be more competitive when there are few sellers, but the effect vanishes as the number of sellers grows large.

We find that when sellers can perform discriminatory pricing, the presence of shoppers ensures that uniform pricing can no longer be sustained as an equilibrium. Specifically, we incorporate shoppers into our model by supposing now that there is an interior search type $t_0 \in (0, \bar{t})$ that is perfectly patient and retain the single crossing condition [\(1\)](#). Buyers with search types $t < t_0$ are called “shoppers” and strictly prefer to search longer. We assume that $U_n(p, t)$ converges to a finite value for all prices and search types as n tends towards infinity.¹⁵ One additional technical assumption of the game is that, if a buyer receives infinitely many price quotes, then they have the option to pay the infimum of these prices. This ensures that a buyer’s

¹⁵For example, utility can take the form $U_n(p, t) = u(p) - (t - t_0) \cdot \sum_{m=1}^n \gamma^{m-1}$ for some $\gamma \in (0, 1)$.

best reply is always well-defined.

Proposition 5. *When buyers comprise a positive fraction of both shoppers and non-shoppers, then all equilibria are Coasian.*

For the intuition, observe that if all sellers charge a constant sequence of positive prices, all nonshoppers will buy the good at the first seller they visit, just as before. Therefore, when a seller detects that a buyer has already performed prior searches elsewhere, it knows that the buyer is a shopper and the seller can increase its profit by deviating to a lower price. Thus, the presence of shoppers leads sellers to cut prices later in the sequence, drawing patient nonshoppers to prolong their search, producing Coasian dynamics.

4 Buyer Welfare under Price Discrimination

We now ask how buyers fare in the Coasian equilibria. Relative to uniform pricing, in the Coasian equilibria sellers can price discriminate by charging higher prices to more impatient buyers. At the same time, the cascading sequence of future prices makes continuing search more desirable, applying downward pressure on earlier prices. When considering the net effect, we find that strategic complementary yields the following sharp welfare comparison.

Proposition 6. *If the game is logconcave, features strategic complements, and demand is responsive, then there is a Coasian equilibrium that delivers each buyer higher utility than every uniform pricing equilibrium.*

Proof. Using notation from the proof of Proposition 3, the set of prices charged in a uniform pricing equilibrium is equal to the set of x values between zero and \bar{p} satisfying $\lambda(x, x) \geq 0$. Because $\lambda(x, x) < 0$ for all $x < R$, all uniform pricing equilibrium involve sellers setting a price at least as high as R . However, in the Coasian equilibria constructed in the proof of Proposition 3, the first price in sequence is strictly below R . As all buyers have the option to receive a strictly higher payoff from buying after the first search in one of these Coasian equilibria than in any uniform pricing equilibrium, these Coasian equilibria all deliver strictly higher welfare for all buyers. \square

In our main examples, there is an even sharper contrast in the welfare differences between uniform pricing and Coasian equilibria given that they satisfy the following regularity condition.

Definition 6. The game features **regular demand** if $\theta_n(p) \cdot p$ is strictly increasing on $[0, \bar{p})$ for all $n \in \mathbb{N}$.

Example 1–Continued. When search costs are additively separable $\theta_n(p) = -\frac{u'(p)}{c'(0)}$, implying that demand is regular when $c'(0) > 0$ and the utility index $u(p)$ is concave.

Example 2–Continued. When there is exponential discounting, then $\theta_n(p) = \frac{u'(p)}{u(p)r'_n(0)}$, implying that demand is regular when $r'_n(0) < 0$ and the utility index $u(p)$ is logconcave.

In settings with regular demand, Proposition 1 guarantees that there is an interval of prices that can be charged in a uniform pricing equilibrium. This condition allows us to clearly state that every Coasian equilibrium leaves all buyers strictly better off than every uniform pricing equilibrium.

Proposition 7. *If the game is logconcave, features strategic complements, and demand is both responsive and regular, then every Coasian equilibrium delivers each buyer higher utility than every uniform pricing equilibrium.*

Proof. When demand is regular, then $\lambda(x, x)$ admits a unique root R and the first price in the sequence of any Coasian equilibrium must lie strictly below R . The conclusion then follows from the argument in the proof of Proposition 6. \square

To understand the intuition behind this result, suppose all sellers besides i charge $p = (p_1, p_2, \dots)$, and seller i 's best reply is $\hat{p}^i(p)$. Strategic complementarity ensures that i 's optimal first price, \hat{p}_1^i , is strictly increasing in the other sellers' second price p_2 . Clearly, when the second price of the other sellers is zero, seller i would still do best to begin with a positive price $\hat{p}_1^i(0) > 0$. Demand regularity can be understood as providing that, whenever i finds it optimal to set its first price equal to others' second price $\hat{p}_1^i = p_2$, then increasing p_2 does not result in a strictly larger increase in \hat{p}_1 . But then we can neatly partition the set of prices so that $\hat{p}_1^i(p_2) - p_2$ is positive for $p_2 \in [0, R)$ and zero for $p_2 \in [R, \bar{p}]$, implying that $p_2 \in [0, R)$

in a Coasian equilibrium and $p'_2 \in [R, \bar{p}]$ in a uniform pricing equilibrium. It then follows that all prices in a Coasian equilibrium lie below those in any uniform pricing equilibrium since $p_2 < R \leq p'_2$ implies $\hat{p}_1^i(p_2) < \hat{p}_1^i(p'_2) = p'_2$.

We now demonstrate that a weaker welfare result can be shown if we depart with the logconcavity and strategic complementarity assumptions, but retain regular demand assumption and impose that $\theta_n(p)$ is independent of n . The latter assumption is satisfied in our running examples.

