

2155

Discussion  
Papers

# Review of Proxy Vector Autoregressive Analysis

Opinions expressed in this paper are those of the author(s) and do not necessarily reflect views of the institute.

#### IMPRESSUM

© DIW Berlin, 2026

DIW Berlin  
German Institute for Economic Research  
Anton-Wilhelm-Amo-Str. 58  
10117 Berlin

Tel. +49 (30) 897 89-0  
Fax +49 (30) 897 89-200  
<http://www.diw.de>

ISSN electronic edition 1619-4535

Papers can be downloaded free of charge from the DIW Berlin website:  
<http://www.diw.de/discussionpapers>

Discussion Papers of DIW Berlin are indexed in RePEc and Econstor:  
<http://ideas.repec.org/s/diw/diwwpp.html>  
<https://www.econstor.eu/handle/10419/10>

# Review of Proxy Vector Autoregressive Analysis

Martin Bruns

University of East Anglia, School of Economics,  
Norwich Research Park, NR4 7TJ, Norwich, United Kingdom  
email: martin.j.bruns@gmail.com

and

Helmut Lütkepohl

DIW Berlin and FU Berlin, Boltzmannstr. 20, 14195 Berlin, Germany  
email: luetkepohlh@gmail.com

This version: January 30, 2026<sup>1</sup>

**Abstract.** In structural vector autoregressive analysis it has become quite popular to identify some structural shocks of interest by external instruments or proxies. This study points out a range of areas where such proxies have been used and sketches the way the proxies have been constructed. It reviews identification and estimation methods that have been considered in this context. Moreover, it points out some features such as heteroskedasticity, nonfundamentalness of the shocks and violations of the standard assumptions for proxies that may result in complications.

*Key Words:* Structural vector autoregression, proxy VAR, local projection, weak instruments, internal instruments, external instruments, fundamental shocks

*JEL classification:* C32

---

<sup>1</sup>The authors thank Sascha Keweloh, Lutz Kilian, Christian Matthes, Alessio Volpicella, Christian Wolf, an Associate Editor and two anonymous referees for helpful comments on an earlier draft of the paper.

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Proxies</b>	<b>2</b>
2.1	General Formal Properties of Proxies . . . . .	3
2.2	Specific Proxies . . . . .	4
<b>3</b>	<b>Identification</b>	<b>10</b>
3.1	External Instruments . . . . .	11
3.2	Internal Instruments . . . . .	12
3.3	Comparison of External and Internal Proxy VAR Approaches . . . . .	13
<b>4</b>	<b>Estimation and Inference</b>	<b>14</b>
4.1	Conventional Frequentist Estimation Methods . . . . .	15
4.2	Local Projections . . . . .	16
4.3	Bayesian Methods . . . . .	19
4.4	Weak Instruments . . . . .	20
4.5	Bootstrap Intervals for Structural Impulse Responses . . . . .	22
<b>5</b>	<b>Complications</b>	<b>24</b>
5.1	Heteroskedasticity . . . . .	25
5.2	Nonfundamental Shocks . . . . .	25
5.3	Non-exogeneity . . . . .	26
<b>6</b>	<b>Conclusions</b>	<b>28</b>

# 1 Introduction

Using instruments to identify structural shocks of interest has become quite popular in recent structural vector autoregressive (VAR) studies. In this literature, instrumental variables are considered that are closely related to the shocks of interest. They are often called proxies and the related studies are referred to as proxy VAR studies. Examples of such proxies are variables that proxy monetary policy shocks (e.g., Gürkaynak, Sack and Swanson (2007)) or fiscal shocks (e.g., Mertens and Ravn (2012)) or uncertainty shocks (e.g., Piffer and Podstawski (2018)). We will review many more proxies for structural shocks in Section 2, where we also provide more references.

Recently researchers have started to use a set of proxies to identify a set of structural shocks (e.g., Jarociński and Karadi (2020), Hou (2024) among many others). In that case, the proxies often do not identify the individual shocks without making further assumptions. For example, identifying restrictions on the impact responses of the variables or even sign restrictions can be used for fully identifying the shocks of interest. We will discuss a range of possibilities specifically useful for proxy VAR analysis in Section 3. Related estimation and inference methods are considered in Section 4.

We assume that the data generating process (DGP) is a  $K$ -dimensional VAR process with lag-order  $p$ ,

$$y_t = \nu + A_1 y_{t-1} + \dots + A_p y_{t-p} + u_t, \quad (1)$$

where the residual process,  $u_t$ , is zero-mean white noise with nonsingular covariance matrix  $\Sigma_u$ ,  $u_t \sim (0, \Sigma_u)$ , such that the  $u_t$  are serially uncorrelated. They are the one-step ahead prediction errors for  $y_t$  based on the information  $\{y_{t-1}, y_{t-2}, \dots\}$ . Residuals  $u_t$  that can be written as a function of  $y_t, y_{t-1}, \dots$ , are sometimes called  $y_t$ -fundamental (see, e.g., Kilian and Lütkepohl (2017, Chapter 17) for more details). The setup in (1) is the reduced-form of the VAR model.

In our benchmark setup, the structural shocks, denoted as  $\mathbf{w}_t = (w_{1t}, \dots, w_{Kt})'$ , are assumed to be linear combinations of the  $u_t$ ,  $\mathbf{w}_t = B^{-1}u_t$ , such that  $u_t = B\mathbf{w}_t$ . In the following we will refer to shocks that are linear combinations of the fundamental reduced-form residuals  $u_t$  as fundamental.<sup>2</sup> The  $(K \times K)$  transformation matrix  $B$  contains the impact effects of the structural shocks on the variables  $y_t$ . The  $\mathbf{w}_t$  are assumed to have a diagonal covariance matrix  $\Sigma_{\mathbf{w}}$  such that  $B\Sigma_{\mathbf{w}}B' = \Sigma_u$ . If the shocks are normalized to have unit variances and, hence,  $\Sigma_{\mathbf{w}} = I_K$ , the impact effects matrix  $B$  has to be such that  $BB' = \Sigma_u$ .

---

<sup>2</sup>In some of the recent literature, this property is referred to as invertibility (see, e.g., Plagborg-Møller and Wolf (2021)). Forni and Gambetti (2014) and Forni, Franconi, Gambetti and Sala (2025) call the property informational sufficiency.

We assume further that the first  $K_1$  shocks,  $\mathbf{w}_{1t} = (w_{1t}, \dots, w_{K_1t})'$ , are of primary interest and have to be properly identified as economic shocks, while the last  $K - K_1$  shocks,  $\mathbf{w}_{2t} = (w_{K_1+1,t}, \dots, w_{Kt})'$ , are not in the focus of the analysis. Accordingly, we partition the vector of shocks as  $\mathbf{w}'_t = (\mathbf{w}'_{1t}, \mathbf{w}'_{2t})$ . The matrix of impact effects,  $B$ , is partitioned correspondingly as  $B = [B_1 : B_2]$ ,  $B_1$  being a  $(K \times K_1)$  matrix and  $B_2$  being of dimensions  $(K \times (K - K_1))$ .

