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in OPEC Market Power –
A Stackelberg Oligopoly
with Fringe

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Endogenous shifts in OPEC market power – A Stackelberg oligopoly with fringe*

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Abstract

This article proposes a two-stage oligopoly model for the crude oil market. In a game of several Stackelberg leaders, market power increases endogenously as the spare capacity of the competitive fringe goes down. This effect is due to the specific cost function characteristics of extractive industries. The model captures the increase of OPEC market power before the financial crisis and its drastic reduction in the subsequent turmoil at the onset of the global recession. The two-stage model better replicates the price path over the years 2003–2011 compared to a standard simultaneous-move, one-stage Nash-Cournot model with a fringe. This article also discusses how most large-scale numerical equilibrium models, widely applied in the energy sector, over-simplify and potentially misinterpret market power exertion.

Keywords: crude oil, OPEC, oligopoly, Stackelberg market, market power, consistent conjectural variations, equilibrium model

JEL Codes: C61, C72, L71

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1 Introduction

For the past four decades, oligopolistic behaviour in the crude oil market has been a recurring theme in the economic literature. Various theories were repeatedly tested to understand and explain the behaviour of the Organization of the Petroleum Exporting Countries (OPEC) or Saudi Arabia, its most prominent member. However, the aggregate of these studies is inconclusive at best and contradictory at worst, as summarized by Smith (2005), Alhajji and Huettner (2000a), and Griffin (1985).

The contribution of this paper is threefold: first, I discuss how current large-scale equilibrium models over-simplify and, in a way, misinterpret market power exertion. Then, I propose a more elaborate approach than the standard Nash-Cournot oligopoly to model strategic behaviour when a competitive fringe is present – namely a two-stage game with several Stackelberg leaders that anticipate the reaction of the fringe. Third, this model is applied to the global crude oil market. By representing strategic behaviour using the two-stage game, the market power of OPEC members increases endogenously as the spare capacity of the fringe (i.e., the non-OPEC suppliers) goes down.

Thereby, this article ties into the discussion of the crude oil price increase over the past decade, culminating in the price spike of 2008. This phenomenon initiated a wide discussion in the academic literature regarding its causes, and whether it was rather driven by speculation or fundamentals of supply and demand. While Kaufmann and Ullman (2009) argue that speculation was an important factor for the price spike, Fattouh et al. (2013), Alquist and Gervais (2013), Hamilton (2009), Smith (2009) and Wirl (2008), amongst others, disagree and identify other, more important drivers: low demand elasticity, strong growth of newly industrialized countries, and insufficient production capacity expansion. The results of this work lend support to the latter view, but add increased market power of OPEC as an explanation.

OPEC first gained notoriety in the seventies and eighties. At that time, optimization and equilibrium models were widely applied, both theoretically (e.g., Salant, 1976; Newbery, 1981) and numerically (e.g., Salant, 1982). These models usually combined a Hotelling-style exhaustible resources approach and Nash-Cournot or Stackelberg market power. Equilibrium models subsequently went out of fashion – for two reasons, I believe: first, the failure of the oil price to follow the path projected by a Hotelling-type model; and second, the debate regarding the consistency of Nash-Cournot equilibria. I will discuss both issues in more detail below.

With the liberalization of the electricity and natural gas markets in Europe, Nash-Cournot equilibrium models were again widely used in large-scale numerical energy market applications. This was due to advances in algorithms and computation power, which allowed to drop many simplifications necessary in the early models. Recent applications for the natural gas market include the World Gas Model (Gabriel et al., 2012b; Egging et al., 2010), GaMMES (Abada et al., 2013), and Gastale (Lise and Hobbs, 2008). There were also a number of models for electricity markets (e.g. Neuhoff et al., 2005; Bushnell, 2003), and – more recently – the global coal markets gained some attention (Trüby and Paulus, 2012; Haftendorn and Holz, 2010).

There are three recent numerical partial equilibrium models for the crude oil market: Aune et al. (2010) present a dynamic equilibrium model in which both production and investment decisions of OPEC are strategic. They emphasize the requirement by financial markets that certain profitability measures are fulfilled

so that investment can take place. Al-Qahtani et al. (2008), in contrast, focus on the role of Saudi Arabia; it is the only player that can behave strategically, while all other OPEC members charge an exogenously determined mark-up on top of marginal production costs.

Huppmann and Holz (2012) propose a spatial model for the crude oil market to compute prices and quantities produced and consumed, as well as trade flows, under different market structure assumptions over the time horizon 2005–2009. The approach includes arbitragers to account for liquid spot markets, which are an important characteristic of the global crude oil market. They find that a non-cooperative Nash-Cournot oligopoly by OPEC suppliers with Saudi Arabia as a Stackelberg leader and a competitive fringe best describes the crude oil market before the financial turmoil and the onset of a global recession in 2008; afterwards, the market was closer to the competitive benchmark.

Almoguera et al. (2011) approach the question of OPEC market power from the empirical side: rather than computing equilibria from fundamental cost and demand functions as it is done in the numerical work of Huppmann and Holz, they estimate the cost and mark-up parameters from a dataset ranging from 1974–2004. They also find evidence that OPEC is a non-cooperative oligopoly with a competitive fringe. However, their approach cannot capture two-stage market power in the Stackelberg sense, and they do not include the possibility of a shift in market power over time; instead, their approach only draws conclusions on the average behaviour over the entire period. These are the two issues I tackle in this work: the two-stage game aspect, where several Stackelberg leaders anticipate the reaction of the competitive fringe; and the changing level of market power exertion depending on the spare capacity of the fringe.

I proceed as follows: first, I give an account of the debate concerning the consistency of a Nash-Cournot oligopoly and, more generally, conjectural variations. Then, I elaborate on the crude oil market and identify three characteristics that make it particularly interesting for the proposed Stackelberg oligopoly setting. Next, I formulate a simple bathtub model and derive conditions for equilibria in four different non-cooperative oligopoly market structures. Finally, I compute quarterly equilibria from 2003–2011 using these models and discuss how the market power of OPEC members changed before and after the financial crisis according to the proposed two-stage oligopoly setting.

2 Modelling market power in quantity games

It is quite natural to model fossil resource markets as a game in quantities. There are two standard cases: all players act perfectly competitive, i.e. they set price equal to marginal cost; and the Nash-Cournot equilibrium, where each player exerts market power, taking into account the reaction of the demand on its decision, while assuming that all rivals do not deviate from their quantity.¹

Bowley (1924) and Frisch (1933) proposed *conjectural variations* (CV) as a way to elegantly model “intermediate” cases of imperfect competition or market power: instead of simply adding the mark-up warranted by a Cournot model on top of marginal costs in the price-setting of a supplier, a parameter is introduced to capture the expectation (or conjecture) of a supplier regarding the

¹One could also model the crude oil market as a game in supply functions (Klemperer and Meyer, 1989), but this would be mathematically challenging given the specific cost function, and distract from the main focus of this paper. I therefore choose to remain in the realm of quantity games.

reaction (or variation of output) of the rivals. Setting this parameter accordingly allows to model a continuum of market power cases, ranging from the perfectly competitive market to the non-cooperative Nash-Cournot equilibrium to the cooperative cartel solution.²

However, an equilibrium computed from such an arbitrarily chosen, exogenous parameter is not based on any economic theory (cf. Figuères et al., 2004). Hence, a consistency problem arises: the conjecture of any agent need not be correct, i.e., it may not coincide with the actual reaction of the rival(s) (Laitner, 1980). In particular, in a Nash-Cournot equilibrium, each player follows the conjecture that all rivals will not react to any deviation. But in fact, when any player deviates from the equilibrium (by making a mistake, for instance), all rivals will also update their decision.

This observation initiated a stream of research, which required the conjectures to be consistent or rational in equilibrium (e.g. Bresnahan, 1981; Perry, 1982). These extensions were subsequently criticized as well, since the rationality argument requires circular reasoning or information about the rival's cost function. To put it simply: in equilibrium, no agent can know how its rivals would respond to any deviation since no deviation ever actually occurs (cf. Makowski, 1987; Lindh, 1992).