Example 1–Continued. When search costs are additively separable, $\theta_n(p) = -\frac{u'(p)}{c'(0)}$.

Example 2–Continued. When there is exponential discounting, then $\theta_n(p) = \frac{u'(p)}{u(p)\delta'(0)}$, implying that $\theta_n(p)$ is independent of n .

The assumption that $\theta_n(p)$ is independent of n is weaker than [Koopmans et al. \(1964\)](#)'s classic stationarity postulate in the context of infinite consumption streams. Specifically, in our framework, [Koopmans et al. \(1964\)](#)'s stationarity axiom implies that if $U_n(p, t) = U_{n+1}(p', t)$ for any $n \in \mathbb{N}$, then $U_{n'}(p, t) = U_{n'+1}(p', t)$ for all $n' \in \mathbb{N}$. This weaker form of stationarity in turn, is stronger than the assumption that $\theta_n(p)$ is independent of n . Examples that satisfy the latter, but not weak stationarity, are nonlinear (e.g., convex) additive search costs.

Proposition 8. *If demand is regular and $\theta_n(p)$ independent of n , then every Coasian equilibrium delivers at least one buyer higher utility than every uniform pricing equilibrium.*

Proof. We show that, if demand is regular and $\theta_n(p)$ independent of n , then the last price quoted in a Coasian equilibrium (i.e., $p_{\bar{n}}$) lies below the lowest price across all uniform-pricing equilibria. We do not tackle existence of either type of equilibria in this proof.

First, if the only uniform-pricing equilibrium features the monopoly price \bar{p} , every Coasian equilibrium has at least one price below \bar{p} by definition.

Second, suppose that multiple uniform-pricing equilibria exist. We know that an interior uniform-pricing equilibrium price must satisfy equation (4) with equality.¹⁶

¹⁶Since logconcavity may fail, we no longer know whether the condition is sufficient to identify an equilibrium price (as it may find a locally optimal price), but it is necessary.

Thus, the lowest price that potentially constitutes a uniform-pricing equilibrium satisfies $p_{min}^u = \frac{1}{f(0)\theta_1(p_{min}^u)} = \frac{1}{f(0)\theta(p_{min}^u)}$, where the second equality follows from the assumption that $\theta_n(p)$ is independent of n .

Now consider finite Coasian equilibria. The last price that is paid by a consumer in a Coasian equilibrium of finite length \bar{n} , $p_{\bar{n}}$, must satisfy an expression similar to equation (7) with equality: if $p_{\bar{n}-1} = x$, we need $p_{\bar{n}}(x)$ to satisfy

$$F(\tau_{\bar{n}-1}(x, p_{\bar{n}}(x))) - p_{\bar{n}}(x) \cdot f(0) \cdot \theta_{\bar{n}}(p_{\bar{n}}(x)) = 0, \quad (8)$$

because, by definition, the last price that is paid in a Coasian equilibrium must be interior. Under the assumption that $\theta_n(p)$ independent of n , using the fact that $p_{min}^u = \frac{1}{f(0)\theta(p_{min}^u)}$, we can rewrite equation (8) as

$$\frac{p_{\bar{n}}(x)\theta(p_{\bar{n}}(x))}{p_{min}^u\theta(p_{min}^u)} = F(\tau_{\bar{n}-1}(x, p_{\bar{n}}(x))). \quad (9)$$

Suppose now that, as a way contradiction, that $p_{min}^u < p_{\bar{n}}(x)$. But then the left side of equation (9) exceeds one because of the regular demand assumption, while the right side is at most one. Thus, it must be that $p_{min}^u > p_{\bar{n}}(x)$ for any $p_{\bar{n}-1} = x$.

Now consider an infinite Coasian equilibrium. Such an equilibrium is described by two monotone decreasing sequences $\{t_n\}_{n \in \mathbb{N}}$ and $\{p_n\}_{n \in \mathbb{N}}$ satisfying $F(t_{n-1}) - F(t_n) - p_n \cdot f(t_n) \cdot \theta(p_{n+1}) = 0$. We know that $t_n \rightarrow 0$ and, as a bounded monotone sequence, p_n must converge to some limit p^* . This means that $0 = \lim_{n \rightarrow \infty} (F(t_{n-1}) - F(t_n) - p_n \cdot f(t_n) \cdot \theta(p_{n+1})) = p^* \cdot f(0) \cdot \theta(p^*)$. Regularity of demand then provides that $p^* < p_{min}^u$. \square

Proposition 8 is a weaker result than Proposition 7 because we show that only some consumers benefit in all Coasian equilibria as compared to the lowest-priced uniform-pricing equilibrium. But in Proposition 8 we part with the logconcavity and strategic complementarity assumptions. Without logconcavity, we simply lose the convenient property that profits are concave so we cannot say that all prices that satisfy the FOCs are in fact equilibria. Without strategic complementarity, the optimal prices of a firm do not move in the same direction in a strong sense. We show by example in Section 4.2 that in the absence of strategic complementarity, the highest prices in a Coasian equilibrium can exceed the price in the lowest-priced



Figure 1: Coasian equilibrium partitioning of the search costs $\cup_{n=1}^{\infty} [t_n, t_{n-1}]$ for quasilinear utility and linear search costs with $t \sim U[0, 1]$.

uniform-pricing equilibrium. In other words, if strategic complementarity fails, Coasian equilibria may redistribute surplus among buyers, but do not improve all buyers' welfare.

4.1 Example of a Coasian equilibrium

To make the equilibrium construction and the potential magnitude of the welfare gains more concrete, let us return to the environment with additive search costs and consider the case in which utility is quasilinear (Example 1). From Proposition 1, we know that, as long as the game satisfies logconcavity, then a strategy profile is uniform pricing equilibrium if and only if each seller charges a price p in the set $[\min\{1/f(0), \bar{p}\}, \bar{p}]$.

