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Taxation, Insurance and Precautionary Labor

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Abstract

We examine optimal taxation and social insurance if insurance markets are imperfect. This requires the development of a theory of labor supply under uncertainty. We show that the case for social insurance is not generally reinforced by adverse selection in insurance markets as social insurance will have welfare-decreasing effects on the labor market. Furthermore, positive and normative implications are highly sensitive to the insurance market equilibrium concept. While for the Rothschild-Stiglitz case social insurance at least alleviates the inefficiency of underinsurance, with a Wilson pooling equilibrium this inefficiency might even be worsened by social insurance. This sheds new light on the question whether social insurance is an appropriate means of redistribution in the presence of an optimally chosen tax schedule.

JEL-classification: H21, H23, H50, D81, J22.

Keywords: Redistributive Taxation, Social Insurance, Adverse Selection, Precautionary Labor.

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1 Introduction

It is known since the seminal work of Mirrlees (1971) that the problem of taxation is fundamentally linked to asymmetric information between government and workers. Only with the reasonable assumption that the government cannot observe individual productivities does the need for income taxation that distorts labor supply arise. The no less influential contributions of Rothschild and Stiglitz (1976) and Wilson (1977) have shown that a similar issue makes the problem of equilibrium in insurance markets a relevant question. If insurance companies cannot observe individual risk types, they are unable to offer individually tailored contracts but have to take into account the possibility of mimicking. Under these circumstances, the market allocation will not in general be efficient.

It is all the more surprising that up to date there is no satisfactory theory that ties those two branches of information economics together. This lack of a unifying theory is particularly astonishing in the light of the fact that taxation and insurance are highly relevant policy matters. With the present contribution, we try to provide such a theory and to highlight important aspects of the interaction between distorting taxation and imperfect insurance markets.

The link between the two strands is a theory of precautionary labor that we develop in section 2. While the theory of optimal income taxation requires labor supply to be endogenous, models of competitive insurance markets with adverse selection imply that not all uncertainty can be resolved. To combine both approaches, we therefore need to derive the determinants of labor supply under uncertainty. Our model is based on the results of Kimball (1990). We show that under a broad range of reasonable assumptions, a precautionary labor supply motive exists, i.e. that the introduction of uncertainty leads individuals to increase their labor supply.

We then incorporate this theory into a model of optimal linear taxation and social insurance similar in spirit to Rochet (1991), Cremer and Pestieau (1996) and Henriet and Rochet (2004). Even without an explicit theory of insurance markets we can show how market imperfections alter the optimality conditions for taxes and transfers. We also prove that, contrary to a first intuitive notion, market imperfections do not generally reinforce the case for social insurance but might even weaken it.

In section 4 we endogenize the insurance market in the developed framework, which requires a modification of the standard theory to allow for endogenous labor supply. This closes the model as it makes it possible to demonstrate all interaction effects between taxation, social insurance, and private insurance markets. Several insights can be drawn from this approach. Most importantly, the positive and normative results are highly sensitive to the underlying equilibrium concept. Social insurance will generally alleviate the inefficiency of underinsurance in the Rothschild-Stiglitz framework but might have strong negative effects on the labor market. A complete renunciation of social insurance hence might be optimal. If equilibrium is of the Wilson pooling type, social insurance might even worsen the inefficiency in the insurance market.

The most similar existing work is the contribution by Boadway, Leite-Monteiro, Marchand, and Pestieau (2004) who were the first to examine optimal taxation with adverse selection and ex-post moral hazard in private insurance markets.[1] Based on Rothschild-Stiglitz separating equilibria they find the case for social insurance strengthened by market inefficiencies.[2] However, their results are based on the assumption that labor supply decisions take place *after* a possible damage has been realized. This reduces the impact of underinsurance on individual decisions to income effects and admittedly makes it difficult to interpret and sign the resulting optimality conditions. We do not want to eliminate the question of labor supply under uncertainty, which we believe to play a decisive role in the understanding of the interaction between taxation and insurance.

2 Labor Supply under Uncertainty

Some prerequisite results concerning individuals' labor supply under uncertainty have to be derived first. This is done by transferring the results of Kimball (1990) to labor supply in order to establish a theory of 'precautionary labor'. While the theory of precautionary savings has received some attention (Sandmo (1970), Abel (1988), Kimball (1990)), to our knowledge there is no elaborate theory of precautionary labor supply. Eaton and Rosen

[1] We refrain from analyzing moral hazard. Other aspects of our model setup are very similar to Boadway, Leite-Monteiro, Marchand, and Pestieau (2004).

[2] 'The introduction of adverse selection has the effect of fostering social insurance.' (p. 20)

(1979) and Eaton and Rosen (1980) examine the case of endogenous labor with wage uncertainty. However, they mostly present only simulation results. Unlike them, we do not model wage risk but an income independent risk to consumption. Labor supply is chosen *before* the risk is realized. This gives rise to the question whether risk induces people to work more or less than they would in case of certainty.

Let $\tilde{\theta}_1$ denote a random variable with expectation zero and variance σ_1^2. The assumption of zero expectation will be relaxed later. The random variable $\tilde{\theta}_2$ is assumed to have the same expectation but a higher variance σ_2^2. Furthermore, as throughout the paper, an additively separable utility function $U(c, L) = u(c) + v(L)$ is assumed, where c denotes consumption and L denotes labor supply.[3] The standard conditions $u'(c) > 0$, $u''(c) < 0$, $v'(L) < 0$ and $v''(L) < 0$ hold. Denote the productivity of an individual by w. Firms can observe w and pay wages according to marginal productivity such that earned income is wL. The individual is assumed to have an additional, exogenous and state independent income T.

The first order condition for labor supply $L^*(\tilde{\theta}_1)$ that maximizes expected utility in the presence of risk $\tilde{\theta}_1$ is

$$w\,\mathrm{E}\left[u'\left(wL^* + T - \tilde{\theta}_1\right)\right] = -v'(L^*), \tag{1}$$

where E is the expectations operator. (1) is a standard condition stating that labor supply is determined as to equalize expected marginal utility and disutility from work. The sufficient second order condition for a maximum is fulfilled. How does risk affect the first order condition? First, optimal labor supply in a situation with the higher risk $\tilde{\theta}_2$ would be equal to $L^*(\tilde{\theta}_1)$ if expected consumption was higher by the *compensating precautionary premium* $\Psi^*(\tilde{\theta}_1, \tilde{\theta}_2)$ defined by

$$\mathrm{E}\left[u'\left(wL^* + T - \tilde{\theta}_1\right)\right] = \mathrm{E}\left[u'\left(wL^* + T - \tilde{\theta}_2 + \Psi^*(\tilde{\theta}_1, \tilde{\theta}_2)\right)\right]. \tag{2}$$

In that case, the optimality condition (1) would not be affected. The compensating precautionary premium will be extremely helpful for modelling

[3]We need the assumption of separability only to keep the exposition of our labor supply theory concise. As shown by Kimball (1990), the results can be transferred to the case of nonseparable utility. In the later sections, separability is not necessary either but only used since we resort to the theory of labor supply under uncertainty there. Note finally that the function v has to be at least twice, u at least three times differentiable.

adverse selection in insurance markets with endogenous labor supply.

Correspondingly, the *equivalent precautionary premium* $\Psi(\tilde{\theta}_1, \tilde{\theta}_2)$ at labor supply $L^*(\tilde{\theta}_1)$ is defined by

$$\mathrm{E}\left[u'\left(wL^* + T - \tilde{\theta}_1 - \Psi(\tilde{\theta}_1, \tilde{\theta}_2)\right)\right] = \mathrm{E}\left[u'\left(wL^* + T - \tilde{\theta}_2\right)\right]. \qquad (3)$$

Its interpretation is as follows. An increase in risk from $\tilde{\theta}_1$ to $\tilde{\theta}_2$ will have the same effect on labor supply as a lump-sum reduction of income by $\Psi(\tilde{\theta}_1, \tilde{\theta}_2)$, since it affects the optimality condition (1) in the same way. Therefore, statements about the adjustment of labor supply provoked by a change of risk can be reduced to statements about the income effect provoked by a decrease of income by Ψ. Note that Ψ^* and Ψ become the same if the change in risk becomes arbitrarily small.

As shown by Kimball (1990), the discussed premiums are simply the compensating and equivalent risk premia developed by Pratt (1964), applied to the first derivative of u. Therefore, the following relationship holds for the introduction of a small risk σ_2^2 starting from an initial situation without uncertainty ($\sigma_1^2 = 0$):

$$\Psi = \Psi^* = -\frac{u'''(wL^* + T)}{u''(wL^* + T)}\frac{1}{2}\sigma_2^2 + o(\sigma_2^2) = \eta(wL^* + T)\frac{1}{2}\sigma_2^2 + o(\sigma_2^2), \qquad (4)$$

where $o(\sigma_2^2)$ denotes a term that is of smaller magnitude than σ_2^2 and η is the *absolute prudence* of the function u.[4]

From equation (4) follow some first implications for the theory of behavior under risk. If η is positive, the introduction of risk will have the same effect on labor supply as a lump sum reduction of income, which decreases the demand for leisure[5] and increases labor supply; the individual has a motive for *precautionary labor*. The size of η indicates how strong this motive is. As shown in Appendix A, a positive prudence is a necessary condition for constant or decreasing risk aversion, both in absolute and relative terms. Therefore, under the common and realistic assumption of non-increasing risk aversion, precautionary labor effects do exist.

So far, the results were derived for the introduction of small risks with expectation zero, starting from a point of certain income. They can be

[4] The proof for this relationship follows from a Taylor series expansion of $u'(wL^* + T)$ around $wL^* + T$ and is omitted here. Refer for example to Pratt (1964), p. 125.