Myopic strategic behaviour

Figuères et al. (2004) argue that, while an equilibrium based on CV may be in some sense arbitrary, it still offers a useful “shortcut” to capture more complex strategic interaction between players within a static framework. However, the applied partial equilibrium models for natural gas and other energy markets mentioned in the introduction depart in one important way from the theoretical models: these application are usually interested in intermediate cases of non-cooperative oligopolistic behaviour, where some suppliers exert market power while others form a competitive fringe. This is commonly captured by assigning different conjectural variations to distinct suppliers, though these models have usually dropped the CV terminology and directly refer to “suppliers exerting Cournot market power” and “competitive or price-taking suppliers” (cf. Gabriel et al., 2012a).

As shown by Ulph and Folie (1980), such an approach may yield rather counter-intuitive effects. They compare a competitive baseline to two models: first, one supplier acts as Cournot oligopolist vis-à-vis a competitive fringe, and treats the quantity supplied by the fringe as given; for the remainder of this discussion, I will refer to such a model as *Myopic Cournot Equilibrium* (MCE).³ The authors show that under certain – not implausible – conditions, the myopic Nash-Cournot oligopolist in the MCE model earns lower profits than if he were to follow a competitive price rule (i.e., price equals marginal cost). This occurs because the myopic oligopolist does not consider that the fringe player will partly offset the quantity withheld.⁴

In the equilibrium of an MCE model, unilateral deviation would not improve the profits of the Nash-Cournot supplier(s); thus, the oligopolists fulfil the Nash equilibrium condition, while the competitive fringe follows the competitive price

²A more extensive review of conjectural variations, including the mathematical formulation commonly used, is given in Haftendorn (2012) and Ruiz et al. (2010).

³The term “myopic” indicates that the oligopolistic supplier does not consider the reaction of the rivals.

⁴A numerical example of this effect is shown in Gabriel et al. (2012a, p. 108).

rule by assumption. But it is rather unsatisfactory to assume that certain suppliers pursue a strategy that leaves them worse off in equilibrium – and then claim that these players exert market power, as it is usually done in partial equilibrium models that apply such an approach. Therefore, one must be rather careful in describing such a model as a representation of market power (cf. Ralph and Smeers, 2006).

A Stackelberg market is a two-stage game

In the second model studied by Ulph and Folie (1980), the supplier that exerts market power is a Stackelberg leader that takes into account the reaction of the fringe, rather than just the quantity it supplies. The model proposed in this work follows the intuition of this two-stage model: several Stackelberg leaders anticipate the reaction of the fringe in their optimization model.⁵

Mathematically, this yields a two-stage problem, and this can be treated formally as a Mathematical Problem under Equilibrium Constraints (MPEC), if there is one player in the upper-level problem, or Equilibrium Problem under Equilibrium Constraints (EPEC), if there are several players that interact non-cooperatively. EPECs have been proposed as the suitable approach to model electricity markets (Ralph and Smeers, 2006; Hu and Ralph, 2007), as well as more general hierarchical games (Kulkarni and Shanbhag, 2013).

In this work, I propose a Stackelberg oligopoly model to properly capture market power exertion by OPEC members. They form a non-cooperative oligopoly amongst each other, but anticipate – in the Stackelberg sense – the reaction of the competitive fringe. This is accomplished by implicitly including the reaction function of the fringe in each oligopolist’s profit maximization problem: the oligopolists have consistent conjectures regarding the fringe. But before turning to the model itself, I discuss several features and characteristics of the crude oil market that make it a particularly interesting and relevant application.

3 The crude oil market

The market structure in the crude oil sector in general and the role of OPEC, in particular, is still surrounded by controversy. As discussed in the introduction, the crude oil market underwent drastic upheaval in 2007–2008, and I believe that the proposed model sheds some light on this. In addition to these more general reasons, there are three aspects that have theoretical and practical import.

Three reasons why oil is interesting

A credible Stackelberg leader

The notion of a two-stage, Stackelberg game, in which one agent decides first taking the reaction of its rivals into account, is quite straightforward in theory. However, when applied to real world problems, one must argue carefully whether the two-stage setting is plausible – put differently, whether the commitment of

⁵One alternative approach to including the first-order optimality conditions of the fringe in a Nash-Cournot oligopoly model is to subtract the quantity supplied by the fringe from the demand, and let the oligopoly face the residual demand curve. This is, however, problematic if the fringe supply cannot easily be computed (cf. Bushnell, 2003).

the leader to maintain its decision is credible. Otherwise, the game would revert, following a tâtonnement process, to a Nash-Cournot equilibrium.

Almoguera et al. (2011) argue that OPEC can signal through its quota changes, both to other suppliers and to traders in the downstream market; this is the case even if the quota is not strictly adhered to by OPEC members (Dibooglu and AlGudhea, 2007). The quota allocations are only changed every couple of months, hence it is rational for other suppliers to assume that their short-term production decision will not affect the quota and hence OPEC output.

Instantaneous reactions & epistemology

The concept of consistent conjectural variations requires, in principle, that each player reacts instantaneously. However, instantaneous reactions are difficult to reconcile with most actual markets, due to rigidities and lack of information. This is not so in the crude oil market. As I have argued, OPEC can credibly commit to a quota for an extended period of time; in contrast, crude oil is traded in very liquid markets at a high frequency, so the followers – not bound by a quota – feel the impact of their output decisions virtually immediately. This is, I believe, sufficiently close to instantaneous to warrant the use of consistent conjectural variations in this application.

There is one further aspect of both Stackelberg leadership and the use of consistent conjectural variations: the requirement that the leader knows the actual reaction of the rivals – and not just the equilibrium quantity as in a standard Nash-Cournot game. Again, the crude oil sector satisfies this requirement: the market for oil-related services – such as suppliers and operators of oilfield equipment, firms specializing in exploration, and business intelligence providers – is quite concentrated and the OPEC members, collectively, have substantial expertise. Hence, it is reasonable to assume that the OPEC members have a rather good understanding of their rivals' operations and cost structure, and can therefore predict their reactions to a price change.

Non-standard cost functions

In most theoretical and applied work on oligopoly theory, either linear or quadratic cost functions are used. This facilitates some proofs, but in combination with linear demand curves leads to a very strong simplification: the derivative of the optimality condition of each player, and hence the derivative of each player's reaction function, is constant. This translates to constant consistent conjectural variations.

When looking at crude oil production costs – and extractive industries in general – one notices that marginal production costs are quite flat for most of the feasible range, but then increase sharply when producing close to capacity. There are both engineering explanations, such as the need for additional equipment, increased wear-and-tear, more complex technology (water or CO₂ injection), as well as economic reasons: pumping oil too quickly leads to a deterioration of reservoir quality and even a decrease of recoverable resources. A production cost function that exhibits these characteristics will be formally introduced below. The important aspect, which is driving the model, is that the consistent conjecture is not constant any more.

The Hotelling rule isn't in the details

The above argument camouflages what many economists may consider a major omission in this work: theory postulates that when supplying a finite resource, its price must rise in lock-step with the rate of interest due to the consideration of inter-temporal arbitrage. This is known as “Hotelling rule” (Hotelling, 1931), and virtually all theoretical models use it in one way or another (e.g., Salant, 1976; Hoel, 1978; Newbery, 1981). Nevertheless, the real crude oil price fails to exhibit an exponential price increase over the long-term. Hart and Spiro (2011) and Livernois (2009) review extensions of the Hotelling rule to rationalize this phenomenon: these include technological progress, a backstop technology, increasing costs relative to remaining reserves, and uncertainty. The authors also cite empirical work that attempts to identify the scarcity rent as postulated by the Hotelling rule. They conclude – quite forcefully – that the Hotelling rule is of minor importance in today’s crude oil market.