Suppose that search types are uniformly distributed between zero and one and that $\bar{p} \geq 1$. In a strict Coasian equilibrium wherein a positive fraction of buyers purchase the good at the n th search for each $n \in \mathbb{N}$, we can partition the set $(0, 1) = \cup_{n=1}^{\infty} [t_n, t_{n-1})$ so that a buyer with search cost $t \in [t_n, t_{n-1})$ buys after n searches. The buyer with search cost $t = t_n$ is made indifferent between buying after the n th search and continuing once more, implying $p_n - p_{n+1} = t_n$. Thus, using the recursive definition of the prices, we have $p_n = \sum_{k=0}^{\infty} t_{n+k}$ for all n .

Using this recurrence and equation which characterizes optimal prices, we get that in the Coasian equilibrium we must have

$$\sum_{k=0}^{\infty} t_{n+k} = t_{n-1} - t_n \quad (10)$$

$$\implies t_n = t_{n-1} - 2t_n + t_{n+1}. \quad (11)$$

which simplifies to $t_n = \frac{t_{n-1} + t_{n+1}}{3}$ with the boundary condition $t_0 = 1$. We solve the

recurrence by solving its characteristic equation, getting $t_n = \left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)^n$. Figure 1 illustrates the implied partitioning of the search costs. Figure 1 demonstrates that the fraction of buyers who buy is decreasing in their search length: the intervals become shorter as we move from the end of the search costs' support (i.e., right of the interval) towards the start (i.e., leftwards). This feature of the partitioning relies on the fact that the search costs are uniformly distributed.

Prices are given by $p_n = \frac{2\left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)^n}{\sqrt{5}-1}$. Figure 2 illustrates prices in the Coasian equilibrium (red dots) and in the uniform-pricing equilibrium with the lowest price (black squares). The figure illustrates the general result in Proposition 6: buyers are better off in the Coasian equilibrium than in the uniform-pricing equilibria because all prices are lower in the Coasian equilibrium than in the uniform-pricing equilibrium with the lowest price. Figure 2 demonstrates that buyers can get a substantial proportion of the total surplus in Coasian equilibria compared to the uniform-pricing equilibrium. If $\bar{p} = 1$, buyers get zero surplus in the unique uniform-pricing equilibrium. Conversely, since the first price in the depicted Coasian equilibrium is $p_1 \approx 0.618$, each buyer gets at least 38% of the surplus in the Coasian equilibrium.

4.2 Importance of Strategic Complementarity

The central tradeoff we uncover is that price discrimination based on search duration produces two effects: It allows sellers to target the most impatient buyers with higher prices, but also introduces Coasian dynamics which drive down all prices. Asking which effect dominates turns out to depend on whether or not the setting features strategic complements. Commonly studied settings, including all of the examples discussed so far, do feature strategic complements, and so the second effect dominates and all buyers may benefit from price discrimination.

It is important to highlight that the strategic complements condition is essential for the results. For example, consider the linear search cost environment and suppose that the search types follow a Pareto distribution $F(t) = 1 - (t+1)^{-\beta}$ for some $\beta > 1$. Notice, $\frac{\partial}{\partial p} \log(1 - F(\tau_n)) = -\beta/(p - p' + 1)$. This example does not satisfy the logconcavity condition, but it does satisfy the weaker -1 -concavity condition which

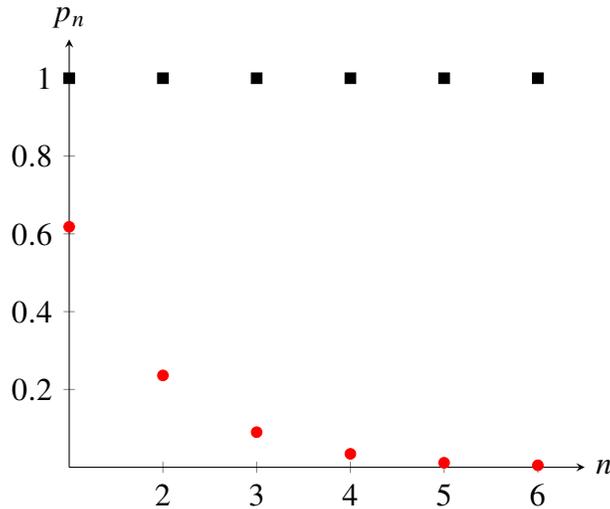


Figure 2: Price charged for a given search duration n in the Coasian equilibrium (red dots) and the uniform-pricing equilibrium with the lowest price (black squares) for $t \sim U[0, 1]$.

is enough for the Coasian equilibrium existence argument supporting Proposition 3 to go through. More substantive is that the example does not feature strategic complements but instead features strategic substitutes (i.e., $\frac{\partial}{\partial p} \log(1 - F(\tau_n))$ is strictly decreasing in p' on \mathbf{P}_n). Perhaps surprisingly, with a slight adjustment to the verbiage to the proof of Proposition 3, it remains that the argument goes through.¹⁷

However, the presence of strategic substitutes leaves ambiguity in the effect of price discrimination on buyer welfare. If we compare Coasian equilibria to the lowest-priced uniform pricing equilibrium, then the most impatient types are always charged higher prices under price discrimination, and made worse off. Sellers can even derive higher profit in a Coasian equilibrium compared with the lowest-priced uniform pricing equilibrium. But because there is always a uniform pricing equilibrium in which sellers extract the entire surplus, in the case of strategic substitutes, there is no clear comparison to be made in terms of whether buyers or sellers benefit from price discrimination.