[5] Our assumption of separable preferences implies that leisure is a normal good.

generalized to cases of risk with nonzero expectation. Changes in risk then simply entail additional income effects. It is still useful to distinguish between pure risk effects via the variance and income effects via expected values. The results can also be generalized to cases of initially risky situations ($\sigma_1^2 > 0$), as long as the prudence does not change its sign over a relevant range of income levels.[6] This is shown in Appendix B, where we assume that a damage of size $(1-\beta)D$ occurs with probability p. The parameter $\beta \in [0,1]$, which can be thought of as the share of risk that is insured, is used to vary the size of the possible damage.[7] It defines a Bernoulli random variable $\tilde{\theta}(\beta)$ with expectation $p\,(1-\beta)D$ and variance $\sigma^2(\beta) = p\,(1-p)[(1-\beta)D]^2$. Increasing β reduces both variance and expected value. Appendix B illustrates that the effect of marginal increases in β on labor supply can now be expressed as

$$\frac{\partial L^*}{\partial \beta} = \frac{\partial L^*}{\partial T}\left[pD - \frac{\partial \Psi}{\partial \beta}\right], \tag{5}$$

where $\partial L^*/\partial T$ denotes the income effect, which is negative. First, higher coverage increases expected income by pD. Second, the change in variance has the same effect as a decrease of income by the corresponding premium Ψ that is raised by an increased insurance coverage β. A general formula for $\partial\Psi/\partial\beta$ can be derived for all levels of β and therefore for situations with preexisting risk. It simplifies to the derivative of (4) with respect to β for the special case of $\beta = 1$. A sufficient condition for $\partial\Psi/\partial\beta$ to be negative at any initial value of β is that the prudence is positive (and vice versa) on the interval of consumption levels spanned by consumption in case of damage and no damage.[8] In that case, reduced variance through higher insurance coverage reduces labor supply since it acts like an increase in income.

Finally, the marginal effect (5) can be integrated so that the relation between prudence and precautionary labor motives remains qualitatively valid for large changes in risk.

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[6]For a detailed discussion of 'preexisting risk' and its implication for the applicability of standard measures of risk aversion see Ross (1981).

[7]For ease of presentation, no insurance premiums are modelled at this point, as they would simply cause additional income effects.

[8]This in turn is fulfilled if the prudence is positive for all consumption levels, which will be the case if risk-aversion is never increasing.

3 Optimal Taxation and Social Insurance

3.1 The Model

This section introduces the basic model and derives conditions for optimal taxation and social insurance in the presence of imperfect insurance markets. This is done without an explicit foundation of the market imperfection. Such questions are postponed to the next section. It will turn out that different concepts explaining imperfection can easily be incorporated. The model structure is similar to Cremer and Pestieau (1996) and Boadway, Leite-Monteiro, Marchand, and Pestieau (2004).

Assume that society consists of N individuals that can be described by two characteristics: their productivity and their probability of experiencing some damage of size D. There are W different productivity levels w_i, $i = 1, ..., W$ and two damage probabilities p_j, $j = L, H$, with $p_L < p_H$. In what follows, the index i will always refer to productivity while j refers to damage probability.

Of all workers in productivity class i, m_{iL} individuals have the low probability p_L of damage while m_{iH} are high risk types with damage probability p_H. The overall number of individuals in class i is therefore $m_i = m_{iL} + m_{iH}$ and the size of the whole population is $N = \sum_{i=1}^{W} m_i$. Denote the *proportion* of individuals in the population that have productivity w_i and damage probability p_j by $n_{ij} = m_{ij}/N$. The population average of the risk probability is then given by $\bar{p} = \sum_{i,j} n_{ij} p_j$. The average probability within productivity group i is $\bar{p}_i = (1/m_i) \sum_j m_{ij} p_j$.

As commonly assumed in the theory of optimal taxation, government maximizes the unweighted utilitarian objective.[9] It can neither observe individual productivities nor damage probabilities[10] but only knows the joint distribution of both characteristics. Hours worked are assumed to be unob-

[9] Introducing individual weights in the welfare function would not fundamentally change the following results. It would simply induce additional weights on the 'social valuation' to be defined below. Doing this would allow to construct the whole second-best Pareto frontier.

[10] As long as the ratio m_{iL}/m_{iH} is not exactly the same in each productivity class, observing damage probability would make some inference about productivity possible. The assumption that damage probabilities are not observable excludes this possibility for government. In addition, if probabilities were observable, there would be no case for social insurance since income taxation could be directly conditioned on risk as was shown by Boadway, Leite-Monteiro, Marchand, and Pestieau (2004).

servable as well, so that distorting taxation of labor income is necessary to redistribute income. The tax schedule is restricted to a constant marginal tax rate τ and a lump-sum transfer T. Additionally, government can force the citizens to insure a share α of the possible damage. Such social insurance is financed by a uniform contribution $\bar{p}\,\alpha D$ by each individual, which ensures that social insurance makes zero profits in expectation.[11] Remaining risk can be insured privately. The contract that individual ij purchases is denoted by $\mathcal{I}_{ij} = (\beta_{ij}, d_{ij})$, where β_{ij} is the privately insured share of the damage and d_{ij} is the premium.

The time structure of the model is as follows. First, government sets its policy $\mathcal{P} = (\tau, T, \alpha)$. Taking $\mathcal{P}$ as given, individuals simultaneously choose their labor supply, pay taxes and social insurance contributions, receive the transfer, and purchase the contract $\mathcal{I}_{ij}$. Finally, the damage occurs according to the given probabilities. After possible payments of social and private insurance, consumption takes place.

3.2 Optimal Government Policy

As we will argue in section 4, each individual receives a finite set of private insurance contract offers from which it chooses the one that yields the highest expected utility. Since labor supply and the purchase of insurance take place simultaneously, the comparison of different contracts involves optimal adjustment of labor supply.[12] Each individual's choice $\mathcal{I}_{ij}$ is therefore the result of a noncontinuous optimization problem. Given $\mathcal{I}_{ij}$, the corresponding optimal labor supply $L^*_{ij}(\tau, T, \alpha, \beta_{ij}, d_{ij})$ can be determined. It is defined by the first order condition

$$\begin{aligned} p_j\, u'\big((1-\tau)w_i L^*_{ij} + T - \overline{p}\,\alpha D - d_{ij} - (1-\alpha-\beta_{ij})D\big) \\ +(1-p_j)\, u'\big((1-\tau)w_i L^*_{ij} + T - \overline{p}\,\alpha D - d_{ij}\big) \\ = -v'(L^*_{ij})/((1-t)\, w_i). \end{aligned} \tag{6}$$

Implicit differentiation of condition (6) yields comparative static effects. Note that the derivative of L^*_{ij} with respect to α includes the effect of the increase

[11] Funding of social insurance through the general budget and therefore the distorting tax τ does not lead to different results if optimal tax rate, transfer and social insurance are determined simultaneously. In that case, the lump-sum transfer T would simply be smaller by the former contribution $\bar{p}\,\alpha D$ while all marginal conditions would be unchanged.

[12] The exact order between the labor supply decision and the purchase of insurance is not relevant as long as individuals have perfect foresight and anticipate future choice options.

in the social insurance contribution. To the contrary, the derivative with respect to β_{ij} does not take into account the change in the private insurance premium. Where needed, the corresponding effect that accounts for this change is marked with the letter A:

$$\left.\frac{\partial L_{ij}^*}{\partial \beta_{ij}}\right|_A = \frac{\partial L_{ij}^*}{\partial \beta_{ij}} - \frac{\partial d_{ij}}{\partial \beta_{ij}}\frac{\partial L_{ij}^*}{\partial T} = \frac{\partial L_{ij}^*}{\partial T}\left[p_j D - \frac{\partial \Psi}{\partial \beta_{ij}} - \frac{\partial d_{ij}}{\partial \beta_{ij}}\right]. \qquad (7)$$

The second equality follows directly from the results in the previous section.

Substituting L_{ij}^* back into the expected utility function yields the indirect expected utility function $V_{ij}^*(\tau, T, \alpha, \beta_{ij}, d_{ij})$. Although the best contract $\mathcal{I}_{ij}$ is chosen out of the set of available contracts, the function V_{ij}^* is not necessarily an optimal value function with respect to (β_{ij}, d_{ij}) in the sense of the Envelope theorem. This would be the case if insurance markets were perfect and each individual would purchase full coverage at an actuarially fair premium. Marginally fair variations in the contract would then not affect V_{ij}^*. With adverse selection, however, individuals will not generally be able to purchase such contracts.[13]

The insurance market equilibrium can be modelled using different equilibrium concepts, some of which will be introduced in the next section. At this point it is only necessary to emphasize that the equilibrium contracts will depend on the policy $\mathcal{P}$, i.e. that $\beta_{ij} = \beta_{ij}(\tau, T, \alpha)$ and $d_{ij} = d_{ij}(\beta_{ij}) = d_{ij}(\tau, T, \alpha)$. Functions that account for equilibrium effects are marked by two asterisks. For labor supply this implies $L_{ij}^{**}(\tau, T, \alpha) = L_{ij}^*(\tau, T, \alpha, \beta_{ij}(\tau, T, \alpha), d_{ij}(\beta_{ij}))$. Indirect utility $V_{ij}^{**}(\tau, T, \alpha)$ is defined analogously. Effects of the policy parameters on utility via the insurance market equilibrium may not be neglected, with the argument given above for the function V_{ij}^*.

Assuming that there is no exogenous revenue requirement, government's optimization problem, written in per-capita terms, is

$$\max_{T,\tau,\alpha} \sum_{i,j} n_{ij}\, V_{ij}^{**}(\tau, T, \alpha) \qquad \text{s.t.} \qquad \sum_{i,j} n_{ij}\,(\tau w_i L_{ij}^{**} - T) = 0. \qquad (8)$$

[13] As before, the derivative of V_{ij}^* with respect to β_{ij} does not account for changes in the premium d_{ij} if not indicated by A.

Assuming interior solutions, the three first order conditions are

$$(T): \quad \sum_{i,j} n_{ij} \frac{\partial V_{ij}^{**}}{\partial T} + \gamma \sum_{i,j} n_{ij} \left(-1 + \tau w_i \frac{\partial L_{ij}^{**}}{\partial T} \right) = 0 \tag{9}$$

$$(\tau): \quad \sum_{i,j} n_{ij} \frac{\partial V_{ij}^{**}}{\partial \tau} + \gamma \sum_{i,j} n_{ij} \left(w_i L_{ij}^{**} + \tau w_i \frac{\partial L_{ij}^{**}}{\partial \tau} \right) = 0 \tag{10}$$

$$(\alpha): \quad \sum_{i,j} n_{ij} \frac{\partial V_{ij}^{**}}{\partial \alpha} + \gamma \sum_{i,j} n_{ij}\, \tau w_i \frac{\partial L_{ij}^{**}}{\partial \alpha} = 0, \tag{11}$$

where γ is the Lagrange multiplier associated with the revenue constraint. It equals the welfare value of a marginal increase in government revenues. Correspondingly, individual utility can be converted to units of revenue by dividing through γ.

In Appendix C the conditions (9)-(11) are transformed to make them interpretable. We use three important concepts. The first is the net social marginal valuation of an individual's income, b_{ij}, well-known from the theory of optimal taxation:

$$b_{ij} = \frac{1}{\gamma} \frac{\partial V_{ij}^*}{\partial T} + \tau w_i \frac{\partial L_{ij}^*}{\partial T}. \tag{12}$$

It captures the effect of an increased transfer T on the objective via the individual's utility and via the effect on the budget constraint through labor supply changes, both measured in terms of government revenues.