This work implicitly assumes that only short-term scarcity rent (i.e., insufficient production capacity and the resulting high marginal costs) is a major driver of oil prices, but that long-term scarcity rent (i.e., rents due to the exhaustibility of crude oil) are negligible. For simplicity, I therefore neglect all inter-temporal considerations other than what can be captured in the production cost function, as discussed above. The model presented in this work is – in each period – a one-shot quantity game comparing different behavioural assumptions. Capacity is fixed and exogenously given; I abstract from investment in new production capacity due to the significant lead-time. I will discuss possible extensions in the last section.

4 A bathtub model

A simple model is used to describe and compare several instances of non-cooperative supplier behaviour in the global crude oil market. As there are several suppliers of crude oil, but only one aggregated demand function and one global price for crude oil, such a model is usually called a bathtub model: several faucets, but only one drain. This simplification is frequently used in crude oil market analysis, in spite of quality differences and transport costs.

There is a set of suppliers that may form an oligopoly, denoted by S . In addition, there is one (aggregated) fringe supplier, f . For general notation relating to all suppliers, I use the indices i, j without stating the set, i.e., to be read as $i, j \in \{S \cup f\}$.

The profit maximization problem of a supplier i can be written as follows:

$$\max_{q_i \in \mathbb{R}_+} p(Q)q_i - c_i(q_i) \quad (1)$$

Here, q_i is the quantity produced by that supplier, while Q is the total quantity supplied to the market. Price depends on total quantity supplied, given by an inverse demand function $p(\cdot)$, and production costs are denoted by $c_i(q_i)$.

The first-order optimality condition (also called Karush-Kuhn-Tucker, or KKT, condition) of supplier i is then given by:

$$p(\cdot) + p'q_i + p' \frac{\partial q_{-i}(q_i)}{\partial q_i} q_i - c'_i(q_i) \leq 0 \perp q_i \geq 0 \quad (2)$$

The production decision of a supplier i implicitly impacts the quantity produced by its rivals; hence, I write $q_{-i}(q_i)$ to express this effect. As is common

in the conjectural variations literature, the term $r_i := \frac{\partial q_{-i}(q_i)}{\partial q_i}$ denotes the conjecture of player i regarding the aggregated reaction of the rivals. Note that this conjecture need not be correct. I distinguish three different conjectures: price-taking behaviour, where the supplier assumes to have no impact on the price; the Cournot conjecture, where the supplier believes to have no impact on the quantity supplied by its rivals, but considers the reaction of the price to his decision; and correct (i.e., consistent) conjectures, where the conjecture coincides with the actual reaction of (some of) the rivals.

For any solution q_i^* to the KKT condition (2) to be indeed a (local) maximum, rather than only a stationary point, one also has to check whether the profit function is concave (at that point).⁶ This holds if the second derivative of the profit maximization problem is lower or equal than 0 (at that point). The condition is stated below; for simplicity, I assume that the second derivative of the price with respect to quantity is zero (i.e., a linear demand function). This will be formalized later.

$$p' \left(2 + 2 \frac{\partial q_{-i}(q_i^*)}{\partial q_i} + \frac{\partial^2 q_{-i}(q_i^*)}{\partial q_i^2} \right) - c_i''(q_i^*) \leq 0 \quad (3)$$

This condition will be discussed in more detail after the specification of the different market power assumptions. Before I proceed, the functional form of the inverse demand and cost functions are specified by the following assumptions:

- A1** The inverse demand function is linear, its slope is negative, and quantities from different suppliers are perfect substitutes, i.e., $p(Q) = a - bQ$, where Q is the total quantity supplied. Parameters a and b are strictly positive.
- A2** The production cost function of each supplier i follows the form proposed by Golombek et al. (1995). It includes a logarithmic term depending on capacity utilization:

$$c_i(q_i) = (\alpha_i + \gamma_i)q_i + \beta_i q_i^2 + \gamma_i(\bar{q}_i - q_i) \ln \left(1 - \frac{q_i}{\bar{q}_i} \right) \quad (4a)$$

$$c_i'(q_i) = \alpha_i + 2\beta_i q_i - \gamma_i \ln \left(1 - \frac{q_i}{\bar{q}_i} \right) \quad (4b)$$

$$c_i''(q_i) = 2\beta_i + \gamma_i \frac{1}{\bar{q}_i - q_i} \quad (4c)$$

$$c_i'''(q_i) = \gamma_i \frac{1}{(\bar{q}_i - q_i)^2} \quad (4d)$$

The cost function parameters α_i , β_i and γ_i are strictly positive for each supplier i . The parameter \bar{q}_i is the maximum production capacity.

Lemma 1. *Under Assumption A2, the range of feasible production quantities is implicitly bounded from above by the capacity \bar{q}_i .*

Lemma 2. *Under Assumption A2, the cost function (4a) and its first, second and third derivative (4b–4d) are strictly positive, strictly monotone and strictly convex for any feasible production quantity $q_i \in (0, \bar{q}_i)$.*

Following Assumption A1, we can rewrite Equation (2):

$$a - b \sum_j q_j - b(1 + r_i)q_i - c_i'(q_i) \leq 0 \perp q_i \geq 0 \quad (5)$$

⁶Quasiconcavity of the profit function cannot be guaranteed in general, hence multiple local maxima may exist; this will be illustrated in Example 8.

Hence, the optimal output decision of supplier i is determined by the price in relation to production costs, adjusted by its (conjectured) impact on the price. This adjustment term can be interpreted as the mark-up that the supplier charges in addition to its marginal costs.

The oligopoly cases

I distinguish four cases of oligopoly: they differ in the assumptions of each supplier regarding the reaction of its rivals to any variation in his output. The first two cases are the “pure” oligopoly theories in the perfect competition and Cournot sense, respectively. The third case is the myopic oligopoly model with a competitive fringe, MCE, as discussed before. These three cases are well studied in theory and frequently applied in numerical equilibrium models.

The fourth case is the addition to the literature by this work: a Nash-Cournot oligopoly, where each oligopolistic supplier has consistent conjectures regarding the reaction of the fringe. Furthermore, the consistent conjecture is not constant, but depends on the quantity supplied by the fringe due to the choice of cost function. As a consequence, the mark-up charged by oligopolistic suppliers in addition to marginal costs changes endogenously depending on the capacity utilization of the fringe.

Perfect competition

Each supplier assumes that his decision does not influence the market price, and he treats the price as a parameter. This is usually called “price-taking behaviour”. Therefore, the first order condition reduces to $p \leq c'_i(q_i) \perp q_i \geq 0$. This can be mimicked in Equation 5 by setting $r_i = -1$.

Nash-Cournot oligopoly

Each supplier, including the fringe, assumes that its actions have no impact on the actions of his rivals: $r_i = 0$. Each supplier acts as a monopolist with respect to the residual demand curve given the quantity supplied by the rivals.

Nash-Cournot oligopoly with fringe

The suppliers that are members of the oligopoly (i.e., OPEC) act as Nash-Cournot players ($r_i = 0 \forall i \in S$); the fringe acts as price-taker ($r_f = -1$). This is the Myopic Cournot Equilibrium (MCE) discussed previously.

Lemma 3. *Under Assumptions A1 and A2, the KKT system (5) has a unique solution in each of the cases Perfect competition, Nash-Cournot oligopoly and Nash-Cournot oligopoly with fringe. The second-order derivative condition (3) holds everywhere on the feasible region.*

Proof. The Jacobian matrix of the KKT system is symmetric and positive definite, hence existence and uniqueness is established (cf. Facchinei and Pang, 2003).

Realizing that r_i is constant by assumption and its partial derivative is thus 0, the second-order derivative condition (3) can be written as follows:

$$-b(2 + r_i) - c''_i(q_i) \leq 0$$

As $r_i \in [-1, 0]$ and $c''_i(q_i) > 0$ following Lemma 2, this condition holds trivially with strict inequality for all $q_i \in [0, \bar{q}_i]$. The profit function of each supplier is thus strictly concave on the feasible region. \square

In all of these cases, each supplier does not distinguish between its rivals; it has an aggregated conjecture regarding the total response. I now turn to an oligopoly that has a more elaborate approach to strategic behaviour.