¹⁷With strategic substitutes, we have $L > R$ and the function $\hat{p}_2(x)$ is strictly decreasing on $[R, L]$.

5 Conclusion

This paper demonstrates that Coasian dynamics can arise in a competitive frictional market. If buyers differ in their patience and sellers can price discriminate based on search length, then buyers benefit in a Coasian equilibrium as compared to uniform pricing. In the Coasian equilibrium, the least patient buyers optimally stop searching first and pay the highest prices. Buyers who are somewhat more patient stop searching next and pay a somewhat lower price, and so on. However, under natural conditions, the ability of sellers to thus price discriminate translates into lower profits than under uniform pricing. This is because all buyers benefit from the expectation of future lower prices so all discriminatory prices are below the uniform price. In other words, sellers are trapped in a version of Coasian dynamics in the Coasian equilibrium.

The Coasian equilibrium has an intuitive structure: people more prone to searching in fact search longer and end up paying lower prices. No buyer has an incentive to pretend to have searched less than he really has as prices decrease in search length. In other words, if buyers could reset their search length by deleting cookies, they would have no incentive to do so. The Coasian equilibrium is also the unique symmetric equilibrium in an extended model where some buyers enjoy shopping. Our model, thus, demonstrates the force of Coasian dynamics in frictional markets, in contrast to [Board and Pycia \(2014\)](#). The driving force behind the Coasian dynamics in our setting is the consumers' heterogeneous patience.

Our model's results contribute to the broader inquiry that explores online privacy and the effects of large-scale data collection in digital markets. In particular, we show that novel opportunities that sellers have in digital markets, such as using advanced tracking technologies, may backfire for them in equilibrium. Thus, revenue managers may have an incentive to use the information they have about buyers sparingly despite sellers' ever-improving abilities to collect and analyse consumer data.¹⁸

The paper can be extended in several dimensions. First, the tracking technology available to the sellers in our model allows them to observe the full extent of

¹⁸[Bourreau and de Streel \(2018\)](#), [Ipsos et al. \(2018\)](#), [Ennis and Lam \(2021\)](#).

a buyer's search history. One potential direction is to consider a more restricted technology in which sellers only observe a noisy signal of the search history, though, we expect that the Coasian dynamics we describe will emerge as long as buyers vary in their patience. Second, allowing for downward-sloping demand would even more starkly demonstrate the extent to which buyers benefit in the Coasian equilibrium as compared to uniform pricing. Third, buyers could search for match values in addition to low prices. If the match values are independent across sellers (as in Board and Pycia (2014)), the force of Coasian dynamics would still lead to decreasing prices that the Coasian equilibrium features. Although the qualitative effect of this extension on prices is clear, the exact welfare results may be of interest. Finally, price discrimination based on the extent of search more generally may be of both empirical relevance and theoretical interest. Search length, depth, breadth and intensity are all indications of buyers' willingness to search, investigate, compare, and buy products, and, thus, offer other potential bases for price discrimination.

A Appendix

A.1 Omitted Proofs

Lemma A.1. *For all $n \in \mathbb{N}$, $\tau_n(p, p')$ is differentiable, increasing in p , and decreasing in p' on the interior of \mathbf{P}_n .*

Proof. Differentiability follows from the implicit function theorem. For all $t \leq \tau_n(p, p')$ the function $U_n(\tilde{p}, t) - U_{n+1}(p', t)$ is strictly decreasing in \tilde{p} , nonpositive at $\tilde{p} = p$, so that $U_n(\tilde{p}, t) - U_{n+1}(p', t) < 0$ for all $\tilde{p} > p$ satisfying $(\tilde{p}, p') \in \mathbf{P}_n$. Therefore, it follows that $\tau_n(\tilde{p}, p') > \tau_n(p, p')$. The remainder of the proof follows by similar arguments. \square

Proof of Proposition 5. In an equilibrium, it cannot be that the infimum of the prices charged by sellers is larger than zero. Otherwise, a seller would have an incentive to charge a lower price at some point along the sequence when the fraction of nonshoppers remaining in the market is small and a positive profit could be made from all visiting shoppers. This implies that that the expected profit from shoppers

is zero and so sellers' optimal pricing only depends on nonshoppers. But then, following the argument of Proposition 2 provides that the equilibrium is in fact a Coasian equilibrium. \square

A.2 Proof of Proposition 4

This section proves a slight generalization of Proposition 4 stated in the following.

Proposition A.1. *Assume $1 - F(\tau_n(p, p'))$ is logconcave in p , $\partial\tau_n(p, p')/\partial p = g_n(\tau_n)$ for some function g_n , $\partial\tau_n(p, p')/\partial p' = \tilde{g}_n(\tau_n)$ for some function \tilde{g}_n , and also that demand is responsive. Then a Coasian equilibrium of every length $\bar{n} \in \{2, 3, \dots\} \cup \{\infty\}$ exists.*

The assumptions that $\partial\tau_n(p, p')/\partial p = g_n(\tau_n)$ and $\partial\tau_n(p, p')/\partial p' = \tilde{g}_n(\tau_n)$ can be understood as requiring that the change in the marginal type when changing the prices only depends on the value of the marginal type. In the additively separable search cost environment with quasilinear utility, these conditions are satisfied because $\partial\tau_n(p, p')/\partial p = -\partial\tau_n(p, p')/\partial p' = \left(\frac{\partial}{\partial t}C(\tau_n(p, p'), n+1) - \frac{\partial}{\partial t}C(\tau_n(p, p'), n)\right)^{-1}$. Also, these assumptions simplify the responsive demand condition to requiring $f(0) \cdot g(0) > 0$.