In the presence of imperfect private insurance markets, government policy has additional effects on the objective via the insurance market equilibrium. Therefore, the concept of net social marginal valuation of an individual's insurance, g_{ij}, is useful:

$$g_{ij} = \frac{1}{\gamma} \left. \frac{\partial V_{ij}^*}{\partial \beta_{ij}} \right|_A + \tau w_i \left. \frac{\partial L_{ij}^*}{\partial \beta_{ij}} \right|_A. \tag{13}$$

As before, it captures the effect of a changing insurance contract via utility and via the budget constraint on the objective. Since (13) is formulated in terms of premium-adjusted effects, its sign and magnitude will depend on the yet undefined change in the private insurance premium.

Finally, the Slutzky decomposition

$$\frac{\partial L_{ij}^*}{\partial \tau} = -w_i L_{ij}^* \frac{\partial L_{ij}^*}{\partial T} - w_i \frac{\partial L_{ij}^c}{\partial w_i^n} = -w_i L_{ij}^* \frac{\partial L_{ij}^*}{\partial T} - \epsilon_{ij} \frac{L_{ij}^*}{1-\tau} \tag{14}$$

is used, where $w_i^n = (1-\tau)w_i$ denotes the net wage, L_{ij}^c is the Hicksian (compensated) labor supply function and $\epsilon_{ij} = w_i^n/L_{ij}^* \times \partial L_{ij}^c/\partial w_i^n$ is the nonnegative elasticity of L_{ij}^c with respect to the net wage. The transformed conditions for optimal government policy are[14]

$$(T): \quad 1 = \bar{b} + \sum_{i,j} n_{ij} g_{ij} \frac{\partial \beta_{ij}}{\partial T} \tag{15}$$

$$(\tau): \quad \frac{\tau}{1-\tau} = \frac{-\mathrm{Cov}\,(wL, b) + \sum_{ij} n_{ij} g_{ij} \left(\frac{\partial \beta_{ij}}{\partial \tau} + \frac{\partial \beta_{ij}}{\partial T} \bar{y} \right)}{\sum_{i,j} n_{ij} w_i L_{ij}^* \epsilon_{ij}} \tag{16}$$

$$(\alpha): \quad \mathrm{Cov}\left(b, \frac{\partial d}{\partial \beta}\right) = -\sum_{i,j} n_{ij} g_{ij} \left(1 + \frac{\partial \beta_{ij}}{\partial \alpha}\right), \tag{17}$$

where $\bar{y} = \sum_{ij} n_{ij} w_i L_{ij}^*$ stands for average labor income and $\bar{b} = \sum_{ij} n_{ij} b_{ij}$ for the population average of b_{ij}. Conditions (15)-(17) generalize the respective conditions that were obtained by Cremer and Pestieau (1996) for the case of perfect private insurance markets. They will be discussed in the next subsections.

3.3 Optimal Transfer T

Consider condition (15). It differs from the condition that Cremer and Pestieau (1996) obtain for the case of perfect insurance markets only by the last term $\sum_{ij} n_{ij} g_{ij}\, \partial \beta_{ij}/\partial T$. This term actually drops out if insurance markets are perfect. The individual contracts are then given by $\mathcal{I}_{ij} = (1-\alpha, p_j(1-\alpha)D)$ as everyone will buy full coverage for the remaining share of the risk at an individually fair premium. These contracts are neither affected by T nor by τ. Condition (15) then states that the marginal gain from increasing the transfer, measured in terms of revenue, should equal its cost.

In the case of imperfect insurance markets, increasing the transfer can have additional effects on the equilibrium contracts $\mathcal{I}_{ij}$ that the individuals can buy. Assume, for example, that a higher transfer increases β_{ij} and d_{ij}

[14] These conditions are perfectly general with one exception. In deriving (17) we use the assumption that the population average of $\partial d_{ij}/\partial \beta_{ij}$ equals $\bar{p}D$. This holds under most insurance market equilibria, including those examined in section 4, but not necessarily in equilibria in which some individuals do not purchase any insurance.

for many individuals with a positive marginal valuation of insurance ($g_{ij} > 0$). Imperfect insurance markets then make a higher transfer T desirable compared to a situation with perfect markets. To the contrary, if higher insurance coverage leads to large reductions in labor supply and therefore $g_{ij} < 0$, imperfect insurance markets weaken the case for high transfers.

3.4 Optimal Tax Rate τ

With the same argument as above, if neither T nor τ have an effect on $\mathcal{I}_{ij}$, as for the case of perfect markets, condition (16) becomes the condition obtained by Cremer and Pestieau (1996).[15] It reflects the trade-off between efficiency and equity that is fundamental to the theory of optimal taxation. The numerator reflects the goal of redistribution since the covariance between income and marginal social valuation can be interpreted as a welfare-based measure of inequality. A large negative correlation makes a higher tax rate more desirable. The denominator captures the distorting effect of taxation. If labor supply reacts strongly to taxation (ϵ_{ij} large), optimal tax rates will be smaller. If taxation did not cause distortions ($\epsilon_{ij} = 0$), redistribution should take place until the correlation between income and marginal social valuation vanishes.

As before, the impact of taxation on the insurance market has to be taken into account in a more general setup. The term $\sum_{i,j} n_{ij} g_{ij} (\partial\beta_{ij}/\partial\tau + \bar{y}\partial\beta_{ij}/\partial T)$ captures these additional effects. Put simply, it states that the tax rate should be higher if it has a desirable impact on the insurance market, e.g. by increasing the insurance coverage for people who have a positive social marginal valuation of insurance. It is worth noting why this effect enters (16) as an (on average) compensated effect, i.e. why we find the term $\bar{y}\partial\beta_{ij}/\partial T$ in brackets. The overall effect of τ on the market contracts is captured by $\partial\beta_{ij}/\partial\tau$. This effect alone might make higher taxes desirable. The additional revenue generated by a marginal increase in τ ($\bar{y}$) can be thought of as the 'negative cost' of such government intervention and is captured by the discussed term.

[15] Cremer and Pestieau (1996) derive the condition with only τ on the LHS but define the elasticity as $\epsilon_{ij} = w_i/L^*_{ij} \times \partial L^c_{ij}/\partial w^n_i$, such that their results is equivalent to ours.

3.5 Optimal Social Insurance α

At first glance, market imperfections should strengthen the case for social insurance since it then alleviates additional efficiency problems. One of the main points of our contribution is to show that this reasoning is not valid.

If insurance markets are perfect, condition (17) reduces to the respective condition in Cremer and Pestieau (1996). Social insurance simply crowds out private insurance ($\partial\beta_{ij}/\partial\alpha = -1$) such that the term on the RHS becomes zero. On the other hand, premiums are actuarially fair and (17) therefore states that the covariance between damage probability and marginal social valuation should be zero.[16] It reflects that social insurance is a nondistorting means of redistribution. High-risk types benefit from social insurance since their private premiums would be larger than the social insurance contribution. The reverse holds for low-risk types, such that increasing α redistributes from low to high risks and lowers the covariance between risk and marginal social valuation. Government should do this until no correlation remains and the potential of social insurance for redistribution is exhausted.[17] This condition was derived under the assumption of an interior solution. It will become an inequality if the optimal α is a corner solution ($\alpha = 1$ if $\mathrm{Cov}(b, p) > 0$ for all values of α and vice versa). The optimal share of social insurance will always be one if high productivity individuals are the ones with lower damage probabilities ($\mathrm{Cov}(w, p) < 0$), which is a reasonable case.[18] With full social insurance, individuals differ only with respect to their productivity, and high risk types will still have the higher marginal social valuation due to their productivity disadvantage. An interior solution or $\alpha = 0$ can only be optimal if productivity and risk are positively correlated. Without social insurance, more productive individuals then have the disadvantage of large private insurance premiums. If this effect is small, it will not be sufficient to induce a positive correlation between risk and social valuation and there should be no social insurance. If the possible damage is large, the risk disadvantage might exceed the productivity advantage and social insurance

[16] Strictly speaking, as the derivative of the private premium with respect to coverage is $p_j D$, the condition states that D times the covariance is equal to zero, which is also obtained by Cremer and Pestieau.

[17] Note the analogy to the previous section where it was argued that a nonzero correlation between social valuation and a variable that the government can (indirectly) condition its policy on can only be optimal if the respective policy instrument causes distortions, which is not the case here.

[18] See Henriet and Rochet (2004) for some empirical evidence.

will be desirable to redistribute from low to high productivity individuals. As α increases, the risk disadvantage becomes less important and an interior solution is reached. The case constructed for such an interior solution is, however, rather unrealistic, such that the model has serious problems with explaining partial social insurance.

Now consider the general version (17) and assume as a starting point that private insurance premiums are still adjusted actuarially fairly ($\partial d_{ij}/\partial\beta_{ij} = p_j D$). Even if the correlation between productivity and risk is negative and the correlation between risk and social valuation therefore positive for all values of α, partial social insurance can be optimal if the RHS of (17) is positive. It is positive if social insurance does increase insurance coverage for individuals who have a large measure of prudence and react by strongly reducing labor supply ($g_{ij} < 0$). Such labor market effects weaken the argument for social insurance compared to situations with perfect insurance markets.[19] The fact that in the most general case it is the derivative of the private premium with respect to the insurance coverage that matters in the correlation in (17) illustrates the logic of redistribution via social insurance: it is not damage probability per se but the comparison between private premiums and social insurance contributions that determines the optimal amount of social insurance.

4 Imperfect Insurance Markets

While the dependence of the insurance contracts $\mathcal{I}_{ij}$ on the policy parameters $\mathcal{P}$ has been left unspecified in the previous section 3, we now proceed to show how such relations emerge naturally from endogenizing the private insurance market equilibrium. Most of the literature on social insurance as a means of redistribution, notably the contributions of Rochet (1991), Cremer and Pestieau (1996) and Henriet and Rochet (2004), is based on the premise of perfect private insurance markets. In this case social insurance crowds out private insurance, but there is no effect of the tax parameters τ and T. In

[19] One could ask whether high labor supply in response to lack of insurance does not itself constitute an inefficiency so that its reduction should *increase* welfare. The fact that this is not true is a typical second-best result. Since taxation distorts labor supply downwards, the opposite distortion through the imperfect insurance market is welfare increasing. The reduction of precautionary labor through social insurance can then in turn decrease welfare.

addition, the individuals' choice of labor supply collapses to a deterministic problem since all households are fully insured. This simplifies the analysis considerably.