Consistent conjecture oligopoly with fringe

The consistency requirement postulates that the conjecture of the player must be “correct”, i.e., it must be equal to the actual reaction of the rivals to a variation in quantity. There is a conceptual difficulty in games with more than two players, as each rival’s reaction in turn depends on the reaction of the other players. Kalashnikov et al. (2011), Ruiz et al. (2010), and Liu et al. (2007) all assume that all suppliers have consistent conjectures regarding all rivals (with the exception of a social welfare-maximizing player in the first article). Then, they derive closed-form expressions for this term under the assumption of quadratic cost functions and a linear demand curve.

In contrast, I use the idea of consistent conjectural variations to model a two-stage oligopoly: an oligopoly takes into consideration the reaction of a competitive fringe, but follows the Cournot conjecture amongst each other. Before formalizing this, I need to introduce some additional notation. Following Liu et al. (2007), the conjecture regarding the aggregated rivals’ reaction can be separated:

$$r_i := \frac{\partial q_{-i}(q_i)}{\partial q_i} = \sum_{j \neq i} \frac{\partial q_j(q_i)}{\partial q_i} =: \sum_{j \neq i} r_{ij}$$

This term states that the aggregated reaction of the rivals can be separated into the sum of individual responses of each rival j , r_{ij} . Now, let $\rho_j(q_i)$ denote the actual reaction function of supplier j to the quantity supplied by supplier i , in contrast to $q_j(q_i)$ previously used, which denotes the conjecture of player i regarding the response of supplier j . Now I can state the assumptions underlying the oligopoly with consistent conjectures regarding the fringe formally.

A3 The oligopoly suppliers $i \in S$ follow the Cournot conjecture amongst each other and have consistent conjectures regarding the fringe:

$$\begin{aligned} \frac{\partial q_j(q_i)}{\partial q_i} &= 0 & \forall j \in S \\ \frac{\partial q_f(q_i)}{\partial q_i} &= \frac{\partial \rho_f(q_i)}{\partial q_i} \end{aligned}$$

The fringe supplier f follows a competitive pricing rule: $r_f = -1$.

In addition, I need a restriction on the cost parameters of the fringe player to simplify the following notation.

A4 The marginal cost of the fringe supplier at zero production is strictly less than the price at maximum production of the oligopoly, which is the minimum possible price if the fringe player does not produce; mathematically:

$$c'_f(0) = \alpha_f < p \left(\sum_{i \in S} \bar{q}_i \right)$$

Lemma 4. *Under Assumptions **A1**, **A2** and **A4**, the fringe supplier always produces a positive quantity q_f in equilibrium.*

Proof. Assume that $q_f = 0$. Starting from Equation (5) and replacing $c'_f(0)$ by $p(\sum_{i \in S} \bar{q}_i)$ according to Assumption **A4** yields the following:

$$a - b \sum_{j \in S} q_j - c'_f(0) > a - b \sum_{j \in S} q_j - a + b \sum_{j \in S} \bar{q}_j = b \sum_{j \in S} (\bar{q}_j - q_j) > 0$$

This is a contradiction to the first-order condition of the fringe supplier. \square

This lemma has an important interpretation: the oligopoly cannot force the fringe supplier out of the market, and q_f will always be positive in equilibrium. Assumption **A4** and Lemma 4 allow us to omit the rather tedious case of establishing conjectures if a supplier is not producing, or of limit-pricing strategies by a monopolist (cf. Hoel, 1978).

Lemma 5. *Under Assumptions **A1**, **A2**, **A3** and **A4**, each oligopoly supplier's conjectural variation equals the reaction of the fringe supplier, and it has the following functional form:*

$$r_i = \sum_{j \neq i} r_{ij} = \frac{\partial q_f(q_i)}{\partial q_i} = -\frac{b}{b + c''_f(q_f)} \in (-1, 0) \quad \forall i \in S$$

Furthermore, r_i is continuous with respect to $q_f \in [0, \bar{q}_f]$.

Proof. Following Assumption **A4** and Lemma 4, the first-order condition for the fringe supplier must hold with equality. Furthermore, this equality implicitly defines the fringe's output as a reaction to the output by firm i .

$$a - b \left(q_i + \sum_{j \in S \setminus \{i\}} q_j + q_f(q_i) \right) - c'_f(q_f(q_i)) = 0 \quad (6)$$

According to Assumption **A3**, each oligopoly supplier conjectures that the other oligopoly suppliers do not react to its output variation; hence, I write q_j rather than $q_j(q_i) \forall j \in S \setminus \{i\}$.

Taking the derivative of Equation (6) with respect to the output of an oligopoly supplier q_i , $i \in S$, and using the implicit function theorem yields the optimal response of the fringe to a variation in output by supplier i :

$$\begin{aligned} -b - b \frac{\partial q_f(q_i)}{\partial q_i} - c''_f(q_f) \frac{\partial q_f(q_i)}{\partial q_i} &= 0 \\ \Rightarrow r_{if} := \frac{\partial q_f(q_i)}{\partial q_i} &= -\frac{b}{b + c''_f(q_f)} \end{aligned} \quad (7)$$

Each oligopoly supplier conjectures that every rival apart from the fringe supplier does not react, hence its conjectural variation term reduces to the conjecture regarding the fringe. Noting that $c''_f(q_f) > 0 \forall q_f \in [0, \bar{q}_f]$ and continuous yields $r_i \in (-1, 0)$ and the continuity of r_i . \square

Following Lemma 5, I can rewrite the system of suppliers' first-order conditions (5) as follows:

$$-a + b \sum_j q_j + b \left(1 - \frac{b}{b + c''_f(q_f)} \right) q_i + c'_i(q_i) \geq 0 \perp q_i \geq 0 \quad \forall i \in S \quad (8a)$$

$$-a + b \sum_j q_j + c'_f(q_f) \geq 0 \perp q_f \geq 0 \quad (8b)$$

Allow me to briefly discuss the term $r_i = r_{if} = -b/b+c'_f(q_f)$, the fringe's reaction, and relate it to earlier consistent conjectural variations literature. This term is similar to the examples discussed by Bresnahan (1981) and others if one uses quadratic or symmetric linear costs. In these cases, Assumption **A5** (see below) will hold trivially in the absence of capacity constraints. The equilibrium would not be as straightforward, however, in the case of asymmetric linear costs; in such a case, one would have to consider limit pricing or other more elaborate formulations.

Furthermore, due to the cost function used here, the fringe's reaction is not constant, and it is here that the proposed model departs from the previous literature – and it is here where two ambiguities arise, compared to the other oligopoly cases discussed before: first, uniqueness is not guaranteed, and the second-order derivative condition (Equation (3)) does not necessarily hold everywhere on the feasible region (i.e. the profit function may not be concave or not even quasi-concave). The second-order derivative condition is included in the theorem below, while the following assumption will provide a condition and test for uniqueness.

A5 Assume that the parameters satisfy the following inequality, where x, y are feasible production vectors:

$$\begin{aligned} & b \left(\sum_j (x_j - y_j) \right)^2 + \sum_j (c'_j(x_j) - c'_j(y_j)) (x_j - y_j) \\ & + b \sum_{i \in S} \left[\left(1 - \frac{b}{b + c''_f(x_f)} \right) x_i - \left(1 - \frac{b}{b + c''_f(y_f)} \right) y_i \right] (x_i - y_i) > 0 \\ & \forall x, y \in \prod_j [0, \bar{q}_j], x \neq y \end{aligned}$$

Theorem 6. *Under Assumptions **A1**, **A2**, **A3** and **A4**, a solution $((q_i^*)_{i \in S}, q_f^*)$ to the KKT system (8) always exists. It is indeed an equilibrium if it satisfies the second-order derivative condition:*

$$-b \left(2 - 2 \frac{b}{b + c''_f(q_f^*)} - \frac{b^2 c'''_f(q_f^*)}{(b + c''_f(q_f^*))^3} q_i^* \right) - c''_i(q_i^*) \leq 0 \quad \forall i \in S \quad (9)$$

*Furthermore, if Assumption **A5** is satisfied, the solution is unique.*

Proof. See Appendix A.