Lemma A.2. *Assume that $1 - H$ is a twice differentiable and logconcave function. Then for any constant $c \geq 0$, the function $c - H$ is logconcave on any interval in the domain satisfying $c > H$.*

Proof. Twice differentiating $\log(c - H(x))$ obtains

$$\frac{d^2}{dx^2} \log(c - H(x)) = \frac{d}{dx} \left[\frac{-\frac{dH(x)}{dx}}{c - H(x)} \right] = -\frac{\frac{d^2H(x)}{dx^2}(c - H(x)) + \left(\frac{dH(x)}{dx}\right)^2}{(c - H(x))^2}$$

and thus, it is enough to show that $\frac{d^2H(x)}{dx^2}(c - H(x)) + \left(\frac{dH(x)}{dx}\right)^2 \geq 0$. By inspection, the inequality holds when $\frac{d^2H(x)}{dx^2} \geq 0$. When $\frac{d^2H(x)}{dx^2} < 0$, then $\frac{d^2H(x)}{dx^2}(c - H(x)) + \left(\frac{dH(x)}{dx}\right)^2 \geq \frac{d^2H(x)}{dx^2}(1 - H(x)) + \left(\frac{dH(x)}{dx}\right)^2 \geq 0$ where the last inequality follows from $-1 - H(x)$ being logconcave. \square

Lemma A.3. *Given the assumptions in Proposition A.1, the following hold.*

(a) $-\frac{c-F(t)}{f(t)g_n(t)}$ is increasing in t whenever $c > F(t)$.

(b) *The game satisfies strategic complements.*

Proof. Beginning with (a), from Lemma A.2, we know that $\log(c - F(\tau_n(p, p')))$ is concave in $p \geq p'$, meaning

$$\frac{\partial}{\partial p} [\log(c - F(\tau_n(p, p')))] = -\frac{f(\tau_n(p, p'))g_n(\tau_n(p, p'))}{1 - F(\tau_n(p, p'))}$$

is decreasing in p . Define $h(t) \equiv -f(t)g_n(t)/(1 - F(t))$. We know that $\frac{\partial h(\tau_n(p, p'))}{\partial p} = \frac{dh}{dt}|_{t=\tau_n(p, p')} \cdot g_n(\tau_n(p, p')) < 0$, implying $\frac{dh}{dt} < 0$. Therefore, $\frac{d}{dt} \frac{1}{h(t)} = -\frac{dh(t)}{dt} \cdot h^{-2}(t) > 0$ and thus (a) holds.

Turning to (b), the desired conclusion is equivalent to $\frac{\partial h(\tau_n(p, p'))}{\partial p'} > 0$, which must hold as $\frac{\partial h(\tau_n(p, p'))}{\partial p'} = \frac{dh}{dt}|_{t=\tau_n(p, p')} \cdot \tilde{g}_n(\tau_n(p, p')) > 0$. \square

Proof of Proposition A.1. Our approach is to show that there exists a nonempty interval $[L^*, R^*]$ for which any sequence pair satisfying (5) requires $p_1 \in [L^*, R^*]$ and also, if $p_1 \in [L^*, R^*]$, then there is a unique corresponding sequence pair satisfying (5). The first step of the proof develops an inductive argument to construct a sequence of nonempty nested intervals $\{[L_n, R_n]\}_{n \in \mathbb{N}}$ such that, for there to exist values $\{p_i\}_{i=1}^n$ and $\{t_i\}_{i=1}^n$ satisfying $\tau_i(p_i, p_{i+1}) = t_i$ and $p_i = [F(t_i) - F(t_{i-1})] / [f(t_i) \cdot g(t_i)]$ for all $1 \leq i \leq n$, it is necessary and sufficient that $p_1 \in [L_n, R_n]$.

Claim. *Define $[L_0, R_0] = \mathbb{R}$, $\hat{t}_0(x) = 0$, and $\hat{p}_1(x) = x$ for all $x \in \mathbb{R}$. For every $n \in \mathbb{N}$, there exists a collection of nonempty nested intervals $\{[L_i, R_i]\}_{i=1}^n$ and strictly increasing functions $\hat{p}_i : [L_{i-1}, R_{i-1}] \rightarrow [0, \bar{p}]$ and $\hat{t}_i : [L_i, R_i] \rightarrow [0, \bar{t}]$ such that, for all $1 \leq i \leq n$, the following conditions hold.*

For all $x \in [L_i, R_i]$:

1. $\hat{p}_i(x) = \frac{F(\tau_i(\hat{p}_i(x), \hat{p}_{i+1}(x))) - F(\hat{t}_{i-1}(x))}{f(\tau_i(\hat{p}_i(x), \hat{p}_{i+1}(x))) \cdot \frac{\partial}{\partial p} \tau_i(\hat{p}_i(x), \hat{p}_{i+1}(x))}$.
2. $\tau_i(\hat{p}_i(x), \hat{p}_{i+1}(x)) = \hat{t}_i(x)$.
3. $\hat{p}_i(L_i) = \frac{F(\tau_i(\hat{p}_i(L_i), 0)) - F(\hat{t}_{i-1}(L_i))}{f(\tau_i(\hat{p}_i(L_i), 0)) \cdot \frac{\partial}{\partial p} \tau_i(\hat{p}_i(L_i), 0)}$.
4. $\hat{p}_i(R_i) = \frac{F(\hat{t}_{i-1}(R_i))}{f(0) \cdot g(0)}$.

Base Case.