The assumption of efficient insurance markets is, however, based on an informational inconsistency. Notably, it is hard to understand why insurance companies are able to observe individual damage probabilities while the government is not. Two arguments have been put forward to justify this assumption. First of all, a similar informational asymmetry between the government and the private sector underlies the classical income tax model by Mirrlees (1971), where firms pay wages according to individual productivities w whereas the government can only observe total income wL.[20] Analogously, one may argue here that although insurers possess some information about individuals, it is not possible for the government to collect it. A second justification goes back to Rochet (1991). In his view, the case for social insurance as a redistributive instrument is strongest if private insurance markets are taken as efficient. Otherwise, there would only be an additional efficiency argument for the introduction of a social insurance system.[21]

We will now examine the validity of this reasoning by assuming that insurance companies have no information on individual risks p_j, but that they can observe the individual productivity levels w_i.[22] This allows us to divide the private insurance market into W sub-markets, one for each productivity level. For each of them we consider a simple adverse selection problem. As is well known, imperfect insurance markets can be modelled using a variety of game theoretic approaches that differ significantly in their positive and normative implications.[23] In the following, we aim at demonstrating how

[20] Rochet and Maderner (1995) have made this point and developed a model of taxation where the government can observe w as do the firms. However, in their model, w is endogenized by a choice of education so that lump-sum taxation contingent on w is not optimal.

[21] The same argument is used by Boadway, Leite-Monteiro, Marchand, and Pestieau (2004) on p. 17.

[22] With this structure, there is still an asymmetry between government and insurers in the sense that the latter can observe productivities whereas the former cannot. However, it is not implausible that insurance companies face the same information set as the firms on the labor market. With our assumption, we at least eliminate any asymmetry within the private sector. In addition, assuming that insurers cannot even observe productivities we would face a bidimensional adverse selection problem which would excessively complicate the analysis. The same informational assumption is taken by Boadway, Leite-Monteiro, Marchand, and Pestieau (2004).

[23] See, for instance, Hellwig (1987) for an overview.

the optimality conditions for tax and social policy from section 3 can be readily applied to very different equilibrium concepts as those developed by Rothschild and Stiglitz (1976) and Wilson (1977).

A similar analysis has been performed by Boadway, Leite-Monteiro, Marchand, and Pestieau (2004). However, they only consider Rothschild-Stiglitz equilibria in the private insurance markets. One of our main results in this section is that the effects identified in this framework are not robust to alternative equilibrium concepts. In addition, our theory of precautionary labor will prove extremely valuable in intuitively understanding all effects of income taxation and social insurance.

4.1 Adverse Selection with Endogenous Labor Supply

Before proceeding to a detailed analysis of the different equilibrium concepts, some general comments on how to include adverse selection in our model are necessary. In the standard models going back to Rothschild and Stiglitz (1976) and Wilson (1977), only insurance coverage and premiums are explained endogenously. In our model, labor supply is an additional endogenous variable. While it has become clear in sections 2 and 3 that labor supply reacts to the parameters of the insurance contract, the relationship also works in the opposite direction: equilibrium insurance contracts cannot be determined without accounting for variations in labor supply. The standard model of adverse selection therefore has to be extended by our theory of precautionary labor. This gives rise to two main complications.

A crucial condition that needs to be fulfilled to make adverse selection problems tractable is the single-crossing property. It implies that the high risk types are characterized by a steeper indifference curve in the (β_{ij}, d_{ij})-space for every given combination of coverage and premium. Put formally, the marginal rate of substitution between coverage and premium

$$\left.\frac{\mathrm{d}d_{ij}}{\mathrm{d}\beta_{ij}}\right|_{V^*=\bar{V}} = -\frac{\partial V^*_{ij}/\partial\beta_{ij}}{\partial V^*_{ij}/\partial d_{ij}} = \frac{\partial V^*_{ij}/\partial\beta_{ij}}{\partial V^*_{ij}/\partial T} = \frac{Du'(c^0_{ij})}{u'(c^0_{ij}) + \frac{1-p_j}{p_j}u'(c^1_{ij})} > 0, \qquad (18)$$

must be increasing in p_j, where

$$\begin{aligned} c^0_{ij} &\equiv (1-\tau)w_i L^*_{ij} + T - \bar{p}\,\alpha D - d_{ij} - (1-\alpha-\beta_{ij})D \quad \text{and} \\ c^1_{ij} &\equiv (1-\tau)w_i L^*_{ij} + T - \bar{p}\,\alpha D - d_{ij} \end{aligned}$$

stand for the consumption levels of individual ij in case of loss and no loss, respectively. As can be seen from (18), this would clearly hold if labor supply did not react to the damage probability p_j. However, as shown in Appendix D, a higher risk p_j induces households to work more. The consumption of high risk types will therefore be higher in both states of nature and their marginal utilities of consumption consequently lower at each point in the (β_{ij}, d_{ij})-space. In the plausible case that preferences are characterized by a positive measure of prudence, this may generate an opposite effect on the marginal rate of substitution (18). To make sure that the single-crossing property holds globally, we therefore would have to assume either increasing risk aversion, which is empirically unappealing, or that the direct effect of the probabilities dominates the indirect effect via labor supply. It is demonstrated in the Appendix that this is the case whenever the ratio p_H/p_L is sufficiently high, which we assume in the following.

A second issue that arises from the endogeneity of labor supply relates to mimicking behavior. Any separating equilibrium involves a binding incentive compatibility constraint in the sense that the contracts are such that one type is indifferent between revealing his type and mimicking the other. In the Rothschild-Stiglitz equilibrium, for instance, this requires that the high risks be indifferent between their full insurance contract and the low risks' contract with less than full insurance but a premium that is more than fair for them. In general, it will be optimal for the high risks to supply different amounts of labor given the two insurance contracts. On the one hand, there is a precautionary motive resulting from different levels of coverage while on the other hand the premium difference generates an income effect.

We can use the results from section 2 to determine the direction of the change in labor supply in case of mimicking. Suppose an individual ij faces two insurance contracts $\mathcal{I}_{ij} = (\beta_{ij}, d_{ij})$ and $\tilde{\mathcal{I}}_{ij} = (\tilde{\beta}_{ij}, \tilde{d}_{ij})$ with $\tilde{\beta}_{ij} < \beta_{ij}$, between which it is indifferent. Then, due to risk aversion we must have $\tilde{d}_{ij} < d_{ij}$. The two levels of optimal labor supply $L^*_{ij} = L^*_{ij}(\tau, T, \alpha, \beta_{ij}, d_{ij})$ and $\tilde{L}_{ij} = L^*_{ij}(\tau, T, \alpha, \tilde{\beta}_{ij}, \tilde{d}_{ij})$ are determined by condition (6). Clearly, we would obtain $\tilde{L}_{ij} = L^*_{ij}$ only in the special case in which

$$\begin{aligned} & p_j\, u'\big((1-\tau)w_i L^*_{ij} + T - \overline{p}\,\alpha D - d_{ij} - (1-\alpha-\beta_{ij})D\big) \\ & + (1-p_j)\, u'\left((1-\tau)w_i L^*_{ij} + T - \overline{p}\alpha D - d_{ij}\right) \\ = \; & p_j\, u'\big((1-\tau)w_i L^*_{ij} + T - \overline{p}\,\alpha D - \tilde{d}_{ij} - (1-\alpha-\tilde{\beta}_{ij})D\big) \\ & + (1-p_j)\, u'\big((1-\tau)w_i L^*_{ij} + T - \overline{p}\alpha D - \tilde{d}_{ij}\big). \end{aligned} \tag{19}$$

If (19) holds, the optimality condition for labor supply is not altered when choosing $\tilde{\mathcal{I}}_{ij}$ instead of $\mathcal{I}_{ij}$.

In section 2 it turned out to be helpful to separate income and variance effects of a change in the coverage β_{ij} before applying the definitions of the compensating and equivalent premiums (2) and (3). The difference of the expected damage between the two contracts is $p_j(\beta_{ij} - \tilde{\beta}_{ij})D$. It is now convenient to decompose the overall premium difference as follows:

$$d_{ij} - \tilde{d}_{ij} \equiv p_j(\beta_{ij} - \tilde{\beta}_{ij})D + x. \tag{20}$$

Hence, the total premium difference consists of a fair part that compensates the change in expected risk and a remaining unfair part denoted by$x$. With this, we are able to apply the concept of the compensating precautionary premium Ψ^* to the present situation. It is implicitly defined by

$$\begin{aligned} & p_j\, u'\big((1-\tau)w_i L^*_{ij} + T - \overline{p}\,\alpha D - d_{ij} - (1-\alpha-\beta_{ij})D\big) \\ & + (1-p_j)\, u'\left((1-\tau)w_i L^*_{ij} + T - \overline{p}\alpha D - d_{ij}\right) \\ = \; & p_j\, u'\big((1-\tau)w_i L^*_{ij} + T - \overline{p}\,\alpha D + \Psi^* - x - \tilde{d}_{ij} - (1-\alpha-\tilde{\beta}_{ij})D\big) \\ & + (1-p_j)\, u'\big((1-\tau)w_i L^*_{ij} + T - \overline{p}\alpha D + \Psi^* - x - \tilde{d}_{ij}\big). \end{aligned} \tag{21}$$

Comparing (21) with (19), we conclude that there is no change in labor supply if the compensating precautionary premium equals the unfair premium difference: $\Psi^* = x$. In addition, given positive prudence and $\tilde{\beta}_{ij} < \beta_{ij}$ we obtain that $\tilde{L}_{ij} > L^*_{ij}$ if $\Psi^* > x$ and vice versa.[24] This result is best understood by going back to the intuition from section 2, where only changes in

[24]Using (3), we could equivalently state the result in terms of the equivalent precautionary premium Ψ as follows:

$$\tilde{L}_{ij} \gtreqless L^*_{ij} \quad \Leftrightarrow \quad \Psi \lesseqgtr x.$$

variance were considered. Here, this is captured by the different shares of insurance coverage while the only remaining additional income effect is due to the unfair part x of the insurance premium difference. If it is positive, it counteracts the precautionary motive.[25] It just compensates it if it equals the compensating precautionary premium.

We are also able to say something on the size of the effects. For instance, if the coefficient of prudence is sufficiently high, then the compensating premium is high and we obtain $\tilde{L}_{ij} > L^*_{ij}$, i. e. the individual supplies more labor in the more risky contract. The same holds if the unfair part x of the premium difference is sufficiently small.