The matrix  $B$  contains structural parameters of the model. The  $k$ -th column of  $B$ , say  $b_k$ , represents the impact effects of the  $k$ -th shock on all the  $K$  variables. Thus, the columns of  $B_1$  contain the impact effects of the shocks of interest,  $\mathbf{w}_{1t}$ . Having  $B_1$ , the latter shocks can be obtained from the reduced-form residuals as<sup>3</sup>

$$\mathbf{w}_{1t} = (B'_1 \Sigma_u^{-1} B_1)^{-1} B'_1 \Sigma_u^{-1} u_t, \quad (2)$$

if the shocks are instantaneously uncorrelated, as assumed in our benchmark setup.

The structural impulse responses of the shocks of interest for propagation horizon  $h$  are known to be  $\Theta_{1,h} = \Phi_h B_1$ , where the  $\Phi_h$  are reduced-form quantities obtained recursively from the  $A_1, \dots, A_p$  VAR slope coefficients as  $\Phi_h = \sum_{j=1}^h \Phi_{h-j} A_j$ , with  $\Phi_0 = I_K$ , for  $h = 1, 2, \dots$ , and  $A_j = 0$  for  $j > p$  (e.g., Lütkepohl (2005, Sec. 2.1.2)). Thus, we need the impact effects and reduced-form quantities to get the structural impulse responses.

This will be the benchmark setup in this study which is assumed to hold unless explicitly mentioned otherwise. In the next section we will first spell out the general assumptions that a variable or set of variables has to satisfy to qualify as a proxy and then we will present a range of examples of proxies that have been used in the recent literature. As mentioned earlier, sometimes a set of proxies identifies only a set of shocks but does not identify them individually. As individual shocks are often needed for the desired analysis, we discuss ways of identifying structural shocks individually in Section 3. Section 4 is devoted to estimation of and inference for identified structural parameters, shocks and impulse responses and Section 5 deals with complications that may arise in proxy VAR analysis if the assumptions of our benchmark setup are not satisfied. Finally, Section 6 concludes.

## 2 Proxies

As mentioned in the introduction, the proxies are supposed to be closely related to the shocks or, phrased in statistical terms, they are constructed to be well correlated to the shocks of interest. Sometimes the proxies are even thought of as the shocks observed with measurement error. In some studies, a proxy  $z_t$  for a

---

<sup>3</sup>The relation follows from the fact that  $w_{kt} = b'_k \Sigma_u^{-1} u_t / b'_k \Sigma_u^{-1} b_k$  (see, e.g., Stock and Watson (2018), Bruns and Lütkepohl (2022, Appendix A.1)) and  $(B'_1 \Sigma_u^{-1} B_1)^{-1} = \Sigma_{\mathbf{w}_1}$ .

shock  $w_t$  is in fact assumed to be generated as

$$z_t = \phi w_t + \eta_t, \quad (3)$$

where  $\phi$  is a scalar and  $\eta_t$  is a measurement error (e.g., Mertens and Ravn (2013), Jentsch and Lunsford (2019), Caldara and Herbst (2019), Bruns and Lütkepohl (2023a)). Other authors assume a richer proxy data generating process than (3), which could, for example, also include lags of the proxy or other model variables on the right hand side (see Stock and Watson (2018) and Plagborg-Møller and Wolf (2021)). We will discuss different models for the proxies in more detail in Section 3.2.

A proxy does not have to be measured at every date in the sample period. Often there are missing values in the proxy series which are typically filled by zeros (see, e.g., Kilian (2008), Piffer and Podstawski (2018), Boer and Lütkepohl (2021), Mertens and Ravn (2013)). In fact, in a number of cases a measurement of the proxy is only available for a sequence of event dates (e.g., Wright (2012), Piffer and Podstawski (2018), Clemens, Michelsen and Rieth (2025), Mertens and Ravn (2013)). Given that the proxy just has to be correlated with the shock, such an instrument may well provide sufficient information to estimate the shock.

## 2.1 General Formal Properties of Proxies

Identification of the structural parameters and, hence, the structural shocks is assumed to be based on a set of  $N$  instrumental variables (proxies)  $z_t = (z_{1t}, \dots, z_{Nt})'$  satisfying

$$\mathbb{E}(\mathbf{w}_{1t} z_t') = \Sigma_{\mathbf{w}_1 z} \neq 0, \quad \Sigma_{\mathbf{w}_1 z} (K_1 \times N), \quad \text{rk}(\Sigma_{\mathbf{w}_1 z}) = K_1 \quad (\text{relevance}), \quad (4)$$

$$\mathbb{E}(\mathbf{w}_{2t} z_t') = 0 \quad (\text{exogeneity}). \quad (5)$$

Sometimes it is also required that

$$\mathbb{E}(\mathbf{w}_{t-j} z_t') = 0 \quad \forall j \neq 0 \quad (\text{lead-lag exogeneity}) \quad (6)$$

(see Stock and Watson (2018, Condition LP-IV) and Miranda-Agrippino and Ricco (2023)). Condition (6) implies that proxies are uncorrelated with future structural shocks. This condition is not overly restrictive since we tend to think of structural shocks as unexpected disturbances. But (6) also implies that the proxies are uncorrelated with past structural shocks. This implication is quite strong and more restrictive, but can be defended for example if the proxies are “pre-cleaned”. Pre-cleaning means that the proxies are the residuals of a preliminary regression of proxies on past endogenous variables and other control variables (see

Miranda-Agrippino and Ricco (2021)). Condition (6) is sometimes useful for deriving asymptotic properties of some estimators, for example, to derive consistency of the LP-IV estimator in Stock and Watson (2018).

It is just required that we have an instrument variable which is correlated with the shock of interest, we do not need the shock itself. The proxy can, in fact, be just a dummy variable. Actually, Boer and Lütkepohl (2021) consider a three-point variable which is 1 or  $-1$  on event dates, depending on whether the shock is positive or negative, respectively, and 0 for all dates without known events.

If just a single shock ( $K_1 = 1$ ) is identified by a scalar proxy ( $N = 1$ ), then conditions (4) and (5) imply that the proxy has to have nonzero correlation with the shock of interest and is uncorrelated with all other shocks. That property is also sometimes assumed for the individual proxies if there are multiple proxies. For the multiple proxy case it is clearly stronger than the conditions (4) and (5) (see also the discussion in Section 3).

## 2.2 Specific Proxies

Stock and Watson (2012) discuss a range of proxies that have been used to identify macroeconomic shocks such as monetary policy shocks, oil market shocks, productivity shocks, uncertainty shocks and financial shocks. The following examples of proxies that have been used for proxy VAR analysis show the ingenuity of researchers in constructing variables that are related to specific economic shocks and qualify as proxies.

We do not attempt a complete survey of all proxies that have been used in the proxy VAR literature but just provide some examples of proxies. Generally, one can use any variable that is correlated with one shock only and uncorrelated with all other shocks as an instrument for a specific shock. Hence, one could also use shocks estimated in previous empirical studies as proxies in subsequent investigations. Estimated shocks are obviously not the true shocks but typically shocks measured with error. Specifically, shocks obtained from DSGE (dynamic stochastic general equilibrium) models are sometimes used as instruments in subsequent proxy VAR studies. For example, Stock and Watson (2012) use shocks from the well-known DSGE model constructed by Smets and Wouters (2007) as proxies for some of the shocks considered in their study.

### Monetary Policy Proxies

Monetary policy analysis is a field where structural VARs have been used routinely. A number of researchers use external instruments or proxies for identifying the monetary policy shocks. The proxies are often based on changes in some forward rate or futures contracts at the time of policy announcements of the central bank.