For the base case, let us verify that the claim holds when $n = 1$. By assumption $\hat{p}_1(x) = x$, which is strictly increasing. We need to identify the set of x values for which there is a $p' \in [0, x]$ satisfying

$$\lambda_1(x, p') \equiv x - \frac{1 - F(\tau_1(x, p'))}{f(\tau_1(x, p')) \cdot \frac{d\tau_1(x, p')}{dx}} = 0. \quad (12)$$

As a consequence of logconcavity and Lemma A.3, the function $\lambda_1(x, p')$ is strictly increasing in x on $[p', \bar{p}]$ and strictly decreasing in p' on $[0, x]$. This means that if an equilibrium involves $p_1 = x$, then p_2 must equal the unique value $\hat{p}_2(x)$ satisfying $\lambda_1(x, \hat{p}_2(x)) = 0$. To determine the values of x for which this condition can hold, if we define L_1 to be the number that satisfies $\lambda_1(L_1, 0) = 0$, then it must be that $x \geq L_1$ to ensure that $\hat{p}_2(x) \geq 0$. We note that $L_1 > 0$ because $\lambda_1(0, 0) = -\frac{1}{f(0) \cdot g(0)} < 0$.

To determine the largest value that x can take, consider the function (12) evaluated at $p' = x$, which simplifies to $\lambda_1(x, x) = x - \frac{1}{f(0) \cdot g(0)}$. Let R_1 denote the root satisfying $\lambda_1(R_1, R_1) = 0$. For all values $x \in [L_1, R_1]$ we have, $\lambda_1(x, 0) \geq 0$ and $\lambda_1(x, x) \leq 0$ and thus there is a $0 \leq p' \leq x$ that satisfies $\lambda_1(x, p') = 0$. Therefore, there is a function $\hat{p}_2 : [L_1, R_1] \rightarrow [0, \bar{p}]$ that is implicitly defined by $\lambda_1(x, \hat{p}_2(x)) = 0$ and is strictly increasing.

Define $\hat{t}_1(x) \equiv \tau_1(x, \hat{p}_2(x))$. Notice that $\hat{t}_1(x)$ satisfies $x - \frac{1 - F(\hat{t}_1(x))}{f(\hat{t}_1(x)) \cdot g_1(\hat{t}_1(x))} = 0$. And so by Lemma A.3, we can conclude that $\hat{t}_1(x)$ is strictly decreasing on $[L_1, R_1]$.

Inductive Case.

Now let us make the inductive hypothesis and assume that the claim holds for $i = n - 1$. We want to demonstrate the claim holds for $i = n$. Specifically, we want to identify a nonempty interval $[L_n, R_n] \subset [L_{n-1}, R_{n-1}]$, a strictly increasing function $\hat{p}_n : [L_{n-1}, R_{n-1}] \rightarrow [0, \bar{p}]$, and a strictly decreasing function $\hat{t}_n : [L_n, R_n] \rightarrow [0, \bar{t}]$ satisfying the four conditions of the claim.

From the inductive hypothesis, $\hat{p}_n(x)$ is defined as the increasing function satisfying $\lambda_{n-1}(\hat{p}_{n-1}(x), \hat{p}_n(x)) = 0$ for all $x \in [L_{n-1}, R_{n-1}]$. We need to identify the values for x for which there exists a solution to

$$\lambda_n(\hat{p}_n(x), p') = \hat{p}_n(x) - \frac{F(\hat{t}_{n-1}(x)) - F(\tau_n(\hat{p}_n(x), p'))}{f(\tau_n(\hat{p}_n(x), p')) \cdot \frac{\partial \tau_n(\hat{p}_n(x), p')}{\partial p}} \quad (13)$$

for some $p' \in [0, \hat{p}_n(x)]$. By Lemma A.4 below, λ_n is strictly increasing in x and strictly decreasing in p' . Therefore, if there exists a Coasian equilibrium of length at least n , then if $p_1 = x$ it must be that $p_{n+1} = \hat{p}_{n+1}(x)$ satisfying $\lambda_n(\hat{p}_n(x), \hat{p}_{n+1}(x)) = 0$.

Similar to before, if we consider $\lambda_n(\hat{p}_n(x), \hat{p}_n(x)) = \hat{p}_n(x) - \frac{F(\hat{t}_{n-1}(x))}{f(0) \cdot g(0)}$, the function is strictly increasing in x on $[L_{n-1}, R_{n-1}]$, is equal to $-\frac{F(\hat{t}_{n-1}(L_{n-1}))}{f(0) \cdot g(0)} < 0$ at $x = L_{n-1}$, and is equal to $\hat{p}_n(R_{n-1}) > 0$ at $x = R_{n-1}$. Hence, there is a unique $R_n \in (L_{n-1}, R_{n-1})$ satisfying $\lambda_n(\hat{p}_n(R_n), \hat{p}_n(R_n)) = 0$. Because λ_n is strictly decreasing in its second argument $\lambda_n(\hat{p}_n(R_n), 0) > 0$ and thus there is a unique value $L_n \in (L_{n-1}, R_n)$ satisfying $\lambda_n(\hat{p}_n(L_n), 0) = 0$. Therefore, there is a strictly increasing function $\hat{p}_{n+1} : [L_n, R_n] \rightarrow [0, \bar{p}]$ that solves $\lambda_n(\hat{p}_n(x), \hat{p}_{n+1}(x)) = 0$.

Define $\hat{t}_n(x) : [L_n, R_n] \rightarrow [0, 1]$ to be the function satisfying $\hat{t}_n(x) \equiv \tau_n(\hat{p}_n(x), \hat{p}_{n+1}(x))$. From Lemma A.3 we can conclude that $\hat{t}_n(x)$ is strictly decreasing. Therefore, the claim holds when $i = n$. By induction, the claim thus holds for all $n \in \mathbb{N}$.