Before discussing the intuition and practical verification of Assumption **A5** for given parameters, I would like to refer to one other potential avenue to prove uniqueness: Sherali et al. (1983) present a model of one Stackelberg leader and a number of Cournot followers. They show that the total quantity supplied by the (lower-level) Nash-Cournot oligopoly is – as a function of the leader's quantity – unique, convex and decreasing under certain assumptions. If, in addition, the cost function of the leader is strictly convex, the leader's problem is a strongly concave maximization problem, and the problem has a unique solution.

However, the logarithmic cost function used in this work to represent the characteristics of extractive industries leads to a violation of the convexity of

the lower-level's response. This would be true for any similar function with the desired properties for extractive industries, e.g., a piece-wise linear approximation of Equation (4b), as well as any model with capacity constraints in the lower level.

When solving a particular application of the model proposed here, how can one determine whether the parameters derived from the data satisfy Assumption **A5**, and hence that the solution obtained is unique? First, note that the Jacobian $J(\cdot)$ of the KKT system (8) is neither symmetric nor necessarily positive definite. Hence, one cannot proceed as easily as in Lemma 3.

I therefore propose an alternative, numerical approach, namely requiring strong monotonicity of the equivalent Variational Inequality (VI, discussed in the Appendix) – this is the interpretation of Assumption (**A5**). The first term is a square, hence positive. Because the marginal cost function is strictly monotone (cf. Lemma 2), the second term is a sum of strictly positive terms. The sign of the third term, however, is ambiguous. Furthermore, the entire term is not convex.

Nevertheless, whether Assumption (**A5**) holds for given parameters of a numerical application can be verified by solving the following optimization problem:

$$\begin{aligned} \min_{\substack{x, y \in K \\ x \neq y}} & b \left(\sum_j (x_j - y_j) \right)^2 + \sum_j (c'_j(x_j) - c'_j(y_j)) (x_j - y_j) \\ & + b \sum_{i \in S} \left[\left(1 - \frac{b}{b + c''_f(x_f)} \right) x_i - \left(1 - \frac{b}{b + c''_f(y_f)} \right) y_i \right] (x_i - y_i) \quad (10) \end{aligned}$$

The objective value is 0 for $x = y$. Hence, if the infimum of problem (10) is also equal to zero, the KKT system (8) has a unique solution. It would be more elegant, of course, to present a closed-form expression of sufficient assumptions for a unique equilibrium. However, I could not (yet) find a practical approach.

5 A simple numerical example

In order to illustrate the differences between the oligopoly cases and the issue of non-uniqueness, I present two simple examples of two-player games. Supplier 2 acts as competitive fringe, while supplier 1 is an oligopolist exerting market power using the different conjectures discussed before: myopic Cournot behaviour (*MCE*), the Stackelberg leader-follower behaviour implemented using consistent conjectures regarding the fringe (*CCV*), and – as a benchmark – perfectly competitive behaviour (*PC*).

The first example serves to illustrate two points: the optimal exertion of market power “converges” to the Nash-Cournot solution when the fringe player reaches its capacity limit; and the profit earned when market power is exerted in the Stackelberg sense is always higher than under myopic Cournot behaviour.

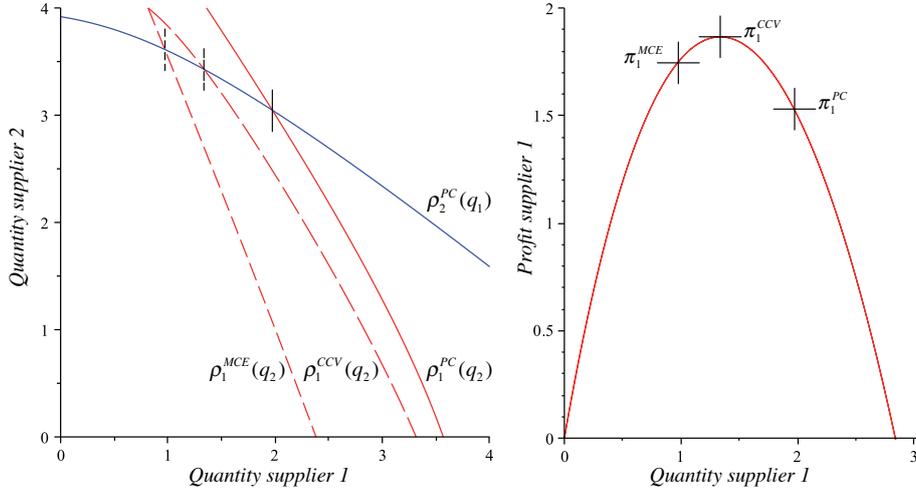


Figure 1: Illustration to Example 7

Example 7. Assume two suppliers 1 and 2, facing the following cost curves and inverse demand function:

$$\begin{aligned}
 c_1(q_1) &= (1 + 1)q_1 + 0.2q_1^2 + 1(4 - q_1) \ln \left(1 - \frac{q_1}{4}\right) \\
 c_2(q_2) &= (1 + 0.6)q_2 + 0.1q_2^2 + 0.6(4 - q_2) \ln \left(1 - \frac{q_2}{4}\right) \\
 p(Q) &= 10 - 1.5Q
 \end{aligned}$$

The capacity limit of each supplier is 4 units.

The reaction functions are shown in the left-hand part of Figure 1: $\rho_i(q_j)$ is the reaction function of player i to the quantity supplied by player j , where the superscripts refer to the market power case.⁷ The equilibria are marked by vertical lines. This figure illustrates how the optimal response of the Stackelberg leader converges from the competitive case, if the fringe (supplier 2 in this example) is not constrained, to the Nash-Cournot reaction function when the fringe approaches its capacity limit.

The right-hand part of Figure 1 illustrates the profit of the oligopolist considering the reaction of the competitive fringe supplier. In this example, the profit generated under the myopic Cournot conjecture is indeed higher than the profit under competitive marginal-cost pricing. Many applied studies discussed in the introduction jump, from this observation, to the conclusion that MCE behaviour is equivalent to the optimal exertion of market power. This figure illustrates that this is not the case: instead, producing more than under the MCE conjecture yields higher pay-off for the oligopolistic supplier, with the maximum attained at the consistent conjectural variations equilibrium.

The second example illustrates that there may exist several solutions at which the first- and second-order conditions are satisfied.

⁷The reaction functions are computed by solving the first-order condition given the quantity of the other player.

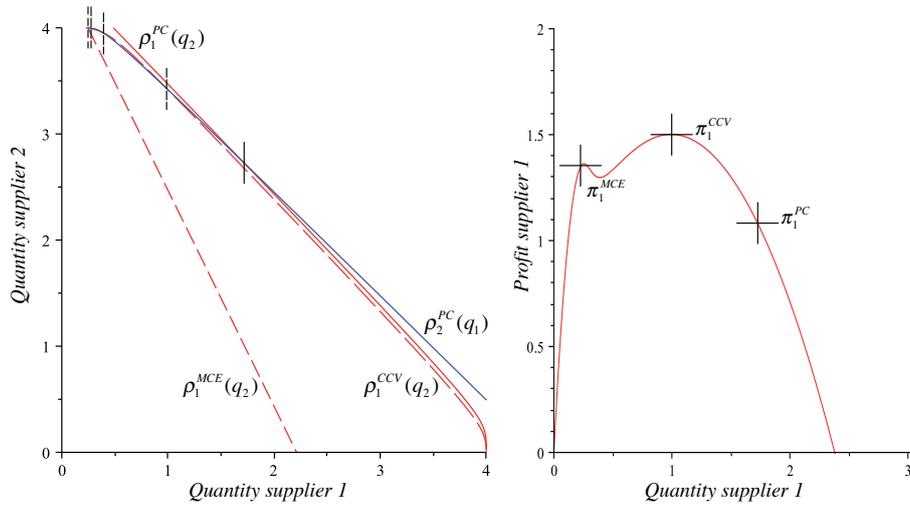


Figure 2: Illustration to Example 8

Example 8. The assumptions regarding the suppliers are identical to Example 7, but the inverse demand function is $p(Q) = 100 - 22Q$. As illustrated in Figure 2, the reaction functions $\rho_2^{PC}(q_1)$ and $\rho_1^{CCV}(q_2)$ now intersect multiple times. There exist two equilibria, and one point where the first-order conditions of both suppliers are satisfied, but the second-order derivative condition is violated. The latter point is actually a local profit minimum, as can be seen in the right-hand part of Figure 2. It is obvious that the profit curve of the oligopolistic supplier is not quasi-concave.