4.2 Rothschild-Stiglitz Separating Equilibrium

In this section we consider separating equilibria as suggested by Rothschild and Stiglitz (1976) in each of the W private insurance markets. Given a productivity group i, a Rothschild-Stiglitz equilibrium is defined as a set of contracts $\mathcal{I}_{ij}$ with the properties that all individuals choose the contract that maximizes their utility, no contract in the set is associated with expected losses and there exists no contract which, when offered in addition to that set, would earn strictly positive expected profits. As Rothschild and Stiglitz have shown, such an equilibrium cannot be pooling in the sense that both risk types choose the same contract. However, a separating equilibrium exists if the share of high risks m_{iH}/m_i is sufficiently high which we shall assume for all i.

The Rothschild-Stiglitz equilibrium consists of two contracts, one for the high and one for the low risks. The high-risk types get their best contract given an individually fair premium. It is the solution of

$$\max_{\beta_{iH}} V^*_{iH}(\tau, T, \alpha, \beta_{iH}, d_{iH}) \quad \text{s.t.} \quad d_{iH} = p_H \beta_{iH} D$$

with the necessary condition

$$\left.\frac{\partial V^*_{iH}}{\partial \beta_{iH}}\right|_A = \frac{\partial V^*_{iH}}{\partial \beta_{iH}} - p_H D \frac{\partial V^*_{iH}}{\partial T} = 0. \tag{22}$$

[25] In case of mimicking an individual with less insurance, we always must have $x > 0$. This follows from the indifference condition $V^*_{ij}(\tau, T, \alpha, d_{ij}, \beta_{ij}) = V^*_{ij}(\tau, T, \alpha, \tilde{d}_{ij}, \tilde{\beta}_{ij})$ and risk aversion. It implies that individual ij must be compensated for the lower coverage $\tilde{\beta}_{ij} < \beta_{ij}$ by a more than fair premium reduction, i. e. $\tilde{d}_{ij} < d_{ij} - p_j(\beta_{ij} - \tilde{\beta}_{ij}) \Leftrightarrow x > 0$.

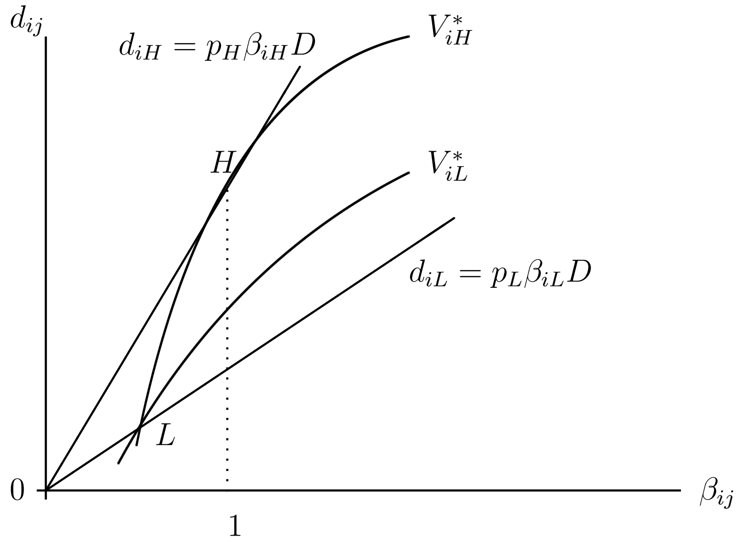

Figure 1: Rothschild-Stiglitz separating equilibrium ($\alpha = 0$)

It is straightforward to show that the resulting contract is given by $\beta_{iH} = 1-\alpha$ and $d_{iH} = p_H(1-\alpha)D$. Hence, the high risks get their first-best contract with full insurance. The low risks' equilibrium contract, by contrast, lies on the low risks' zero-profit line and is such that the high risks' incentive compatibility constraint is just binding. Formally, β_{iL} solves

$$V^*_{iH}(\tau, T, \alpha, 1-\alpha, p_H(1-\alpha)D) = V^*_{iH}(\tau, T, \alpha, \beta_{iL}, p_L\beta_{iL}D). \qquad (23)$$

The LHS of (23) indicates the utility the high risks derive from their own contract. The incentive compatibility constraint requires that they cannot do better by mimicking the low-risk types, i. e. by choosing the contract $(\beta_{iL}, p_L\beta_{iL}D)$. Utility from mimicking is given by the RHS of (23) and shall be denoted with the shortcut $\tilde{V}_{iH}$ in the following. Clearly, the low risks are underinsured. Figure 1 illustrates the equilibrium for the case $\alpha = 0$.

The two conditions (22) and (23) allow us to simplify the optimality conditions (15) to (17) from section 3. First of all, as all individuals pay an actuarially fair premium, the premium adjusted effect of insurance coverage on labor supply reduces to

$$\left.\frac{\partial L^*_{ij}}{\partial \beta_{ij}}\right|_A = -\frac{\partial L^*_{ij}}{\partial T}\frac{\partial \Psi}{\partial \beta_{ij}} < 0, \qquad (24)$$

which is a pure precautionary effect. In addition, (24) completely vanishes for the high risks as $\partial\Psi/\partial\beta_{ij} = 0$ in case of full insurance. Next, we obtain

$$\left.\frac{\partial V_{ij}^*}{\partial\beta_{ij}}\right|_A = \frac{\partial V_{ij}^*}{\partial\beta_{ij}} - p_j D \frac{\partial V_{ij}^*}{\partial T}.$$

Whereas this is again zero for the high risks because of (22), the low-risk types derive positive utility from a marginal premium-compensated increase in coverage. As can be seen in Figure 1, the slope of their indifference curve in contract L must be steeper than their zero profit line, which implies

$$\left.\frac{\mathrm{d}d_{iL}}{\mathrm{d}\beta_{iL}}\right|_{V^*=\bar{V}} = \frac{\partial V_{iL}^*/\partial\beta_{iL}}{\partial V_{iL}^*/\partial T} > p_L D. \tag{25}$$

With these results, we are able to rewrite the net social marginal valuation of insurance g_{ij} as defined by (13) for the two risks types as follows:

$$g_{iL} = \frac{1}{\gamma}\left(\frac{\partial V_{iL}^*}{\partial\beta_{iL}} - p_L D \frac{\partial V_{iL}^*}{\partial T}\right) + \tau w_i \frac{\partial L_{ij}^*}{\partial T}\frac{\partial\Psi}{\partial\beta_{iL}} \tag{26}$$

and

$$g_{iH} = 0. \tag{27}$$

There are two counteracting welfare effects of variations in insurance coverage for the low risks captured in (26). First, any actuarially fairly adjusted increase in β_{iL} is clearly welfare enhancing. Given positive prudence, however, the reduced risk also induces them to supply less labor, which reduces income tax revenue and therefore welfare.[26] Hence, the sign of g_{iL} is generally ambiguous and depends notably on the size of the coefficient of prudence. The net social marginal valuation of insurance for the high-risk types is zero. This is a direct implication of the fact that they obtain their first-best insurance contract.

The comparative static effects of the policy parameters on insurance coverage remain to be discussed. This is the last step to close our model. We demonstrate in the following subsections how the optimality conditions (15) to (17) can be interpreted under the present equilibrium concept.

[26] Two similar effects have been found by Boadway, Leite-Monteiro, Marchand, and Pestieau (2004). Their 'labour market efficiency term' is hard to qualify and interpret, however, as it is based on counteracting income effects in the different states of nature.

4.2.1 Optimal Transfer T

The high-risk types' equilibrium insurance coverage is $\beta_{iH} = 1 - \alpha$ in the Rothschild-Stiglitz equilibria, which of course implies $\partial\beta_{iH}/\partial T = 0$. Hence, there are no effects of the transfer on the high risks' insurance. By contrast, variations in T in general influence the low risks' insurance contracts as can be seen from differentiating (23) implicitly:

$$\frac{\partial\beta_{iL}}{\partial T} = -\frac{\partial V_{iH}^*/\partial T - \partial\tilde{V}_{iH}/\partial T}{\partial\tilde{V}_{iH}/\partial\beta - p_L D\partial\tilde{V}_{iH}/\partial T}. \tag{28}$$

First, we can show that the denominator of (28) is positive. Indeed, (25) and the single-crossing property imply together

$$\frac{\partial\tilde{V}_{iH}/\partial\beta}{\partial\tilde{V}_{iH}/T} > \frac{\partial V_{iL}^*/\partial\beta}{\partial V_{iL}^*/T} > p_L D.$$

In signing the numerator of (28), it is helpful to make use of the first order condition (6) which provides us with the following two relations:

$$\frac{\partial V_{iH}^*}{\partial T} = -\frac{v'(L_{iH}^*)}{(1-\tau)w_i} \quad \text{and} \quad \frac{\partial\tilde{V}_{iH}}{\partial T} = -\frac{v'(\tilde{L}_{iH})}{(1-\tau)w_i}, \tag{29}$$

where L_{iH}^* denotes the high risks' optimal labor supply in case they choose the contract intended for them and $\tilde{L}_{iH}$ stands for their optimal labor supply when they imitate the low risks. Due to the assumption of increasing marginal disutility of labor, we obtain

$$\frac{\partial\beta_{iL}}{\partial T} > 0 \quad \Leftrightarrow \quad \tilde{L}_{iH} > L_{iH}^*.$$

An increased lump-sum transfer relaxes the incentive compatibility constraint (23) if the high risks increase their labor supply when imitating the low risks in the insurance market. This is the case if their preferences are characterized by a high measure of prudence. Using our insights from section 4.1, we can even derive a more explicit result. There we concluded that more labor is supplied in the riskier contract if the compensated precautionary premium exceeds the unfair premium difference and vice versa. Transferring this to the Rothschild-Stiglitz equilibria, we can determine the comparative static effect as follows:

$$\frac{\partial \beta_{iL}}{\partial T} \gtreqless 0 \quad \Leftrightarrow \quad \Psi^* \gtreqless (p_H - p_L)\beta_{iL} D.$$

Finally, the optimality condition (15) now reads

$$\bar{b} + \sum_i n_{iL} \left(\frac{1}{\gamma} \left(\frac{\partial V^*_{iL}}{\partial \beta_{iL}} - p_L D \frac{\partial V^*_{iL}}{\partial T} \right) - \tau w_i \frac{\partial L^*_{iL}}{\partial T} \frac{\partial \Psi}{\partial \beta_{iL}} \right) \frac{\partial \beta_{iL}}{\partial T} = 1. \tag{30}$$

Compared to the standard result $\bar{b} = 1$, (30) accounts for the additional effects caused by a marginally higher transfer T on the underinsured population. If the precautionary labor motive is strong, the low risks get more insurance coverage. On the one hand, this enhances welfare since underinsurance is inefficient. However, as less labor is supplied, total welfare may decrease, which is more likely the more prudent the households are. Our theory of precautionary labor therefore proves crucial in determining whether imperfect insurance markets justify a higher or lower lump-sum transfer compared to the framework of Cremer and Pestieau (1996).