For the U.S. this approach was already used by Kuttner (2001) and has been applied more recently, for example, by Gürkaynak et al. (2007), Gertler and Karadi (2015), Barakchian and Crowen (2013), and Miranda-Agrippino and Ricco (2021). For the UK that approach was applied by Cesa-Bianchi, Thwaites and Vicendoa (2020) and Braun, Miranda-Agrippino and Saha (2025). Altavilla, Brugnolini, Gürkaynak, Motto and Ragusa (2019) used it for the euro area and Kubota and Shintani (2025) for Japan.

Some researchers such as Wright (2012) have even used the changes of Treasury futures at different maturities and consider some aggregate of them as proxy. Wright used the first principle component of 2-, 5-, 10- and 30-year futures. See also Barakchian and Crowen (2013) for a related approach.

As central banks have used a range of tools for monetary policy in addition to short-term interest rate changes, some studies base the monetary policy analysis on a set of proxies for different types of shocks. For example, for the U.S., Jarociński and Karadi (2020) account for the fact that the Fed not only uses interest rate changes as policy tools but also communication to the public. They consider two shocks, a conventional monetary policy shock and a central bank communication shock, which they identify by two different proxies as well as sign restrictions. They actually try different sets of proxies to capture the two different types of monetary policy tools. Miranda-Agrippino and Ricco (2021) make a related argument showing that commonly used monetary policy proxies contain information both about the shock of interest and the state of the economy. They propose a new high-frequency proxy for the U.S. projecting market-based monetary policy surprises on their own lags as well as Greenbook forecasts to ensure both non-predictability of the proxy as well as orthogonality to the state of the economy.

For the euro area, Altavilla et al. (2019) consider even four monetary policy shocks, a target shock capturing conventional monetary policy action, a timing and a forward guidance shock capturing different aspects of European Central Bank (ECB) communication and a quantitative easing shock which is meant to reflect the policy action in the quantitative easing period. They identify the shocks by four proxies constructed as rotated principle components on a set of changes in yields of risk-free rates of seven different maturities based on two intra-day windows on days of ECB Governing Council meetings. The first period covers the press-release time and the second window is the period of the press conference following each ECB Council meeting. The rotated principle components are used as proxies. Related work is also reported by Martínez-Hernández (2020) and Ricco, Savini and Tuteja (2024).

For the U.S., Swanson (2021) uses principal components constructed from the changes around FOMC meetings of a set of federal funds futures, Eurodollar fu-

tures, Treasury bond yields with different times to maturity, the S&P 500 and two exchange rates as a basis for three different proxies for monetary policy action: conventional policy based on the federal funds rate (FFR), forward guidance, and large-scale asset purchases (LSAP). He rotates the three PCs such that the resulting three factors used as proxies are such that (1) changes in forward guidance have no effect on the current FFR, (2) changes in LSAPs have no effect on the FFR and (3) the LSAP factor is as small as possible in the period before the zero lower bound was reached by the interest rate. Related work for the UK is reported by Braun et al. (2025). Clearly, the construction of proxies for monetary policy shocks has reached a high level of sophistication by now.

### **Fiscal Proxies**

Mertens and Ravn (2013) construct two quarterly proxies for U.S. fiscal shocks for the period 1950 - 2006, one for personal income tax shocks and one for corporate income tax shocks. Based on work by Romer and Romer (2009a, 2009b), they consider a series of event dates where changes in tax liabilities have occurred. They eliminate all dates for which the tax changes were known some time in advance and retain only those dates where the implementation lag of the changes in tax liabilities is less than one quarter. They construct the personal income tax proxy as the sum of changes in personal income tax liabilities and in employment tax liabilities divided by the personal taxable income of the previous period. Similarly the corporate tax proxy is constructed as changes in corporate tax liabilities in period  $t$  divided by corporate profits in period  $t - 1$ . Mertens and Ravn (2013) use these proxies in a structural VAR study of the impact of tax shocks on the U.S. economy. A related proxy measuring tax changes relative to gross domestic product (GDP) was constructed and used by Romer and Romer (2010).

Moreover, proxies have been constructed for government spending shocks. For example, Fisher and Peters (2010) construct an instrument for government spending shocks by considering the stock returns of military contractors in times where exogenous decisions on military spending have occurred.

### **Risk and Uncertainty Proxies**

Uncertainty is regarded as important for economic development. Thus, measuring its impact on the economy is of interest. Bloom (2009) constructs proxies for uncertainty shocks as 0-1 variables that assume the value of 1 at special event dates where stock market volatility was above a certain threshold and that are 0 otherwise. Carriero, Mumtaz, Theodoridis and Theophilopoulou (2015) use his uncertainty measure in a proxy VAR analysis.

Piffer and Podstawski (2018) construct a proxy for economic uncertainty by considering the variations in the price of gold on a set of event dates where uncertainty was expected to be high. For instance, they use as event dates the 9/11 attack (11 Sept. 2001), Iraq's invasion of Kuwait that started the first Gulf War (2 Aug. 1990), the Black Monday on the stock market (19 Oct. 1987) or the massacre at the Tienanmen Square (3 June 1989).

### **Oil Market Proxies**

For a study of the crude oil market, Känzig (2021) constructs one proxy for an oil supply news shock by considering the oil futures price changes in a tight window around OPEC announcements about their production plans. He also uses another proxy for shortfall of OPEC oil production shocks caused by exogenous political events such as wars or civil disturbances. That proxy is based on work by Kilian (2008) and Bastianin and Manera (2018).

Degasperi (2023) argues that the oil supply news proxy of Känzig (2021) contains information about both future oil supply and future oil demand. Similarly to Jarociński and Karadi (2020) he proposes to decompose Känzig's proxy into a supply and a demand side proxy according to its correlation with the stock market. Moreover, Kilian (2024) notes that the availability of oil futures prices has changed during the time period used by Känzig for constructing his proxy. Kilian also proposes another way of aggregating the changes in futures prices on announcement days to monthly frequency for the proxy series. Constructing a proxy by accounting for such issues, Kilian (2024) finds that the impulse responses to a shock identified by the proxy change substantially.

Montiel Olea, Stock and Watson (2021) consider a proxy for oil supply shocks using measurements from Kilian (2008) of shortfalls in OPEC oil production associated with wars and civil disruptions.

### **Productivity Proxies**

Productivity is an important variable for macroeconomic development and, hence, productivity shocks are important in many structural VAR studies. While productivity shocks are often identified in alternative ways, there are also studies that use proxies for identifying productivity shocks. For example, Stock and Watson (2012) consider the productivity shock from the Smets and Wouters (2007) DSGE model as an instrument for productivity.

Lunsford (2015) investigates the responses of the U.S. economy to TFP shocks and constructs two related proxies, a consumption TFP proxy and an investment TFP proxy. He considers two utilization-adjusted TFP measures constructed by Fernald (2014). One of them measures the TFP of the consumption sector ex-

cluding durable goods and the other one measures the TFP of durable goods and equipment investment. Two proxies are then constructed as the residuals from regressing consumption TFP and investment TFP on a constant and four lags of the endogenous macroeconomic variables considered in his VAR model. The resulting TFP proxies were also used in studies by Arias, Rubio-Ramírez and Waggoner (2021) and Bruns and Lütkepohl (2025).