It is straightforward that in this example, a Stackelberg leader that satisfies the epistemological qualifications (i.e., has sufficient knowledge to exert market power in the sense discussed here) would choose the equilibrium where it produces more, earning π_1^{CCV} . It is less clear, however, whether a numerical solver would find this equilibrium. If it were to terminate in the other equilibrium, that would be unfortunate; if, however, it would terminate in the other solution, this would be outright wrong, as this is not a Nash equilibrium. Indeed, when solving this problem in GAMS and setting the starting values for the PATH solver accordingly, all three intersections of the ρ_1^{CCV} and ρ_2^{PC} curves could be obtained as results, and the solver claimed optimality in all cases.

6 A crude oil application

Let's now turn to the actual question of this paper: endogenous market power of OPEC suppliers over the past years. OPEC membership changed over the time period under investigation. To avoid shifts in the capacity share of OPEC, I assume the following countries to be OPEC members over the entire period: Algeria, Angola, Ecuador, Iran, Iraq, Kuwait, Libya, Nigeria, Qatar, Saudi Arabia, Venezuela, and the United Arab Emirates.⁸

⁸Angola and Ecuador (re-)joined in 2007; Indonesia suspended membership in 2009; OPEC website (http://www.opec.org/opec_web/en/about_us/25.htm, accessed Feb 14, 2013).

Data

In contrast to the model proposed in Huppmann and Holz (2012), I use quarterly data. The problem with any analysis of the crude oil market is the lack of available, reliable and consistent data, as pointed out by Smith (2005). Regarding quantities produced and consumed, I rely exclusively on IEA data, in particular the “Quarterly Statistics” (IEA, 2012, and earlier versions). Production data in the Quarterly Statistics are disaggregated by country and three oil types: crude, natural gas liquids (NGL), and non-conventional.⁹ IEA frequently updates their published data, so I only use data from publications at least 6 quarters after the fact. The categories *Global biofuels production* and *processing gains*, which are not assigned to a country in the IEA reports, are treated as if produced by independent suppliers. This data is complemented with information from the monthly IEA Oil Market Reports (OMR).¹⁰

To derive production capacity, I use the following methodology: for each period, country and oil type (crude, NGL, and non-conventional), production is such that the average over the preceding and following four quarters is 95% of capacity. If actual production is above this value due to a short-term spike, I assume that production is at 98% of capacity in this period.¹¹

For OPEC countries, the IEA publishes a measure called *sustainable production capacity* (SPC) in the OMR. The definition of the sustainable production capacity is that the production level can be reached within 30 days and be sustained for 90 days. This fits nicely with the quarterly data that I use for this analysis. If available, I use this data rather than the capacity derived from the above methodology.¹²

This methodology guarantees three important aspects: first, I capture the trends in each country and oil type; second, in each period reference production is below capacity. Third, and most importantly, this approach yields aggregate capacity time series in line with those reported by the IEA and EIA, even though spare capacity in 2008 is most likely over-estimated.

Production costs are divided into two parts: production/lifting costs, on the one hand, are derived from Aguilera et al. (2009), who estimate average production costs for a large number of fields worldwide. I assume that these costs increase by 5 % p.a.; unconventional oil is assumed to be twice as expensive as crude oil in every country. On the other hand, crude oil has to be shipped to market, and low-quality crude is traded at a discount. Shipping costs are a function of the oil price, therefore each country is assigned a scalar (ranging from 1–3 based on distance to markets and oil quality), and a linear trade cost term based on the actual crude oil price and multiplied with that scalar is added to the cost function.

To obtain the linear inverse demand curve, I use actual quarterly demand (from IEA, as above) and the global average crude oil price (obtained from Datastream, a Thomson Reuters information service) as reference demand points.

⁹Crude oil production in the “Neutral zone” are shared equally between Saudi Arabia and Kuwait, in line with IEA methodology.

¹⁰Oil market report website (<http://omrpublic.iea.org/>, accessed Feb 1, 2013).

¹¹For Iraq (2003) and Lybia (2011), actual production is assumed to be at 98% of capacity in each period to account for the war-related production stops. To consider hurricane-induced outages in the Gulf of Mexico, I assume that during the third and fourth quarter of 2005, total production capacity of the United States was reduced by 0.5 mb/d, and during the third quarter of 2008, it was reduced by 1 mb/d (cf. OMR Oct 10, 2008 and OMR Jan 17, 2006).

¹²There are instances where due to updates, actual production reported a year after the fact is higher than reported capacity in the quarter immediately after the fact. In this case, the estimate derived from actual production is used.

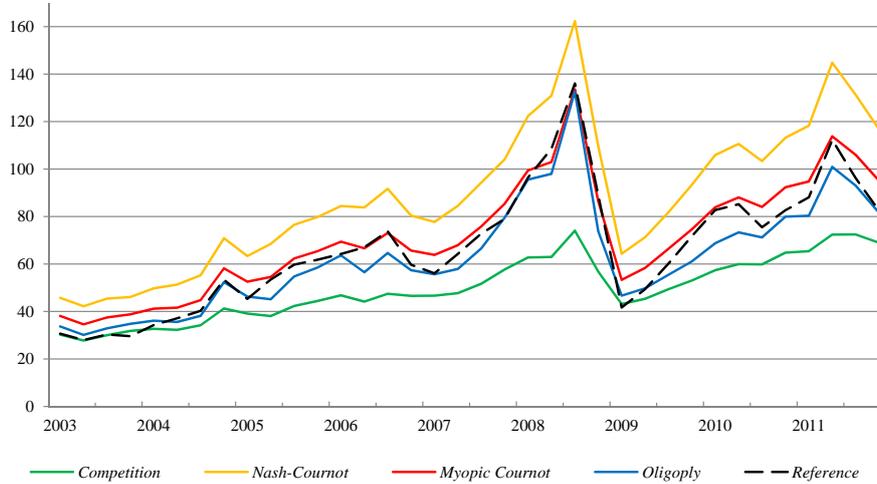


Figure 3: Equilibrium price by market power case, and reference (in US\$/bbl)

The curve is then fitted assuming a demand elasticity at the reference of point of -0.10 , as discussed by Hamilton (2009).

Results

This section presents the numerical results for the four market power cases: Perfect competition (*Competition*); Nash-Cournot oligopoly (all suppliers exert market power, *Nash-Cournot*)¹³; Nash-Cournot oligopoly with fringe (OPEC members are Cournot players in the MCE sense, *Myopic Cournot*); and consistent conjecture oligopoly with fringe (OPEC members have consistent conjectures regarding the fringe, *Oligopoly*).

The equilibrium prices computed in each of the four market power cases and the reference crude oil price are shown in Figure 3; the order of results is intuitive: *Nash-Cournot* yields the highest prices, while *Competition* exhibits the lowest. *Myopic Cournot* is in between these two extremes, albeit it moves in relative lockstep to the first two cases. In contrast, the price according to the *Oligopoly* case fluctuates between the competitive and the myopic Cournot case. In particular, it matches reasonably well the actual price path over the time period: close to competitive in 2003, converging to the myopic Cournot case until 2008, and a reversion to the competitive price benchmark after the onset of the financial crisis and the global recession.

The market power conjecture in the two-level, Stackelberg model is shown in Figure 4. The consistent conjecture of OPEC of its market power increases steadily until the third quarter of 2008, then drops drastically, and increases again over the time period 2010-2011.