4.2.2 Optimal tax rate τ

Again we find no effect of tax policy on the high risks' insurance contracts. As for the low risks, implicit differentiation of (23) yields

$$\frac{\partial \beta_{iL}}{\partial \tau} = -\frac{\partial V^*_{iH}/\partial T\,(-w_i L^*_{iH}) - \partial \tilde{V}_{iH}/\partial T\,(-w_i \tilde{L}_{iH})}{\partial \tilde{V}_{iH}/\partial \beta - p_L D \partial \tilde{V}_{iH}/\partial T}. \tag{31}$$

Again, (31) has the opposite sign of its numerator. Substitution of the optimality conditions (29) and some rearrangements yield as condition for the tax effect being positive

$$\frac{v'(L^*_{iH})}{v'(\tilde{L}_{ih})} > \frac{\tilde{L}_{iH}}{L^*_{iH}}.$$

Due to concavity of v this is equivalent to

$$\frac{\partial \beta_{iL}}{\partial \tau} > 0 \quad \Leftrightarrow \quad \tilde{L}_{iH} < L^*_{iH}. \tag{32}$$

This reveals that the sign of $\partial \beta_{ij}/\partial \tau$ will always be the opposite of the sign of $\partial \beta_{ij}/\partial T$. It is therefore ambiguous how the additional effects of taxation on the insurance market affect the first order condition for the optimal rate

τ compared to the result of Cremer and Pestieau (1996). Both cases in which imperfect insurance markets call for higher and for lower tax rates are imaginable.

4.2.3 Optimal Social Insurance α

We now examine the case for social insurance in the presence of separating equilibria. Clearly, as the high risks are always fully insured, social insurance completely crowds out their private insurance: $\partial\beta_{iH}/\partial\alpha = -1$. Hence, in total there are again no effects on the high risks insurance and the optimality condition (17) simplifies to

$$D\,\mathrm{Cov}(b,p) = -\sum_i n_{iL} g_{iL} \left(1 + \frac{\partial\beta_{iL}}{\partial\alpha}\right). \tag{33}$$

(33) indicates that two factors determine whether the argument for social insurance is reinforced compared to the results of Cremer and Pestieau (1996). First, it depends on whether the low risks' total insurance coverage is increased or decreased in response to a change in α. Implicitly differentiating (23), we obtain

$$\frac{\partial\beta_{iL}}{\partial\alpha} = -\frac{\partial\tilde{V}_{iH}/\partial\beta - \left((p_H - \bar{p})\partial V^*_{iH}/\partial T + \bar{p}\,\partial\tilde{V}_{iH}/\partial T\right) D}{\partial\tilde{V}_{iH}/\partial\beta - p_L D \partial\tilde{V}_{iH}/\partial T}. \tag{34}$$

The denominator is known from (28) where it was shown to be positive. In addition, due to

$$(p_H - \bar{p})\frac{\partial V^*_{iH}}{\partial T} + \bar{p}\frac{\partial\tilde{V}_{iH}}{\partial T} > p_L \frac{\partial\tilde{V}_{iH}}{\partial T}$$

the numerator is smaller than the denominator in (34), which establishes

$$\frac{\partial\beta_{iL}}{\partial\alpha} > -1. \tag{35}$$

Thus, a marginal increase in social insurance unambiguously mitigates the inefficiency in the private insurance market. However, this does not imply that the case for social insurance is reinforced. Inspection of (33) reveals that the sign of g_{iL} is crucial as well. If households are very prudent, the negative welfare effect resulting from decreased labor supply dominates the positive effect from higher insurance coverage and g_{iL} is negative. It could then even be optimal to completely renounce on the introduction of a social insurance system despite the desired redistributive and efficiency impacts in the insurance market. This result demonstrates that the argument put forward by Rochet (1991) mentioned above is not valid. The desirability of social insurance in the presence of perfect private insurance markets does not imply its desirability given insurance market imperfections.

4.3 Wilson Pooling Equilibrium

In order to examine to what extent the results in the previous subsection 4.2 depend on the particularities of the Rothschild-Stiglitz equilibrium concept, we now consider an alternative notion of equilibrium going back to Wilson (1977). To qualify as a Rothschild-Stiglitz equilibrium a set of contracts must be such that no contract outside this set generates strictly positive expected profits. This requirement is modified in the concept of Wilson (1977). Here, there must not exist any contract outside the equilibrium set which would earn strictly positive profits after the unprofitable contracts in the original set have been withdrawn. As was shown by Wilson (1977), if a Rothschild-Stiglitz separating equilibrium exists, then it is also an equilibrium in the sense of this modified notion. Otherwise, the Wilson concept allows for the existence of a pooling equilibrium where both risk types choose the same contract. We will consider this second case in the following.

Given a productivity group i, the Wilson pooling equilibrium is such that both risk types choose the low risks' preferred contract on the zero profit line for the whole group. Consequently, it is determined by the program

$$\max_{\beta_i} V_{iL}^*(\tau, T, \alpha, \beta_i, d_i) \quad \text{s. t.} \quad d_i = \bar{p}_i \beta_i D, \tag{36}$$

where β_i is the private insurance coverage for both risk types. The first order condition is

$$\left.\frac{\partial V_{iL}^*}{\partial \beta_i}\right|_A = \frac{\partial V_{iL}^*}{\partial \beta_i} - \bar{p}_i D \frac{\partial V_{iL}^*}{\partial T} = 0, \tag{37}$$

which, using the definition of V_{iL}^*, is equivalent to

$$p_L(1-\bar{p}_i)u'(c_{iL}^0) - (1-p_L)\bar{p}_i u'(c_{iL}^1) = 0. \tag{38}$$

As before, c_{iL}^0 and c_{iL}^1 in (38) stand for the consumption levels of type iL in

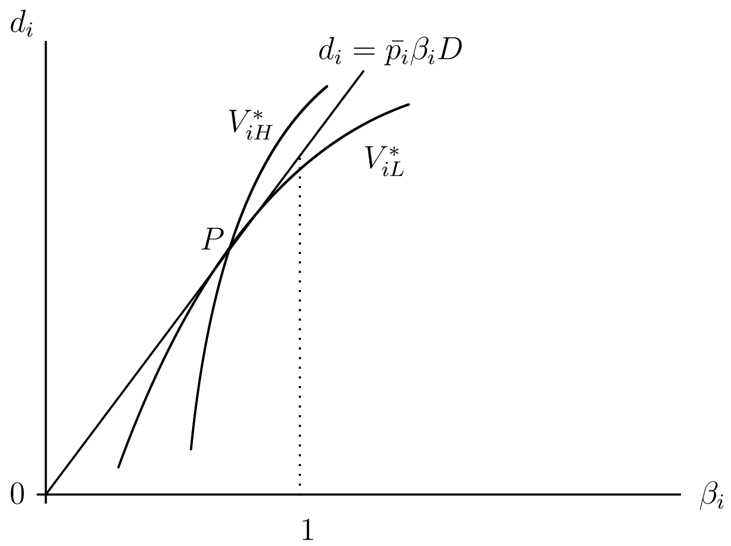

Figure 2: Wilson pooling equilibrium ($\alpha = 0$)

case of loss and no loss, respectively, with optimized labor supply. Clearly, (38) implies

$$\frac{u'(c_{iL}^1)}{u'(c_{iL}^0)} = \frac{p_L(1-\bar{p}_i)}{\bar{p}_i(1-p_L)} < 1$$

and thus $\beta_i < 1-\alpha$ for all i. We therefore find two fundamental differences to the properties of the Rothschild-Stiglitz equilibria. First, all individuals pay an individually unfair premium $\bar{p}_i\beta_i D$. This is because pooling equilibria involve cross-subsidization from low to high risks. Second, and more importantly, the whole population is now underinsured. Figure 2 provides a graphical illustration for the case $\alpha = 0$.

Turning first to the effect of a premium adjusted change in β_i on labor supply, we obtain

$$\left.\frac{\partial L_{ij}^*}{\partial \beta_i}\right|_A = \left((p_j - \bar{p}_i)D - \frac{\partial \Psi}{\partial \beta_i}\right)\frac{\partial L_{ij}^*}{\partial T}. \tag{39}$$

Compared to (24), we find an additional income effect from the unfair premium. In the Wilson equilibria, the high risks pay a premium which is more than fair for them. Their labor supply is hence reduced given that leisure is a normal good. Together with the negative precautionary effect we find that higher insurance coverage unambiguously reduces the high-risk types' labor supply. By an analogous argument, the effect is in general undetermined for the low risks but also negative if they are sufficiently prudent.

Similarly, the change in expected utility derived from a compensated variation in insurance coverage is now given by

$$\left.\frac{\partial V_{ij}^*}{\partial \beta_i}\right|_A = \frac{\partial V_{ij}^*}{\partial \beta_i} - \bar{p}_i D \frac{\partial V_{ij}^*}{\partial T}.$$

This vanishes for the low risks because of (37) and is positive for the high risks due to the assumed single-crossing property. Thus, in a reversal of our findings based on the separating equilibria, only the high risks derive a direct utility gain from increased coverage.

With these two insights, we can discuss the net social marginal valuation of insurance g_{ij} for the two types. For the low risks, we are left with

$$g_{iL} = \tau w_i \left((p_L - \bar{p}_i) D - \frac{\partial \Psi}{\partial \beta_i} \right) \frac{\partial L_{iL}^*}{\partial T}, \tag{40}$$

which is the ambiguous labor supply distortion explained above. The high risks in turn are characterized by

$$g_{iH} = \frac{1}{\gamma} \left(\frac{\partial V_{iH}^*}{\partial \beta_i} - \bar{p}_i D \frac{\partial V_{iH}^*}{\partial T} \right) + \tau w_i \left((p_H - \bar{p}_i) D - \frac{\partial \Psi}{\partial \beta_i} \right) \frac{\partial L_{iH}^*}{\partial T}. \tag{41}$$

Their net social marginal gain from insurance captures the two opposite effects on utility and labor supply and is therefore ambiguous as well. Yet, both terms again crucially depend on the strength of the precautionary motive.