## Financial Proxies

Economic conditions crucially depend on credit supply. Therefore credit supply shocks are financial shocks of central interest for economic policy. Hence, researchers have constructed measures for credit supply shocks. A number of them were constructed and used in proxy VAR studies.

Based on firm-level data, Gilchrist and Zakrajšek (2012) construct an index of credit spreads and propose a component of the index, the excess bond premium (EBP), as a proxy for credit supply. The EBP is viewed as a measure of changes in the risk aversion of the financial sector. It has actually been used in a number of macroeconomic structural VAR studies as one of the endogenous variables to capture changes in the likelihood of an upcoming recession.

Mumtaz, Pinter and Theodoridis (2018) compare these and a number of other measures for credit supply shocks as structural VAR proxies using a simulated DSGE setup. They include, for example, measures proposed by Bassett, Chosak, Driscoll and Zakrajšek (2014) and Jermann and Quadrini (2012) in their comparison. Bassett et al. (2014) develop an indicator of changes in the supply of bank intermediated credit based on changes in lending standards as reported by the Federal Reserve Board’s Senior Loan Officer Opinion Survey on Bank Lending Practices (SLOOS). Instead Jermann and Quadrini (2012) construct a financial shock series as a residual in a DSGE model framework for the U.S.. Moreover, Mumtaz et al. (2018) construct a textual measure of credit supply shocks by counting the number of occurrences of the words “credit crunch” and “tight credit” in nine U.S. newspapers.

Mumtaz et al. (2018) also consider other identification schemes such as sign restrictions and identification through heteroskedasticity for credit supply shocks and find notable differences in the impulse responses to credit supply shocks based on the different proxies and also other identification devices. Thus, it is important for empirical work which identification strategy and which proxy is used to identify credit supply shocks.

## Climate-related Proxies

A growing literature constructs climate-related indicators to trace out the effects of climate shocks on the macroeconomy at monthly or quarterly frequency. In this setting the exogeneity condition (5) implies that a climate-related indicator is uncorrelated with contemporaneous non-climate shocks such as monetary or fiscal policy shocks or demand/supply shocks. This condition can often be argued for more easily than for other, non-climate proxies.

Some studies employ climate related proxies based on natural disasters. For example, Kim, Matthes and Phan (2025) investigate the effect of severe weather on macroeconomic variables by using a monthly Actuaries Climate Index capturing the frequency of severe weather events such as extremely low temperatures, high temperatures, heavy precipitation, drought, high wind, and sea level change. This proxy is then used as the first variable in the VAR model that also incorporates a set of standard macroeconomic measures. The severe weather shock is identified using a recursive scheme, which corresponds to the internal proxy VAR approach discussed in Section 3.2. Using a non-linear model, the authors find that severe weather shocks have significantly, negatively, and persistently affected economic activity in the U.S. in recent years, unlike 60 years ago when no significant effects were found. A proxy based on the Actuaries Climate Index is also used by Liao, Sheng, Gupta and Karmakar (2024) in a dynamic factor model.

A related idea is used by Eickmeier, Quast and Schüller (2024) to construct a natural disaster index for the U.S. containing hydrological, meteorological or geophysical disasters for the period 2000M1 till 2019M12. The baseline index is equal to the number of disasters in a given month and zero in months without a disaster. The impact of the disaster shock identified via this proxy on macroeconomic aggregates but also financial variables is estimated using a local projection approach (see Section 4.2). In related work, Usman, Fernández and Parker (2025) aggregate the occurrence of heatwaves, precipitation extremes, droughts, and floods to quarterly dummy variables and trace out their macroeconomic effects on European Union countries in a local projection framework.

In another set of studies, proxies are employed that are based on temperature deviations from their historic means. For example, Bilal and Känzig (2024) construct a yearly global temperature shock and a local temperature shock for 173 countries over the last 120 years. They do so, following the smoothing technique in Hamilton (2018), by regressing temperature two years ahead on two lags as of the current period and use the innovations of this regression as temperature proxies. This isolates shocks that typically persist for several years. They then employ this temperature proxy in a local projection framework and find that the effects of temperature shocks on GDP are up to six times larger than previously thought.

Ciccarelli, Kuik and Martínez Hernández (2024) investigate the effects of tem-

perature shocks on four European countries using as a proxy the monthly deviation in temperature from its historical mean in the period 1991-2020. They estimate country-specific VARX models where the exogenous temperature indicator is interacted with a seasonal dummy (see Section 4.3 for a discussion of using proxies within a VARX model). They find, for example, that temperature increases are inflationary in particular in summer and in warmer euro area countries.

Baleyte, Bazot, Monnet and Morys (2024) construct a temperature anomaly indicator as the deviation of temperature in a given month from the average over the period 1950 till 1980. They then use this proxy as a direct shock measurement, allowing the impulse response to a temperature shock to depend on the current temperature state in a smooth-transition local projection framework. In a similar setup Lucidi, Pisa and Tancioni (2024) investigate the effects of temperature anomalies on inflation in a panel local projection approach for euro area countries. Cevik and Gwon (2024) estimate three-variable VARs for six countries separately ordering temperature deviations first and identifying a temperature shock via a recursive ordering in line with an internal proxy VAR approach.

### **Proxies for Non-target Shocks**

Caldara and Kamps (2017) show for the case of tax and government spending shocks that, if the researcher takes a stand on the policy rule, then fiscal policy shocks can be identified using non-fiscal proxies, that is, proxies for non-target shocks. In a five-variate model the authors use three non-fiscal proxies capturing total factor productivity, oil shocks, and monetary policy shocks to identify the coefficients of a fiscal policy rule. They then recover the tax shock and the government spending shock as residuals imposing the additional restriction that government spending does not respond contemporaneously to taxes. An advantage of this approach could be that it allows identifying a policy shock even if no plausible instruments for this shock are available. However, the approach requires taking a stand on both the number and nature of the non-policy shocks since these need to be identified using non-policy proxies. This might make the approach less robust to the omission of potentially important non-policy shocks or variables in the model. Still, if the elements of the policy rule are broadly agreed upon, then the use of non-policy proxies could be an appealing alternative when no policy-proxies are available.

## **3 Identification**

The identification problem in structural VAR analysis arises from the fact that the requirement of uncorrelated structural shocks does not uniquely identify the impact

effects matrix  $B$  and, hence, does not determine unique shocks. Standardizing the shock variances to one, uncorrelated shocks imply that  $B$  has to satisfy  $BB' = \Sigma_u$ . If  $B$  satisfies that condition, then any orthogonal transformation  $BQ$ ,  $Q$  being an orthogonal matrix, also satisfies it. Thus, further assumptions or restrictions are needed to identify the structural shocks.

Occasionally one may be interested in the joint effect of a set of shocks. In that case it may not be necessary to identify the shocks individually and the shocks may even be correlated. For example, in monetary policy analysis, if there are two types of monetary policy shocks, e.g., interest rate setting and communication, only the joint effect may be of interest. In that case it may be sufficient to identify only linear combinations of the shocks rather than the individual shocks.