The aggregate supply of OPEC is shown in Figure 5.¹⁴ In any model with

¹³For this case, the fringe was disaggregated by country; otherwise, one Cournot player would control around 60% of capacity and this would not be a plausible benchmark.

¹⁴The OPEC production capacity reduction in 2003 is due to the war in Iraq, and the drop in 2011 is due to the war in Lybia.

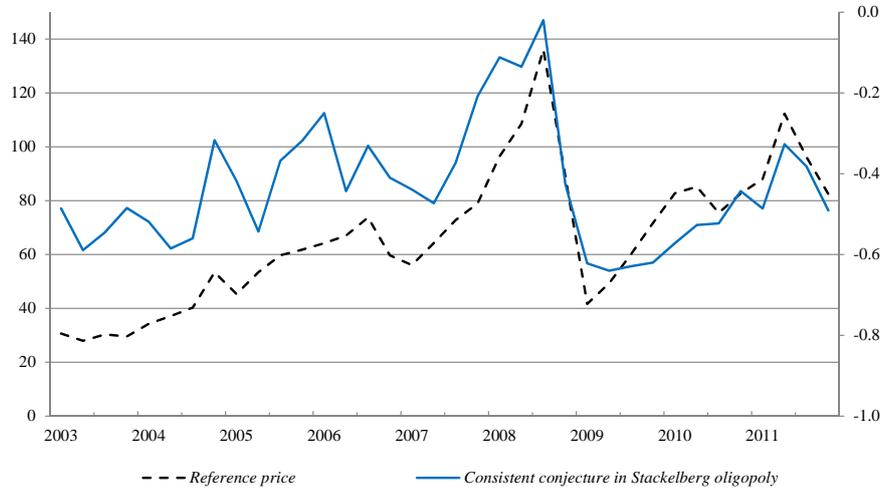


Figure 4: Crude oil reference price (left axis, in US \$/bbl), and market power conjecture by OPEC oligopolists regarding the fringe (right axis; Cournot conjecture: 0; competitive behaviour: -1)

fixed endogenous conjectures, the share of a certain supplier to total supply is roughly constant; this is due to the characteristic that the mark-up charged by each Cournot player is a constant multiplied by this supplier's own production quantity. Hence, in a demand contraction, all suppliers reduce approximately by the same relative amount. This can be seen in the numerical results in the three constant conjecture cases.

When the market power conjecture is endogenous, however, there is a countervailing effect: demand is reduced, hence there is a downward pressure on supply; at the same time, the fringe has more spare capacity when it reduces its supply, thereby leading to a lower consistent conjecture. The supplier therefore reduces the mark-up on marginal costs that it demands, and this has an expansionary effect on its supply. This expansionary effect outweighs the contraction in the current application, as can be seen over the course of the year 2008 in the simulation results. This cannot be reconciled with the actual events in that time period. In general, no market power case seems to fit the observed production levels. One could, of course, calibrate the input data and fine-tune underlying assumptions such that one oligopoly theory fits the data; but this is not the objective of this work. I conclude that the endogenous exertion of market power by OPEC suppliers may have played a role; but other factors were certainly driving the market, too, which cannot be captured by this approach.

Numerical implementation, optimality, and uniqueness

The three oligopoly cases described above (Equations 5 and 8) as well as Problem (10) are implemented in GAMS using the PATH and CONOPT solvers, respectively. One shortcoming of the consistent-conjecture oligopoly model is that the uniqueness of equilibrium cannot be easily guaranteed, as discussed in Example 8 above. Assumption **A5** does indeed not hold for most periods in this

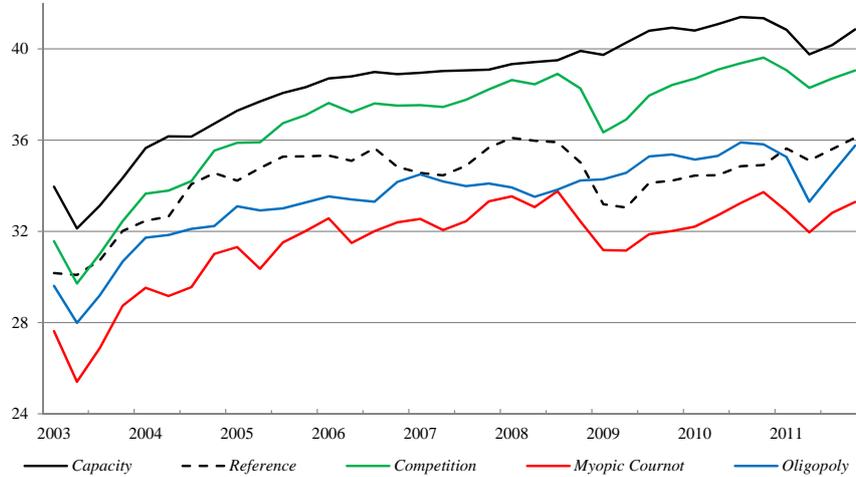


Figure 5: Aggregate OPEC supply by market power case, and reference supply (in mbb/d)

application. In order to test numerically for multiple equilibria, each simulation was initialized from various starting values, and I always obtained the same equilibrium. This supports the notion that there exists only one equilibrium in the numerical application. The second-order derivative condition holds for all oligopolists at the equilibrium in each period.

7 Conclusions

This article argues that non-cooperative strategic behaviour – as it is frequently modelled in large-scale numerical equilibrium models – is used in a flawed way when combining dominant firms and a competitive fringe. The standard approach in applied models forces players to follow a strategy that may leave them worse off in equilibrium compared to simple price-taking behaviour, but the results are then nevertheless interpreted as these players “exerting market power”. To remedy this inconsistency, and to properly model an oligopoly exerting market power considering the reaction of a competitive fringe, I propose a two-level model. Several oligopolists compete non-cooperatively following the Nash-Cournot assumption amongst each other, but take the reaction of the fringe into account – they anticipate the reaction of the fringe in the Stackelberg sense; hence the term *Stackelberg oligopoly* to describe this game. The optimal mark-up charged by the oligopolists is determined by including the consistent conjectural variation regarding the fringe in each oligopolist’s profit maximization problem. As a consequence, the optimal level of quantity withholding – i.e., market power exertion – is endogenized in the model.

Representing the crude oil market and OPEC as a dominant-firms oligopoly in the Stackelberg sense is plausible for two reasons: the OPEC quota is a credible signalling and commitment device towards the fringe; and the liquid spot markets allow virtually instantaneous reactions between prices and output

changes. In order to capture the specific characteristics of extractive industries, a logarithmic cost function is used; marginal costs increase sharply when producing close to capacity. As a result, the reaction of the fringe to a price change depends implicitly on its capacity utilization – the lower its spare capacity, the lower its reaction. Therefore, OPEC members can more easily exert market power when the fringe produces close to capacity, since their loss of market share from the reaction of the fringe is small.

I compare the two-stage Stackelberg oligopoly model to the standard equilibrium models commonly used in large-scale numerical applications: perfect competition, a Nash-Cournot oligopoly, and a myopic Nash-Cournot oligopoly with a competitive, price-taking fringe. As the focus lies squarely on supplier behaviour, I cannot make a statement regarding the causes of the increased demand in 2008 – speculation or fundamentals. Nevertheless, according to the numerical results, the Stackelberg oligopoly approach can replicate quite well the price path over the past decade: starting from a competitive level in 2003, converging to a price level elevated above marginal costs until 2008 as warranted by an OPEC oligopoly with fringe, and then dropping drastically when demand contracted with the onset of the financial crisis and a global recession. This observation of a decline in market power is in line with the conclusions of Huppmann and Holz (2012), though the two-stage model is able to explain the shift in market power endogenously through the high level of spare capacity following the price collapse in the fall of 2008 and the global recession. This high level of spare capacity reduced the optimal mark-up charged by OPEC suppliers and the Stackelberg oligopoly equilibrium was closer to the competitive benchmark.