As in section 4.2, we are finally able to close the model by considering the comparative static effects of the policy parameters on private insurance coverage. A general comment on how to obtain these effects is necessary. Obviously, the dependence of β_i on the policy variables can be determined by differentiating (37). This involves an additional complication compared to our analysis in section 4.2 based on the incentive compatibility constraint (23). There, the envelope theorem allowed us to ignore indirect effects via labor supply. This is no longer possible in the present framework since (37) is not expressed in terms of levels of indirect utility, but of marginal rates of substitution. These are influenced by labor supply and hence the envelope theorem does not apply. The need to account explicitly for all indirect effects through L_{iL}^* in differentiating (37) will complicate our comparative static analysis.

4.3.1 Optimal Transfer T

For the effect of a marginal increase in the transfer T we obtain

$$\frac{\partial \beta_i}{\partial T} = -\frac{p_L(1-\bar{p}_i)u''(c_{iL}^0) - \bar{p}_i(1-p_L)u''(c_{iL}^1)}{SOC}\left(1 + (1-\tau)w_i \frac{\partial L_{iL}^*}{\partial T}\right), \quad (42)$$

where $SOC < 0$ is the second order condition of (36). In case of positive prudence, we have $u''(c_{iL}^0) < u''(c_{iH}^1) < 0$ but $p_L(1-\bar{p}_i) < \bar{p}_i(1-p_L)$, so that even the first factor cannot be signed. However, with a strong precautionary motive it is negative which extends to the whole effect if the counteracting indirect income effect on labor supply is not dominating. Again assuming that the prudence is sufficiently high, the optimality condition (15) then calls for a higher transfer compared to the standard result $\bar{b} = 1$ since it reduces coverage and this is welfare enhancing under these circumstances.

4.3.2 Optimal Tax Rate τ

Implicit differentiation of (38) using (42) yields

$$\frac{\partial \beta_i}{\partial \tau} = -w_i L_{iL}^* \left(1 + \frac{\epsilon_{iL}}{1 + (1-\tau)w_i \partial L_{iL}^*/\partial T}\right)\frac{\partial \beta_i}{\partial T}, \quad (43)$$

where ϵ_{iL} denotes the elasticity of the compensated labor supply with respect to the net wage. The second term in brackets in (43) accounts for the effect of the labor distortion on the marginal rate of substitution of the low risks and hence on equilibrium insurance coverage. In general this is nonzero as the Envelope theorem does not apply. Its sign cannot be determined generally, however, as the sign of its denominator is ambiguous. As for the case of a Rothschild-Stiglitz equilibrium, market imperfections might call both for a higher or a lower optimal tax rate τ.

4.3.3 Optimal Social Insurance α

The case for social insurance is considerably modified when pooling equilibria in the private insurance market are considered. As becomes obvious from the optimality condition

$$D\,\mathrm{Cov}(b_{ij}, \bar{p}_i) = -\sum_{ij} n_{ij} g_{ij}\left(1 + \frac{\partial \beta_i}{\partial \alpha}\right), \quad (44)$$

a first difference to (33) relates to the redistributive impact of social insurance. Instead of $\mathrm{Cov}(b_{ij}, p_j)$, we find the covariance between the net social marginal valuation of income and the *average* damage probabilities in the productivity groups. Clearly, social insurance is now no longer able to redistribute between high and low risks directly as this is already achieved by cross-subsidization in the pooling equilibria. However, it is still possible to redistribute across productivity classes as long as they are characterized by different average risks and hence different private insurance premia. This is captured in $\mathrm{Cov}(b_{ij}, \bar{p}_i)$.

Second, for the effect of α on private coverage we obtain

$$\begin{aligned}\frac{\partial \beta_i}{\partial \alpha} &= -D\frac{(1-\bar{p})(1-\bar{p}_i)p_L u''(c^0_{iL}) + \bar{p}\bar{p}_i(1-p_L)u''(c^1_{iL})}{SOC} \\ &\quad +(1-\tau)w_i\frac{\partial L^*_{iL}}{\partial \alpha}\frac{\partial \beta^*_i/\partial T}{1+(1-\tau)w_i\partial L^*_{iL}/\partial T}. \end{aligned} \tag{45}$$

This expression again demonstrates that the indirect effects on labor supply are relevant and make the overall impact very complicated. We are now unable to rule out $\partial\beta_i/\partial\alpha < -1$ and hence that an increase in social insurance has a negative influence on the efficiency of the insurance market. Such an effect would be favored by a sufficiently high coefficient of prudence, implying that the first factor in (42) is negative, and a strong negative income effect $\partial L^*_{iL}/\partial T$. In this case, the second term in (45) is negative (note that the first one is always negative) and $\partial\beta_i/\partial\alpha < -1$ is possible. This result weakens the argument by Rochet (1991) mentioned above even more.

In sum, we find that the characterization of optimal taxes and social insurance is highly sensitive to the underlying equilibrium concept. Many effects and their normative implications are modified when considering pooling instead of separating equilibria. It is all the more encouraging to see that our model framework is general and flexible enough to be easily applied to these different approaches to the insurance market. We are even able to include the possibility of a co-existence of pooling and separating equilibria for different productivity groups. This could be the case with the Wilson concept and the assumption that separating equilibria exist on some but not all the W markets. We would then simply have to combine the effects found in sections 4.2 and 4.3 in our general optimality conditions.

5 Conclusion

We have aimed at developing a theory of optimal taxation and social insurance in the presence of imperfect private insurance markets. While the problem of taxation requires modelling the households' choice of labor supply endogenously, inefficient insurance markets imply that they have to take this decision under risk. Hence, a theory of labor supply under uncertainty provides the basis for our analysis. As we have shown, there exists a motive for precautionary labor supply under meaningful circumstances.

We then integrated this theory into a model of taxation and social insurance with imperfect insurance markets. This allowed us to show how the optimality conditions based on efficient insurance markets have to be modified. Notably, the strength of the precautionary labor effect turned out to be crucial in determining whether taxes or social insurance should be higher or lower compared to the earlier results of Cremer and Pestieau (1996).

Considering specific equilibrium concepts in the insurance markets, we finally demonstrated the applicability of our model. We draw two main conclusions from this analysis. First, the positive and normative results are highly sensitive to the equilibrium concept used. In particular, the redistributive effects of social insurance depend on whether separating or pooling equilibria exist in the insurance market. Second, contrary to some arguments found in the literature, the case for social insurance is not necessarily reinforced by the existence of insurance market imperfections. Social insurance might even worsen the inefficiency of the insurance market and will have welfare reducing effects in the labor market.

Our paper has raised a variety of unexplored issues. A straightforward extension of our model would be to allow for non-linear income taxation and social insurance. However, we doubt that this would change the results significantly as the intuition for the redistributive effects is independent from the assumed linearity. Further questions relate to the theory of adverse selection with endogenous labor supply that we have developed. What additional interactions between fiscal policy, private insurance and the labor market could emerge? And how do standard results such as the possibility of a Pareto-improving introduction of partial mandatory insurance need to be modified if variable labor supply is accounted for? These issues remain to be addressed by future research.

6 Appendix

6.1 Appendix A: Risk Aversion and Prudence

The Arrow-Pratt coefficient of absolute risk aversion for the consumption part $u(c)$ of the utility function is defined as

$$r_A(c) = -\frac{u''(c)}{u'(c)} > 0. \tag{46}$$

Absolute risk aversion is therefore constant or decreasing if

$$\frac{\partial r_A(c)}{\partial c} = -\frac{u'(c)u'''(c) - [u''(c)]^2}{[u'(c)]^2} \leq 0. \tag{47}$$

A necessary condition for this to be fulfilled is that the numerator of (47) is positive. For this in turn it is necessary that $u'''(c)$ is positive. Therefore, whenever absolute risk aversion is constant or decreasing, the third derivative of the utility function is positive and therefore the prudence is positive as well. A negative $u'''(c)$ and therefore a negative prudence $\eta(c) = -u'''(c)/u''(c)$ is a sufficient condition for increasing absolute risk aversion, which is not a realistic assumption.

Analogously, the coefficient of relative risk aversion is given by

$$r_R(c) = -\frac{c\,u''(c)}{u'(c)} > 0. \tag{48}$$

Relative risk aversion is constant or decreasing if

$$\frac{\partial r_R(c)}{\partial c} = -\frac{u'(c)[c\,u'''(c) + u''(c)] - c\,[u''(c)]^2}{[u'(c)]^2} \leq 0. \tag{49}$$

With the same argument as above this can only be fulfilled if the numerator is positive, which requires $u'''(c)$ to be positive. The argument therefore also applies to relative risk aversion.

6.2 Appendix B: The Precautionary Premiums

Assume that a damage D, of which a share β is insured (free of charge), occurs with probability p. This defines a Bernoulli random variable with expectation $p\,(1-\beta)D$ and variance $\sigma^2(\beta) = p\,(1-p)[(1-\beta)D]^2$.

The condition for optimal labor supply $L^*(\beta)$ is

$$w[p\,u'(wL^* + T - (1-\beta)D) + (1-p)\,u'(wL^* + T)] = -v'(L^*). \tag{50}$$

The income effect can be derived by implicitly differentiating (50):

$$\frac{\partial L^*}{\partial T} = -\frac{w\,[pu''(wL^* + T - (1-\beta)D) + (1-p)u''(wL^* + T)]}{SOC}, \tag{51}$$

where SOC stands for the second derivative of the objective with respect to L and is negative since the sufficient condition for a maximum is fulfilled. Therefore, the income effect is negative, which implies that leisure is a normal good. This results from the assumption of separable utility.

Implicit differentiation of (50) w.r.t. β yields after some rearrangements

$$\frac{\partial L^*}{\partial \beta} = \frac{\partial L^*}{\partial T} pD - \frac{w[u''(wL^* + T - (1-\beta)D) - u''(wL^* + T)]}{SOC} p(1-p)D. \tag{52}$$

The income effect due to decreased expected damage is already visible as the first term on the RHS of (52). Substituting $\Delta u''(\beta)$ for $u''(wL^* + T) - u''(wL^* + T - (1-\beta)D)$ and using the income effect (51) and the fact that the first derivative of the variance $\sigma^2(\beta)$ is given by

$$\frac{\partial \sigma^2}{\partial \beta} = -2p(1-p)(1-\beta)D^2, \tag{53}$$

the effect (52) can be transformed to

$$\frac{\partial L^*}{\partial \beta} = \frac{\partial L^*}{\partial T}\left[pD - \frac{\partial \Psi}{\partial \beta}\right], \tag{54}$$

where

$$\frac{\partial \Psi}{\partial \beta} = \left(-\frac{\Delta u''(\beta)/(1-\beta)D}{p\,u''(wL^* + T - (1-\beta)D) + (1-p)\,u''(wL^* + T)}\right)\left(\frac{1}{2}\frac{\partial \sigma^2}{\partial \beta}\right). \tag{55}$$

To see why (55) denotes the derivative of Ψ, consider a marginal variation of β at β close to 1 (full insurance). The first term on the RHS then becomes the prudence η and (55) boils down to the derivative of (4) with respect to β. The first term on the RHS of (55) is therefore a generalized, secant version of the prudence, applicable to situations with preexisting risk. It has the sign of $\Delta u''(\beta)$. This in turn is equal to the sign of u''' or the conventional prudence, respectively, if these do not change their sign on the range between $wL^* + T - (1-\beta)D$ and $wL^* + T$. In that case, the whole term $\partial\Psi/\partial\beta$ is negative if the prudence is positive, and vice versa.