### 3.1 External Instruments

In order to identify structural shocks of interest, external instruments or proxies are helpful because the relevance and exogeneity conditions (4)/(5) imply

$$\mathbb{E}(u_t z_t') = B\mathbb{E}(\mathbf{w}_t z_t') = B_1 \Sigma_{\mathbf{w}_1 z}. \quad (7)$$

In other words, the parameters,  $B_1$ , needed to identify the shocks of interest, have to satisfy the additional relations (7).

If there is a scalar proxy ( $N = 1$ ),  $B_1$  is just a column vector and  $\Sigma_{\mathbf{w}_1 z}$  is a scalar. Thus,  $\mathbb{E}(u_t z_t')$  is a scalar multiple of the impact effects of the shock of interest which is sufficient for our purposes because the size of the shock that is considered will typically be chosen by the user when a specific analysis is performed. When the shock size is given, the corresponding vector  $B_1$  can be determined. This is obvious if the shock size is specified, for example, by assuming that a specific variable responds to the shock by one unit on impact. In that case one element of  $B_1$  is fixed so that (7) fully determines the other elements of  $B_1$ . In some cases a shock size of one standard deviation is considered. Also in this case it is easy to see that such an additional piece of information is sufficient to fully identify the vector  $B_1$  in our benchmark model setup.

If there are multiple shocks of interest and multiple proxies, the relation (7) can still be helpful for identifying linear combinations of the shocks or the individual shocks if there is additional structure for  $\Sigma_{\mathbf{w}_1 z}$ . For example, if there are exactly as many proxies as there are shocks of interest ( $N = K_1$ ) and each proxy individually satisfies the relevance and exogeneity conditions, then  $\Sigma_{\mathbf{w}_1 z}$  is a diagonal matrix that even over-identifies the shocks individually (see Bruns, Lütkepohl and McNeil (2025)).

In general, in the context of proxy VAR analysis, identifying restrictions for  $B_1$  and, hence, the shocks have been imposed on  $B_1$  directly in the form of exclusion or

other just-identifying restrictions (e.g., Mertens and Ravn (2013), Känzig (2021)) or sign restrictions (e.g., Jarociński and Karadi (2020), Giacomini, Kitagawa and Read (2022), and Braun and Brüggemann (2023)), heteroskedasticity (Carriero, Marcellino and Tornese (2024) and Angelini, Fanelli and Neri (2024)).

Alternatively, one may place restrictions on  $\Sigma_{\mathbf{w}_1 z}$ . They may be just-identifying (e.g., Lunsford (2015) and Lakdawala (2019)) or over-identifying (e.g., Bertsche and Braun (2022), Schlaak, Rieth and Podstawski (2023), Bruns et al. (2025)). In fact, one may even set-identify the shocks by sign restrictions on  $\Sigma_{\mathbf{w}_1 z}$  (e.g., Piffer and Podstawski (2018), Ludvigson, Ma and Ng (2021) and Arias et al. (2021)).

### 3.2 Internal Instruments

In some of the proxy VAR literature, the instruments or proxies are internalized into the VAR model by considering an augmented model such as

$$\begin{pmatrix} z_t \\ y_t \end{pmatrix} = \begin{pmatrix} \nu^z \\ \nu^y \end{pmatrix} + \begin{bmatrix} A_1^{zz} & A_1^{zy} \\ A_1^{yz} & A_1^{yy} \end{bmatrix} \begin{pmatrix} z_{t-1} \\ y_{t-1} \end{pmatrix} + \dots + \begin{bmatrix} A_p^{zz} & A_p^{zy} \\ A_p^{yz} & A_p^{yy} \end{bmatrix} \begin{pmatrix} z_{t-p} \\ y_{t-p} \end{pmatrix} + \begin{pmatrix} u_t^z \\ u_t^y \end{pmatrix}, \quad (8)$$

where  $u_t^{aug} = (u_t^z, u_t^y)'$  is a zero-mean white noise process with covariance matrix denoted by  $\Sigma_u^{aug}$ . Equivalently, one could consider a VAR( $p$ ) for  $(y_t', z_t)'$ . Such an augmented VAR setup makes sense because, if the  $z_t$  variables are closely related to the  $y_t$ , it may be important to include them in the model to avoid omitted variables bias in the impulse responses or other quantities used for structural analysis. If important variables are omitted, the  $u_t^y$  residuals may actually not be fundamental in the sense that they cannot be recovered from past and present  $y_t$ . In that case shocks based only on  $u_t^y$  will also not be fundamental and, hence, one of the basic assumptions of our model setup would be violated. This aspect is an important argument in favour of the augmented VAR model (8). We will discuss nonfundamentalness and noninvertibility issues more extensively in Section 5.2. An augmented VAR setup is used, e.g., by Bańbura, Giannone and Reichlin (2010), Caldara and Herbst (2019), Arias et al. (2021), Angelini and Fanelli (2019), Jarociński and Karadi (2020), Kilian, Plante and Richter (2025) and Plagborg-Møller and Wolf (2021).

The external approach has several advantages over this internal approach, for example, it does not require specifying a model for the proxies and it has fewer parameters to estimate and, hence, potentially more precise estimators. Nevertheless, the internal approach is often used because it can, for example, be easily combined with Bayesian methodology. Internalizing the proxies as in model (8), not only opens up estimation by standard VAR methods (see Section 4), but may also change the way the structural shocks are identified. In fact, in the literature

using the augmented VAR model (8), the shocks of interest are often identified by assuming a recursive scheme (see, e.g., Bańbura et al. (2010)). In other words, a Cholesky decomposition  $\text{chol}(\Sigma_u^{aug})$  of  $\Sigma_u^{aug}$  is used to get the  $N + K$  shocks as

$$\mathbf{w}_t^{aug} = (\text{chol}(\Sigma_u^{aug}))^{-1} u_t^{aug}, \quad (9)$$

the first  $K_1$  of which are regarded as the shocks of interest,  $\mathbf{w}_{1t}$ .

The shocks obtained in this way are contemporaneously uncorrelated by construction. However, as  $\text{chol}(\Sigma_u^{aug})$  is a lower-triangular matrix, the same is true for its inverse. Hence, the shocks  $\mathbf{w}_{1t}$  are linear combinations of the  $u_t^z$ . In fact, if the proxies are white noise, as they sometimes are by construction (see Section 2), then the  $u_t^z$  may just be the mean-adjusted proxies. In that case, the shocks  $\mathbf{w}_{1t}$  are linear combinations of the mean-adjusted proxies. Note that it is no problem that a single shock is correlated with more than one proxy. The crucial condition is that the shocks must still be instantaneously uncorrelated which is ensured by choosing them as in (9). It is also worth mentioning that the matrix of impact effects,  $B_1$ , is of course, uniquely determined if this setup is used. It is the lower left-hand ( $K \times K_1$ ) dimensional submatrix of  $(\text{chol}(\Sigma_u^{aug}))^{-1}$ . Thus, there are differences between the external and the internal proxy VAR approaches. They can also lead to the same impulse response, however, as we will discuss next.

### 3.3 Comparison of External and Internal Proxy VAR Approaches

For the case, where an individual shock is identified by a single proxy, Plagborg-Møller and Wolf (2021) present general conditions under which the external and internal approaches are equivalent in population. Recall that if a single proxy identifies one shock of interest, the shock will be just-identified by both approaches.