Obviously, the lack of reliable data on the crude oil market makes any application on this sector particularly difficult. Furthermore, the assumption of straightforward profit maximization by each supplier ignores a wide array of other potential objectives: targeting a certain level of revenue (Alhajji and Huettner, 2000b); preventing high oil prices to discourage substitution efforts; and the complex negotiations within OPEC, where quota allocations are based on (stated) reserves. This may explain why the quantity results are ambiguous and do not strongly favour one market structure as an explanation. In general, numerical equilibrium models are quite sensitive to underlying assumptions. Nevertheless, the aim of this article is to offer a better approach to model market power exertion when a fringe is present, and to determine whether endogenous market power may be a factor in explaining the crude oil price path over the past decade. I claim that in this respect, the model and the numerical application succeed.

Three avenues for future research are opened up through this work. First, the assumption of a one-shot Nash-Cournot oligopoly among OPEC members should be replaced by a richer model of collusion. This may follow a “bureaucratic cartel” (Smith, 2005) or a “Nash-bargaining cartel” (Harrington et al., 2005). The former considers the rigidities and dynamics of intra-OPEC negotiation; the latter includes cartel-stability considerations, such that each cartel member must have an incentive to remain in the cartel, rather than simply maximizing total revenue of the entire group. This is particularly relevant since OPEC does not have a formal compensation mechanism. Furthermore, intertemporal optimization by crude oil suppliers should be considered; not necessarily in a Hotelling-type model, but by including endogenous investment in new production capacity as a strategic decision. This should ideally be implemented in a game-theoretic approach using closed-loop equilibria (cf. Murphy

and Smeers, 2005).

Second, endogeneity of the conjectural variations and hence the mark-up on top of marginal costs (i.e., market power exertion) should be implemented in long-term numerical equilibrium models. These are widely used to analyse scenarios and investment requirements, most prominently in natural gas (e.g., Gabriel et al., 2012b; Egging et al., 2010; Lise and Hobbs, 2008). However, such models are usually calibrated to reflect a certain situation in the base year, and the assumptions regarding the conjectural variations of players are then assumed to remain fixed for the entire simulation horizon. A model where market power exertion is endogenous and contingent on the capacity of rivals may be a significant extension to these models.

Third, the methodology of using the reaction of the followers in an equilibrium model to properly capture the dominant-firm aspect can be applied to other sectors: equilibrium problems under equilibrium constraints (EPEC) are now widely proposed as appropriate to model hierarchical markets in general (Kulkarni and Shanbhag, 2013), and the electricity market in particular (Ralph and Smeers, 2006). In the power market, supply curves steepen when generation is close to capacity (cf. Hortaçsu and Puller, 2008; Chen et al., 2006), and capacity constraints are an important factor in this sector, leading to kinks in the reaction functions and thus theoretical as well as algorithmic problems. I believe that a consistent conjecture Stackelberg formulation may be a natural way to circumvent the multiplicity of equilibria in EPECs and offer a way to compute numerical solutions more easily.

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A Mathematical Appendix

Proof of Theorem 6

This proof consists of three parts: first, I derive the second-order derivative condition stated in the Theorem. Then, I show existence of a solution to the KKT system via the equivalent Variational Inequality. Last, uniqueness of the solution is shown if Assumption **A5** holds.

Equation (3) states:

$$p' \left(2 + 2 \frac{\partial q_{-i}(q_i^*)}{\partial q_i} + \frac{\partial^2 q_{-i}(q_i^*)}{\partial q_i^2} \right) - c_i''(q_i^*) \leq 0$$

Continuing the proof of Lemma 5, it follows that:

$$\begin{aligned} -b - b \frac{\partial q_f(q_i)}{\partial q_i} - c_f''(q_f) \frac{\partial q_f(q_i)}{\partial q_i} &= 0 \\ -b \frac{\partial^2 q_f(q_i)}{\partial q_i^2} - c_f'''(q_f) \left(\frac{\partial q_f(q_i)}{\partial q_i} \right)^2 - c_f''(q_f) \frac{\partial^2 q_f(q_i)}{\partial q_i^2} &= 0 \\ \Rightarrow \frac{\partial^2 q_f(q_i)}{\partial q_i^2} &= - \frac{b^2 c_f'''(q_f)}{(b + c_f''(q_f))^3} \end{aligned} \quad (11)$$

Inserting this term into Equation (3), in combination with the assumptions, yields the stated second-order derivative condition for the oligopoly suppliers. The profit maximization function of the fringe supplier f is strictly concave following the reasoning of Lemma 3.

Existence of a solution is shown by looking at the equivalent Variational Inequality (VI) to the KKT system (8).

This is to find a vector $q^* = \left[(q_i^*)_{i \in S}, q_f^* \right]^T \in K$ such that:

$$\begin{aligned} F(q^*)(q - q^*) &= \\ &= \left[\begin{array}{c} \left(-a + b \sum_j q_j^* + b \left(1 - \frac{b}{b + c_f''(q_f^*)} \right) q_i^* + c_i'(q_i^*) \right)_{i \in S} \\ -a + b \sum_j q_j^* + c_f'(q_f^*) \end{array} \right]^T \begin{pmatrix} (q_i - q_i^*)_{i \in S} \\ q_f - q_f^* \end{pmatrix} \geq 0 \quad \forall q \in K \end{aligned} \quad (12)$$

The set K is the Cartesian product of each suppliers' feasible quantity decisions (cf. Lemma 1); however, the marginal cost function $c_j'(q_j)$ and hence F is not defined at $q_j = \bar{q}_j$, and therefore, F is not continuous at the limit. To circumvent this problem, a bound is introduced on the produced quantity. Choose \tilde{q}_j such that:

$$a < c_j'(\tilde{q}_j) \text{ and } \tilde{q}_j < \bar{q}_j.$$

Such a bound obviously exists for every supplier. Producing a quantity greater than \tilde{q}_j would violate the complementarity condition; hence, I can safely restrict the supplier's feasible region to $q_j \in [0, \tilde{q}_j]$. Furthermore, $c_j'(q_j)$ and $c_j''(q_j)$ are continuous on that range.

Let n denote the number of oligopoly suppliers. Now, I can formally define the feasible region of VI (12):

$$K = \prod_j [0, \tilde{q}_j] \subset \mathbb{R}_+^{n+1}$$

Because K is closed, convex and compact, and F is continuous on K , the solution set to the VI is non-empty (cf. Facchinei and Pang, 2003, Corollary 2.2.5).

Uniqueness of the solution can be shown through strict monotonicity of F on K , defined as:

$$(F(x) - F(y))^T (x - y) > 0 \quad \forall x, y \in K, x \neq y$$

Here, x and y vector elements of the feasible region K .

$$\begin{aligned} & \left[\begin{array}{c} \left(b \sum_j (x_j - y_j) + b \left(1 - \frac{b}{b + c_f''(x_f)} \right) x_i + c_i'(x_i) - b \left(1 - \frac{b}{b + c_f''(y_f)} \right) y_i - c_i'(y_i) \right)_{i \in S} \\ b \sum_j (x_j - y_j) + c_f'(x_f) - c_f'(y_f) \end{array} \right]^T \begin{pmatrix} (x_i - y_i)_{i \in S} \\ x_f - y_f \end{pmatrix} = \\ & = b \left(\sum_j (x_j - y_j) \right)^2 + \sum_j (c_j'(x_j) - c_j'(y_j)) (x_j - y_j) \\ & \quad + b \sum_{i \in S} \left[\left(1 - \frac{b}{b + c_f''(x_f)} \right) x_i - \left(1 - \frac{b}{b + c_f''(y_f)} \right) y_i \right] (x_i - y_i) > 0 \end{aligned}$$

This condition is stated in Assumption **A5**. K is closed and convex, and F is strictly monotone under this assumption, so there exists at most one solution (cf. Facchinei and Pang, 2003, Theorem 2.3.3).

Combining the results of existence and (at most) uniqueness yields that the solution to the VI is indeed unique. If it satisfies the second-order derivative condition, it is the unique equilibrium of the Stackelberg oligopoly problem with a competitive fringe. \square