By the nested interval theorem, the intersection $[L^*, R^*] \equiv \bigcap_{n=1}^{\infty} [L_n, R_n]$ is nonempty. Therefore, a strategy profile is a Coasian equilibrium with $\bar{n} = \infty$ if and only if there is a value of $x \in [L^*, R^*]$ for which sellers charge $p_n = \hat{p}_n(x)$ for all $n \in \mathbb{N}$ and buyers performing optimal search given that they believe that the sellers will charge these prices. This provides that there is a Coasian equilibrium of infinite length. We can also conclude that there is a Coasian equilibrium of length n by setting $x = R_n$, $p_i = \hat{p}_i(x)$ for all $i \leq n$ and $p_i = \hat{p}_n(x)$ for all $i \geq n$ and having buyers perform optimal search given that they believe sellers will charge these prices. \square

Lemma A.4. $\lambda_n(x, p')$ defined by (13) is increasing in x and decreasing in p' .

Proof.

$$\begin{aligned} \frac{\partial \lambda_n}{\partial x} &= \frac{d\hat{p}_n}{dx} - \frac{f(\hat{t}_{n-1})}{f(\tau_n)g(\tau_n)} \frac{d\hat{t}_{n-1}}{dx} - \frac{d}{dt} \left(\frac{F(\hat{t}_{n-1}) - F(t)}{f(t)g(t)} \right) \Big|_{t=\tau_n} \cdot g(\tau_n). \\ \frac{\partial \lambda_n}{\partial p'} &= -\frac{d}{dt} \left[\frac{F(\hat{t}_{n-1}) - F(t)}{f(t)g(t)} \right] \Big|_{t=\tau_n} \cdot \tilde{g}(\tau_n) \end{aligned}$$

Note $\frac{\partial \lambda_n}{\partial x} > 0$ follows because $\frac{\partial \hat{p}_n}{\partial x} > 0$, $\frac{d\hat{t}_{n-1}}{dx} < 0$, and Lemma A.3. $\frac{\partial \lambda_n}{\partial p'} < 0$ follows from Lemma A.3. \square

B Dynamic Inconsistency

In this appendix, we show how our results can be extended when we generalize the model to allow for dynamic inconsistency. We incorporate dynamically inconsistent preferences by redefining payoffs as follows. Consider a buyer of type t who has already searched n sellers. Such a buyer's payoff from searching $n + k$ sellers and buying the good at price p is $U_n(p, t, k)$. We maintain that utility is strictly decreasing in price, $U_1(p, t, 0) = u(p)$ for all types $t \in [0, \bar{t}]$, $u(\bar{p}) = 0$ for some $\bar{p} > 0$, and that type $t = 0$ is perfectly patient, with $U(p, 0, k)$ being constant in k . We maintain the convention that if a buyer of type t faces a sequence of prices $\mathbf{p} = \{p_1, p_2, \dots\}$ and has performed n searches, then the buyer's utility from continuing to search infinitely many firms is a well-defined limit $\lim_{k \rightarrow \infty} U_n(\inf \mathbf{p}, t, k)$. The single crossing condition now takes the following form. For all prices in $[0, \bar{p})$ and $k \in \mathbb{N}$,

$$U_n(p, t, 0) - U_n(p', t, k) \geq 0 \implies U_n(p, t', 0) - U_n(p', t', k) > 0, \text{ for all } t < t'. \quad (14)$$

Notice that this generalization can now incorporate dynamic inconsistency as, in general, the way a buyer evaluates future decisions may differ from how they evaluate the decisions once they arrive at that future point in time; that is, $U_n(p, t, k)$ may differ from $U_{n+k}(p, t, 0)$. For example, the model can incorporate the usual quasi-hyperbolic β - δ preferences, satisfying $U_n(p, t, 0) = v - p$ and $U_n(p, t, k) = \beta \cdot \delta(t)^k (v - p)$ for all $k \geq 1$ with $\beta \in (0, 1)$. Hyperbolic discounting can be captured with $U_n(p, t, k) = \frac{1}{1+t \cdot k} \cdot u(p)$.

Equilibrium. Buyers are *sophisticated* in that they know how their preferences will evolve over time, but cannot commit their future selves to behave in accordance to their current preferences (O'Donoghue and Rabin, 1999). Our equilibrium notion adapts the classic approach of Strotz (1955) and Pollak (1968) whereby strategies

constitute a PBE of an auxiliary game in which the buyer at any search length is a distinct non-cooperative player.

Welfare. In the presence of dynamic inconsistency, a buyer may disagree with its future selves about which outcomes are preferable. For our purposes, we simply measure a buyer’s welfare by the utility as assessed by the buyer from the vantage point of the first search $U_1(p, t, k)$. While it is beyond the scope of this paper, future work may perform a welfare analysis using alternative measures, say when investigating when buyers of every search length are better off under price discrimination (Kodritsch, 2015).

Results. We argue that Propositions 1 through 6 continue to hold under this generalization. Then we show how, restricting our focus to the usual “present biased” forms of dynamic inconsistency, Propositions 7 and 8 continue to hold as well.

Under this generalization, the argument supporting Proposition 2 goes through unchanged, and thus every symmetric discriminatory equilibrium is Coasian, featuring strictly decreasing prices along the active search path, and no lower prices thereafter. That is, if \bar{n} is the supremum of the search lengths taken with positive probability, then p_n is strictly decreasing in the search length for $n \leq \bar{n}$ and $p_n \geq p_{\bar{n}}$ for all $n \geq \bar{n}$. Moreover, sellers receive positive demand from buyers of each search length $n \leq \bar{n}$.