For the more general case of several shocks and proxies, Bruns and Lütkepohl (2025) compare the external and internal proxy VAR approaches as presented in Sections 3.1 and 3.2 in detail. They point out that the shocks are different. As we have seen in the previous sections, the shocks from the external VAR approach are linear combinations of the residuals  $u_t$  of the VAR model (1), while the shocks from the internal VAR approach are linear combinations of the  $u_t^z$  residuals from the augmented VAR model (8) and may even be linear combinations of the mean-adjusted proxies. Moreover, in the external approach, the shocks may not be individually identified by the proxies while in the internal approach they are just-identified.

Despite these differences, the impulse responses obtained from the two approaches may be identical. More precisely, Bruns and Lütkepohl (2025) show that, if there are exactly as many proxies as shocks of interest ( $N = K_1$ ) and

some technical conditions for the generating mechanism of the proxies hold, the impulse responses from the two approaches are identical up to scale if the following conditions are jointly satisfied:

- (a) No lags of the proxies appear in the  $y_t$  equations, that is, the  $z_t$  are not Granger-causal for the  $y_t$ .
- (b) The proxies are instantaneously uncorrelated.
- (c) Each shock in  $\mathbf{w}_{1t}$  is correlated with one proxy only.

Bruns and Lütkepohl (2025) also point out statistical methods that can be used to check these three conditions in practice. Note that the scale of the shocks obtained from the two approaches may differ. That is not a problem in practice because a researcher has to pick the size of the shock for the specific analysis anyway. Hence, the shocks used in an empirical analysis will be of the same scale regardless of the identification method.

Since forecast error variance decompositions (FEVDs) are functions of the impulse responses, the two approaches also provide equal FEVDs if the impulse responses are the same. In contrast, historical decompositions which are also often used as analysis tools in structural VAR studies, will in general not be identical even if the impulse responses are because they also depend on the shocks from the two approaches which will usually be different even if the impulse responses are the same.

## 4 Estimation and Inference

Estimation of the reduced-form of the VAR model is straightforward and can be done by least-squares (LS) or standard Bayesian methods and, as the structural impulse responses etc. can be determined once the impact effects and the reduced-form parameters are available, the main challenge is to estimate the structural parameters  $B_1$ . For this purpose, various estimation methods for proxy VARs have been used, to some extent depending on the identifying restrictions. If, for instance, exclusion restrictions on the impact effects are available, maximum likelihood (ML) or generalized method of moments (GMM) methods may be suitable while Bayesian methods are typically used if sign restrictions are involved. Estimation of the structural impulse responses in the context of proxy VAR analysis is sometimes based on local projection (LP) methods which were first proposed by Jordà (2005) and since then have been refined for use in proxy VAR analysis. In the following these methods are briefly reviewed.

## 4.1 Conventional Frequentist Estimation Methods

If identifying restrictions are available for  $B_1$ , one could set up a Gaussian likelihood function and optimize subject to the restrictions. Thereby standard ML estimators for the parameters of interest are obtained. The estimation problem involves a substantial number of parameters, however, given that there are not only the structural parameters but also the reduced-form VAR parameters in the likelihood function. Since the identifying restrictions may be such that the restricted likelihood maximisation may be involved, other estimation methods for proxy VARs are often considered.

Because we have sets of moment conditions involving the parameters of interest,  $B_1$ , GMM estimation is a plausible possibility. The following  $KN$  moment conditions are available:

$$\mathbb{E}(u_t z_t' - B_1 \Sigma_{\mathbf{w}_1 z}) = 0. \quad (10)$$

In addition, Bruns et al. (2025) use that  $\Sigma_{\mathbf{w}}$  is a diagonal matrix to show that

$$\mathbb{E}[\text{vh}(B_1' \Sigma_u^{-1} u_t u_t' \Sigma_u^{-1} B_1)] = 0, \quad (11)$$

where  $\text{vh}(\cdot)$  is a column vector containing the elements below the main diagonal of the square matrix in the argument. Hence, there is a further set of  $\frac{1}{2}K_1(K_1 - 1)$  moment conditions.

As the matrices  $B_1$  and  $\Sigma_{\mathbf{w}_1 z}$  in general contain  $KK_1$  and  $K_1N$  free parameters, respectively, the  $KN + \frac{1}{2}K_1(K_1 - 1)$  moment conditions in (10) and (11), in general, do not suffice to identify all the parameters of interest. Hence, the estimation problem is under-identified if there are no further restrictions on  $B_1$  and  $\Sigma_{\mathbf{w}_1 z}$ . For example, Lakdawala (2019) considers a triangular  $\Sigma_{\mathbf{w}_1 z}$  matrix which just-identifies the shocks individually. Altavilla et al. (2019) assume that there are  $N = K_1$  proxies and each of them is correlated with one shock only and not correlated with any of the other shocks such that  $\Sigma_{\mathbf{w}_1 z}$  is a diagonal matrix. In that case, the moment conditions even over-identify the shocks of interest if  $K_1 > 1$ .

In practice, the moment conditions (10) and (11) depend on the reduced-form VAR parameters  $\nu, A_1, \dots, A_p$  and  $\Sigma_u$  as well because they involve the reduced-form residuals,  $u_t$ , which also have to be estimated. Bruns et al. (2025) set up a GMM objective function  $J(\eta)$ , based on the moment conditions (10) and (11) only, replacing the  $u_t$  and  $\Sigma_u$  by LS estimates. Substituting  $u_t$  and  $\Sigma_u$  by LS estimates is no problem if the restrictions on the parameters are just-identifying. However, if over-identifying restrictions are available, the GMM objective function has to take into account the nuisance parameters from the reduced-form VAR model. Bruns et al. (2025) derive an expression of the GMM weighting matrix that accounts for the nuisance parameters such that an asymptotically efficient estimator for  $B_1$  is

obtained. Thereby they also get a standard asymptotic  $\chi^2$ -distribution of Hansen's  $J$ -test based on the GMM objective function if over-identifying restrictions are available. The test is useful to check the validity of assumed over-identifying moment conditions.

A related GMM approach was proposed by Gregory, McNeil and Smith (2024). Their approach works if at least  $K - 1$  shocks are identified by proxies, a condition which is often not satisfied in practice. Angelini and Fanelli (2019) assume a parametric model for the proxies such as (3) and propose a minimum distance method that minimizes the distance of the structural parameters from the reduced-form parameters. Their parametric model for the proxies is not useful for many proxies that have been used in the literature because it does not capture the situation that a proxy has nonzero values only for a set of event dates during the sample period.

## 4.2 Local Projections

Local projection (LP) is an alternative estimation method for structural VARs and in particular for proxy VARs. As mentioned earlier, it was originally proposed by Jordà (2005) to estimate impulse responses in VAR analysis and has been further developed into a popular method for structural estimation since then. Bruns and Lütkepohl (2022) provide a review and comparison of a range of LP estimators for proxy VAR models. In this section, we heavily draw from that article.