One can apply the product rule for integration to (55) to obtain a generalized version of (4) for the case of the described Bernoulli risk:

$$\Psi\left(\tilde{\theta}(\beta_0), \tilde{\theta}(\beta_1)\right) = \left[\eta(\beta)\frac{\sigma^2(\beta)}{2}\right]_{\beta_0}^{\beta^1} - \int_{\beta_0}^{\beta_1} \frac{1}{2}\,\sigma^2(\beta)\frac{\partial\,\eta(\beta)}{\partial\beta}\,\mathrm{d}\beta. \tag{56}$$

In (56), $\eta(\beta)$ represents the generalized prudence. Note that changes of$L^*(\beta)$ have to be taken into account when integrating. Still, the qualitative relation between prudence and the premium Ψ remain the same even for large changes in β.

6.3 Appendix C: The Optimal Policy

The first order condition for the optimal transfer is

$$\sum_{i,j} n_{ij}\frac{\partial V_{ij}^{**}}{\partial T} + \gamma\sum_{i,j} n_{ij}\left(-1 + \tau w_i \frac{\partial L_{ij}^{**}}{\partial T}\right) = 0. \tag{57}$$

The effect of T on L_{ij}^{**} can be reduced to effects on L_{ij}^{*} by explicitly taking into account its effect on the private insurance market equilibrium:

$$\frac{\partial L_{ij}^{**}}{\partial T} = \frac{\partial L_{ij}^{*}}{\partial T} + \left.\frac{\partial L_{ij}^{*}}{\partial \beta_{ij}}\right|_A \frac{\partial \beta_{ij}}{\partial T}. \tag{58}$$

The same decomposition can be applied to V_{ij}^{**} and is of course also applicable to the effects of the other government parameters. Using this, as well as the two concepts of marginal social valuation (b_{ij} and g_{ij}) defined in section 3, the optimality condition (15) follows after some rearrangements.

Consider next the first order condition for the optimal level of social insurance:

$$\sum_{i,j} n_{ij}\frac{\partial V_{ij}^{**}}{\partial \alpha} + \gamma\sum_{i,j} n_{ij}\,\tau w_i \frac{\partial L_{ij}^{**}}{\partial \alpha} = 0. \tag{59}$$

Again, the effects of α on L_{ij}^{**} and V_{ij}^{**} can be reduced to effects on L_{ij}^{*} and

V_{ij}^*. After some rearrangements, one obtains

$$\sum_{i,j} n_{ij} \left(\frac{1}{\gamma} \frac{\partial V_{ij}^*}{\partial \alpha} + \tau w_i \frac{\partial L_{ij}^*}{\partial \alpha} \right) + \sum_{i,j} n_{ij} g_{ij} \frac{\partial \beta_{ij}}{\partial \alpha} = 0. \tag{60}$$

The first term on the LHS of (60) can further be transformed by noting that effects of α on labor supply can be expressed as effects of β_{ij} as follows:

$$\frac{\partial L_{ij}^*}{\partial \alpha} = \left. \frac{\partial L_{ij}^*}{\partial \beta_{ij}} \right|_A + (d'_{ij} - \bar{p}D) \frac{\partial L_{ij}^*}{\partial T}, \tag{61}$$

where d'_{ij} stands short for $\partial d_{ij} / \partial \beta_{ij}$. Equation (61) follows from the fact the changes in social and private insurance differ only with respect to their different premiums. The same decomposition holds for indirect utility. Substituting this into (60) yields

$$\sum_{i,j} n_{ij} g_{ij} \left(1 + \frac{\partial \beta_{ij}}{\partial \alpha} \right) + \sum_{i,j} n_{ij} b_{ij} (d'_{ij} - \bar{p}D) = 0. \tag{62}$$

After adding and subtracting $\sum_{i,j} n_{ij} \bar{b} (d'_{ij} - \bar{p}D)$ one obtains

$$\sum_{i,j} n_{ij} g_{ij} \left(1 + \frac{\partial \beta_{ij}}{\partial \alpha} \right) + \sum_{i,j} n_{ij} (b_{ij} - \bar{b})(d'_{ij} - \bar{p}D) + \bar{b} \sum_{i,j} n_{ij} (d'_{ij} - \bar{p}D) = 0. \tag{63}$$

The last term on the LHS of (63) is equal to zero since aggregate profits of insurance companies in a competitive market equilibrium are zero. Therefore, higher insurance coverage for all individuals will be accompanied by adjustments in the premiums such that additional revenues equal additional expected insurance payments on the population average.[27] The optimality condition (17) now follows.

Finally, the first order condition for the optimal tax rate is

$$\sum_{i,j} n_{ij} \frac{\partial V_{ij}^{**}}{\partial \tau} + \gamma \sum_{i,j} n_{ij} \left(w_i L_{ij}^{**} + \tau w_i \frac{\partial L_{ij}^{**}}{\partial \tau} \right) = 0 \tag{64}$$

[27]This holds under a vast majority of insurance market equilibria, among them those discussed in section 4. However, cases are imaginable in which it is not be fulfilled, for example if insurance companies were only able to make price offers and low risks would not purchase any coverage. The optimality condition (17) is therefore not applicable to such cases but a modified condition would have to be derived.

Following the same first steps as before yields

$$\sum_{i,j} n_{ij} g_{ij} \frac{\partial \beta_{ij}}{\partial \tau} + \sum_{i,j} n_{ij} \left(\frac{1}{\gamma} \frac{\partial V_{ij}^*}{\partial \tau} + \tau w_i \frac{\partial L_{ij}^*}{\partial \tau} \right) + \sum_{i,j} n_{ij} w_i L_{ij}^* = 0. \tag{65}$$

Applying the Envelope theorem, the effect of τ on V_{ij}^* boils down to a pure income effect. The tax rate effect on labor supply can be transformed using the Slutzky decomposition (14). After some rearrangements one obtains

$$\frac{\tau}{1-\tau} = \frac{\sum_{i,j} n_{ij} g_{ij} \frac{\partial \beta_{ij}}{\partial \tau} + \sum_{i,j} n_{ij} w_i L_{ij}^* - \sum_{i,j} n_{ij} b_{ij} w_i L_{ij}^*}{\sum_{i,j} n_{ij} w_i L_{ij}^* \epsilon_{ij}}. \tag{66}$$

According to the first order condition (15) for the optimal transfer T, the term $\bar{b} + \sum_{ij} n_{ij} g_{ij} \, \partial \beta_{ij} / \partial T$ equals one and can therefore simply be multiplied in the second sum in the numerator of (66). After some rearrangements, the optimality condition (16) follows.

6.4 Appendix D: The Single-Crossing Property

By implicitly differentiating the first order condition of the individual labor choice problem (6), we obtain

$$\frac{\partial L_{ij}^*}{\partial p_j} = -\frac{(1-\tau) w_i \left(u'(c_{ij}^0) - u'(c_{ij}^1) \right)}{SOC} > 0 \tag{67}$$

because of risk aversion and the second order condition $SOC < 0$. Hence, the high risk types supply more labor than the low risks at any given contract with less than full insurance. To examine how this affects the marginal rate of substitution between coverage and premium, we therefore need to evaluate the sign of

$$\frac{\mathrm{d}}{\mathrm{d}L_{ij}^*} \left(\left. \frac{\mathrm{d}d_{ij}}{\mathrm{d}\beta_{ij}} \right|_{V^* = \bar{V}} \right) = \frac{(1-\tau) w_i p_j (1-p_j) D \left(u''(c_{ij}^0) u'(c_{ij}^1) - u''(c_{ij}^1) u'(c_{ij}^0) \right)}{\left(p_j u'(c_{ij}^0) + (1-p_j) u'(c_{ij}^1) \right)^2}. \tag{68}$$

Risk aversion implies $u'(c_{ij}^1) < u'(c_{ij}^0)$. Hence, if we have $u''(c_{ij}^0) > u''(c_{ij}^1)$ (note that both are negative) then the sign of (68) is unambiguously positive. This is saying that if the third derivative of u is negative, then the high risks always have the higher marginal rate of substitution because the indirect effect via labor supply runs in the same direction as the direct effect. In this case, the single-crossing property holds.

However, as shown in Appendix A, $u'''(.) < 0$ is a sufficient condition for increasing risk aversion and therefore a rather implausible property of preferences. Unfortunately, in the more realistic case of positive prudence $u'''(.) > 0$ the sign of (68) cannot be determined without further assumptions since the indirect effect of the higher risk via the increased labor supply may decrease the marginal rate of substitution. In order to make sure that the single-crossing property is fulfilled we therefore need to assume that the direct effect dominates the indirect one. A sufficient condition for this is that the ratio p_H/p_L is high. To see this, divide the marginal rate of substitution (18) of the high risks by that of the low risks. After rearranging, this yields

$$\frac{p_H}{p_L}\left(\frac{u'(c_{iH}^0)}{u'(c_{iL}^0)}\frac{p_L u'(c_{iL}^0) + (1-p_L)u'(c_{iL}^1)}{p_H u'(c_{iH}^0) + (1-p_H)u'(c_{iH}^1)}\right). \tag{69}$$

Obviously, if p_H approaches one and p_L approaches zero, then the ratio p_H/p_L tends to infinity while the bracketed term converges to

$$\frac{u'(c_{iH}^0)/u'(c_{iH}^1)}{u'(c_{iL}^0)/u'(c_{iL}^1)},$$

which is strictly positive as the size of the labor adjustment can only be finite.[28] Hence, in this case, the direct effect is always dominating and the high risks have the steeper indifference curves in the (β_{ij}, d_{ij})-plane.

[28]To be more precise, the term is strictly positive as long the labor adjustment is such that marginal utility from consumption does never become zero or infinity for any type.

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