The characterization of uniform pricing equilibria (Proposition 1) and Coasian equilibria of length $\bar{n} = 2$ (Proposition 3) also go through unchanged under this more general formulation. The reason is that, upon making the first search, every imperfectly patient buyer ($t > 0$) expects that its future selves will face a constant sequence of prices and thus all future selves will halt search and make a purchase. For Proposition 4, the additively separable search cost environment remains a special case of the generalized model, and so its conclusion is maintained. The simple argument for Proposition 5 showing that only Coasian equilibria can exist continues to hold unchanged within the generalization. Finally, as the welfare result in Proposition 6 compares uniform pricing equilibria with the Coasian equilibrium of length

$\bar{n} = 2$ constructed in Proposition 3, its conclusion also still holds.

The conclusions of Propositions 7 and 8 rely on all Coasian equilibria having the property that more patient buyers always search at least as long as those who are less patient. This ensures that those buyers who are indifferent between buying after n searches and continuing further strictly prefer to halt at $n + 1$ searches. Here, we argue that this property is guaranteed to be satisfied as long as we narrow down the type of dynamic inconsistency that can occur to take the form of *weak present bias* (Schweighofer-Kodritsch, 2018). For all $n \in \mathbb{N}$, $k, k' \geq 0$, and prices in $[0, \bar{p})$

$$U_n(p', t, k) - U_n(p, t, 0) \geq 0 \implies U_{n-k'}(p', t, k + k') - U_{n-k'}(p, t, k') \geq 0. \quad (15)$$

A buyer is defined to be weakly present biased if the fact that the buyer's "self" after n searches prefers to continue for k more searches and buy at price p' (rather than stop immediately and buy at price p) implies that all the buyer's "earlier selves" would also prefer stopping after $n + k$ searches at price p' rather than n searches at price p . The hyperbolic and quasi-hyperbolic discounting examples mentioned above satisfy weak present bias. Another example is a variant of exponential discounting in which buyers experience declining patience: $U_n(p, t, k) = \delta(t, n)^k u(p)$ with $\delta(t, n)$ decreasing in both arguments.

Claim. Given the above assumptions, a Coasian equilibrium of length \bar{n} is characterized by a partition $(0, \bar{t}) = \cup_{n=1}^{\bar{n}} (t_n, t_{n-1})$, where buyers of type $t \in (t_n, t_{n-1})$ search n sellers and a buyer of type $t = t_n$ is indifferent between n and $n + 1$ searches.

Proof. As stated above, every Coasian equilibrium of length \bar{n} features: (i) a sequence of prices $\mathbf{p} = \{p_n\}_{n \in \mathbb{N}}$ for which p_n is strictly decreasing in the search length for $n \leq \bar{n}$ and $p_n \geq p_{\bar{n}}$ for all $n \geq \bar{n}$, and (ii) positive demand for each search length $n \leq \bar{n}$. We first show that a buyer of type t must search at least as long as a less patient buyer of type $t' > t$.

Toward a contradiction, suppose that, in equilibrium a type $t' > t$ searches strictly longer than a buyer of type t . Let $b \geq 2$ denote the smallest search length at which the strategy of buyer t' involves making a purchase. Let a be the largest search length less than b at which the strategy of buyer t involves making a purchase. Finally, let c denote the smallest search length at least as large as b at which the

strategy of buyer t involves making a purchase. Thus, we have $a < b \leq c$.

Suppose that c is finite. Because t' chooses to continue after a searches, knowing that its future self will make a purchase after b searches, we have $U_a(p_a, t', 0) \leq U_a(p_b, t', b - a)$. By single crossing, this implies that a more patient type would be willing to search as long $U_a(p_a, t, 0) < U_a(p_b, t, b - a)$. Also, because, after b searches, t prefers to continue, knowing that its future self will purchase after c searches, it follows that $U_b(p_b, t, 0) \leq U_b(p_c, t, c - b)$. Due to present bias, it follows that, from the vantage point of having performed a searches, the type t buyer would prefer to continue searching c times rather than halt at b times, i.e., $U_a(p_b, t, b - a) \leq U_a(p_c, t, c - b + b - a) = U_a(p_c, t, c - a)$. But then we arrive at a contradiction because $U_a(p_a, t, 0) < U_a(p_b, t, b - a) \leq U_a(p_c, t, c - a)$, meaning that type t strictly prefers to *not* buy at a searches, but to continue and let its future self buy at c searches. Notice that the same argument can be applied when either both b and c are infinite or when b is finite and c is infinite. Thus, we can conclude that, in any Coasian equilibrium, a buyer of type $t < t'$ searches at least as long as a buyer of type t' .

Combining this with Proposition 2, we can further conclude that, in a Coasian equilibrium of length \bar{n} , the type space can be partitioned $(0, \bar{t}) = \cup_{n=1}^{\bar{n}} (t_n, t_{n-1})$, where buyers of type $t \in (t_n, t_{n-1})$ search n sellers. Thus, if a buyer of type t_n is indifferent between halting at n searches and continuing, in equilibrium, its future self must choose to make a purchase at $n + 1$ searches, implying $U_n(p_n, t_n, 0) = U_n(p_{n+1}, t_n, 1)$. \square

The arguments supporting Propositions 7 and 8 rely on the fact that equation (6) holds in a Coasian equilibrium. This equality, in turn, is determined by the fact that there is a single buyer type indifferent between buying in the first search and continuing further and that this buyer type expects its future self to make a purchase on the second search. The above claim verifies that, when preferences are weakly present biased, then this property continues to hold in any Coasian equilibrium and thus, the welfare results in Propositions 7 and 8 follow as well.

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