The LP form of the VAR model (1) is

$$y_{t+h} = \nu_h + A^{(h+1)}Y_{t-1} + v_{t+h}^{(h)}. \quad (12)$$

The intercept vector  $\nu_h$  depends on the integer  $h$ ,  $Y'_{t-1} = (y'_{t-1}, \dots, y'_{t-p})$  is a  $Kp$ -dimensional vector of lagged dependent variables,

$$v_t^{(h)} = u_t + \Phi_1 u_{t-1} + \dots + \Phi_h u_{t-h} \quad (13)$$

is a weighted sum of the reduced-form errors  $u_t, \dots, u_{t-h}$  and

$$A^{(h)} = [A_1^{(h)}, \dots, A_p^{(h)}]$$

is the  $(K \times Kp)$  dimensional matrix consisting of the first  $K$  rows of  $\mathbf{A}^h$ ,  $\mathbf{A}$  being the companion matrix

$$\mathbf{A} = \left[ \begin{array}{cccc|c} A_1 & A_2 & \dots & A_{p-1} & A_p \\ \hline I_{(p-1)K} & & & & 0_{K(p-1) \times K} \end{array} \right].$$

Suppose we are interested in the impulse responses of one of the shocks in  $\mathbf{w}_{1t}$  with impact effects in one of the columns of  $B_1$ , say  $b$ . Then the impulse responses of interest can be obtained as

$$\theta_h = \Phi_h b, \quad h = 0, 1, \dots, \quad (14)$$

where  $\Phi_h = A_1^{(h)}$ . Thus, the impulse responses can be estimated as  $\hat{\Phi}_h \hat{b}$ , where  $\hat{\Phi}_h = \hat{A}_1^{(h)}$  can be obtained as some linear estimator from the LP form of the model (12). Given that the error term  $v_{t+h}^{(h)}$  is autocorrelated one may prefer a GLS estimator (see, e.g., Lusompa (2023)). The estimator  $\hat{b}$  may be obtained via the related proxy.

Breitung and Brüggemann (2023) and Montiel Olea and Plagborg-Møller (2021) propose to use a lag augmented model,

$$y_{t+h} = \nu_h + A^{(h+1)}Y_{t-1} + A_{p+1}^{(h+1)}y_{t-p-1} + v_{t+h}^{(h)}, \quad h = 0, 1, \dots, H-1. \quad (15)$$

Here  $A^{(h+1)}$  is a  $(K \times Kp)$  dimensional matrix while  $A_{p+1}^{(h+1)}$  is  $(K \times K)$ . Lag-augmentation has been used by Toda and Yamamoto (1995), Dolado and Lütkepohl (1996) and Dufour, Pelletier and Renault (2006) to fix unit root asymptotics.

If the true DGP is a VAR( $p$ ), then the coefficient matrices of the additional lag are known to be zero, i.e.,  $A_{p+1}^{(h+1)} = 0$ ,  $h = 0, 1, \dots, H-1$ , and estimating the lag-augmented model by LS implies an inefficiency. Montiel Olea and Plagborg-Møller (2021) show that the resulting lag-augmented LP estimator is more robust to unit roots and near unit roots and therefore has advantages for inference although adding a lag with zero coefficients generates an inefficiency.

The model (15) can be reparameterized as

$$y_{t+h} = \nu_{h-1} + \Theta_h \mathbf{w}_t + A_*^{(h)}Y_{t-1} + v_{t+h}^{(h-1)},$$

where  $A_*^{(h)}$  is a  $(K \times Kp)$  matrix and  $\Theta_h = \Phi_h B$  is the matrix of structural impulse responses at propagation horizon  $h$ . If  $\mathbf{w}_t$  were available, this model could be used for estimation. In practice, the  $\mathbf{w}_t$  have to be replaced by estimates, of course. If we are just interested in the  $i$ -th column of the  $\Theta_h$  matrix, we just need an estimator of the  $i$ -th component of  $\mathbf{w}_t$  because, under our assumptions, the components of  $\mathbf{w}_t$  are uncorrelated and dropping orthogonal regressors from the equations for LS estimation does not affect the estimators of the other parameters. Hence,  $\theta_h$ , the  $i$ -th column of  $\Theta_h$ , can be estimated by LS using the model

$$y_{t+h} = \nu_h + \theta_h \hat{w}_{it} + A_*^{(h)}Y_{t-1} + v_{t+h}^{(h-1)} \quad \text{for } h = 1, \dots, H. \quad (16)$$

Here  $\hat{w}_{it}$  is an estimate of  $w_{it}$ . Bruns and Lütkepohl (2022) review a range of proposals how to estimate  $w_{it}$  and how to deal with the autocorrelation in the error term of the model. These result in different estimators of the structural impulse responses. Bruns and Lütkepohl (2022) perform a large-scale simulation comparison of a number of the resulting estimators and conclude that the standard proxy VAR approach with external proxies is preferable to the LP approach in terms of estimation precision, if a finite-order VAR process is a good approximation to the

true DGP and the proxy is well correlated with the shock of interest. However, Plagborg-Møller and Wolf (2021) show that LP-estimation or a VAR with internal instruments may provide valid impulse response estimates even if the shocks are nonfundamental which can be an advantage of the latter methods. Note that nonfundamentalness can be induced by omitted variables which are a possibility in applied work.

In this context an interesting result was established by Stock and Watson (2018) who assume that the proxies satisfy the relevance, the exogeneity, and the lead-lag exogeneity conditions. For a scalar proxy  $z_t$  for the first shock  $w_{1t}$  and mean-adjusted  $y_t$ , they note that, using  $z_t$  as an instrument, the standard IV estimator of the coefficient in the linear model

$$y_{t+h} = \theta_h y_{1t} + u_{t+h}^{(h)}, \quad (17)$$

is

$$\hat{\theta}_h(IV) = \left( \sum_{t=1}^{T-h} z_t y_{1t} \right)^{-1} \sum_{t=1}^{T-h} z_t y_{t+h}. \quad (18)$$

Clearly, the regressor in (17) is in general correlated with the error term and, hence, simple LS regression is inconsistent. In contrast, the IV estimator,  $\hat{\theta}_h(IV)$ , converges in probability to  $\theta_h$  because the instrument is uncorrelated with the error term,  $u_{t+h}^{(h)}$ , which contains leads and lags of  $u_t$  and  $y_t$  and, hence, of  $\mathbf{w}_t$ . Stock and Watson (2018) show that the estimator  $\hat{\theta}_h(IV)$  is consistent and asymptotically normal under general conditions.

Montiel Olea, Plagborg-Møller, Qian and Wolf (2024) show that LP estimators are more robust than VARs to model misspecification such as too short lag length, missing variables, inappropriate aggregation and measurement error. VARs can lead to severe distortions of the impulse responses even if the misspecification is small, unless the VAR has long lag-length. A broader discussion on when and how to use VAR and LP approaches is summarised in Montiel Olea, Plagborg-Møller, Qian and Wolf (2025). It includes scenarios where, unlike in this review, a finite-order VAR process might not be a good approximation to the DGP and attention is not restricted to a proxy identification scheme. The authors point out that VAR and LP models tend to deliver similar impulse responses for horizons up to the VAR lag length, i.e. for  $h \leq p$ , but start to differ for longer horizons. They conclude that VAR models should be used only with lag lengths long enough to ensure equivalence with LP. They offer guidance for applied researchers in the form of a number of lessons learned. Baumeister (2025) challenges some of Montiel Olea et al. (2025)'s conclusions and provides modified lessons. Regarding the choice of lags, Baumeister (2025) agrees to use a rich lag structure if the focus is on

higher horizon impulse responses, but points out that users of LP methods tend to rely on auxiliary VAR models to guide the choice of the number of lags. In addition, Bayesian VAR models are suitable to incorporate a rich lag structure because they can deal with the associated curse of dimensionality by relying on additional information in the form of prior distributions (see also the next section). Li, Plagborg-Møller and Wolf (2024) perform a small sample comparison of LP and VAR estimators of impulse responses based on a very large number of data generating processes. They conclude that using bias-corrected LP methods is a good strategy if small bias is the main objective while VARs are preferable when estimation precision, e.g., small variances are given priority.

### 4.3 Bayesian Methods

Bayesian analysis requires specification of a prior for the parameters and the likelihood for the data and then derive the posterior for the quantities of interest such as impulse responses. As there are well-established standards for VAR models how to proceed in Bayesian analysis, Bayesian approaches to proxy VARs start from the internal instrument parametrisation of the model in equation (8). Thereby one can take advantage of conventional priors etc.. The augmented model (8) requires the researcher to take a stance on a specific model generating the proxies  $z_t$ , contrary to the external instrument parametrisation, where less structure is imposed on the generation of proxies and which can therefore be thought of as semi-parametric.

Typically, block zero restrictions are imposed on the autoregressive slope coefficients to sharpen inference. Caldara and Herbst (2019), Bahaj (2020), and Drautzburg (2020) set  $A_i^{zz} = A_i^{yz} = A_i^{zy} = 0$ ,  $i = 1, \dots, p$ , in the augmented model (8) implying (i) no serial correlation in the proxies, (ii) no Granger-causality of  $y_t$  for the proxies, and (iii) no Granger-causality of the proxies for  $y_t$ . Arias et al. (2021) restrict only  $A_i^{zy} = 0$ , implying no Granger-causality of  $y_t$  for the proxies.

If multiple proxies are used to identify multiple shocks, then, without further just-identifying restrictions, the model is set-identified in the sense that orthogonal transformations of the impact effects of the shocks of interest are equally compatible with the data. As pointed out by Baumeister and Hamilton (2015) in the context of sign restrictions, the prior distributions over set-identified parameters matter even asymptotically. For example, in a frequentist setting, using two proxies to identify a government spending and a tax shock, Mertens and Ravn (2012) employ an impact zero restriction to disentangle these two shocks. In a Bayesian context, this restriction could be called “dogmatic” in the sense that it is never revised even asymptotically. If such a restriction were controversial however, there could be interest in an approach, which does not condition on a single, unrevisable prior distribution over the set-identified parameters of the model. That is why Giacomini et al. (2022) propose a prior-robust approach to identifying mul-

multiple shocks using multiple proxies. Rather than a single prior distribution, this approach considers a class of priors over the set-identified parameters subject to identifying restrictions. Giacomini et al. (2022) provide conditions under which there exists a frequentist interpretation for the resulting impulse response bands. Other papers argue that even in a set-identified context, as long as the identified set is sufficiently narrow, standard Bayesian methods are still suitable. This is discussed for example in Arias, Rubio-Ramírez and Waggoner (2025) and Inoue and Kilian (2026) for the case of sign-restricted VARs.

If FEVDs are the statistics of interest, then Fusari, Marlow and Volpicella (2024) show estimation and inference procedures for cases where multiple proxies set-identify multiple structural shocks. They derive the lower and upper bounds for the set of FEVDs and show equivalence between a Bayesian and a frequentist approach.

Bayesian proxy VARs are potentially useful to incorporate non-linearities in the model. The reason is that a non-linear specification with a potentially large number of parameters benefits from Bayesian shrinkage methods and a proxy identification scheme leads to a particularly tractable sampling procedure. This is shown in Bruns and Piffer (2024) who use a VAR model with proxies as exogenous variables rather than the augmented model in (8) and propose a Proxy Smooth Transition VAR model to trace out the non-linear dynamics of structural shocks in different regimes. Bruns and Piffer (2024) illustrate the importance of estimating rather than calibrating the parameters governing the non-linearities of the model, suggest a prior distribution for them, and provide a convenient factorisation of the posterior distribution to sample them in a computationally feasible way. The proxy is added as an exogenous regressor which simplifies inference and still allows for the impact of the identified structural shock to vary across regimes already on impact. Another example of a non-linear proxy VAR is Paul (2020) who proposes a time-varying parameters Bayesian proxy VAR for U.S. monetary policy. As an alternative to non-linear VAR models, non-linear LP approaches are discussed for example in Gonçalves, Herrera, Kilian and Pesavento (2024).

#### 4.4 Weak Instruments

The assumption of instrument relevance states that a proxy is useful for identifying the effects of a structural shock if it contains sufficient information about this shock, see equation (4). In other words, the proxy and the structural shock need to have a sufficiently high degree of correlation. This assumption can easily be violated in practice, e.g., if the proxy is observed only infrequently or if a large share of its variation is made up of measurement error. In such cases standard inference becomes invalid.

Following the setting in Montiel Olea et al. (2021) and restricting attention to

a single shock,  $w_{1t}$ , identified via a single proxy,  $z_t$ , we rewrite (4) as

$$\mathbb{E}(w_{1t}z_t) = \sigma_{w_1z,T}, \quad (19)$$

where the sample size  $T$  in the subscript of the covariance  $\sigma_{w_1z,T}$  indicates that this quantity is allowed to vary with  $T$  in this setup. If the population covariance is nonzero and does not depend on the sample size, such that  $\sigma_{w_1z,T} = \sigma_{w_1z} \neq 0$ , one can argue that the proxy is strong because it will satisfy the population requirements for proxies eventually even in sample, if the sample is large enough. As researchers typically have to deal with small samples, a better approximation to the actual situation may be obtained if one assumes that the covariance in (19) gets smaller with the sample size and is zero asymptotically. This local-to-zero asymptotics has also been developed for proxies. Staiger and Stock (1997) call the proxy weak if  $\sigma_{w_1z,T} = a/T$  for a nonzero real number  $a$  and, hence,  $\sigma_{w_1z,T} = a/T \rightarrow 0$  for  $T \rightarrow \infty$ .

At least two problems arise in this case: First, standard bootstrap-based confidence intervals (see Section 4.5) have incorrect coverage, i.e., the confidence bands obtained from standard inference are too narrow, inaccurately suggesting a precisely estimated effect. And second, the impulse response estimator,  $\theta_h$ , is biased towards the first column of  $\text{chol}(\Sigma_u)$ , where  $\text{chol}(\cdot)$  is the lower-triangular Cholesky decomposition of the argument with the shock of interest ordered first. It is therefore no comfort to obtain similar impact effects estimates under a recursive identification scheme and a proxy-based scheme, but rather this may indicate a potentially weak proxy.

To detect weak instruments, a number of tests have been proposed: Stock, Wright and Yogo (2002) propose evaluating the first-stage  $F$ -statistic of a regression of  $y_{1,t}$  on  $z_t$ , where the impact response of one variable, ordered first without loss of generality, is not equal to zero. The regression should include as control variables lags of  $y_t$  and potentially  $z_t$ , as discussed in Stock and Watson (2018). If a rule of thumb such as  $F > 10$  is violated, the proxy is considered to be weak. Montiel Olea and Pflueger (2013) present a heteroskedasticity-robust version of this test and Stock and Yogo (2005) extend the test in a general setting to the case of multiple instruments. Lewis and Mertens (2026) assess the weak instrument bias in impulse response functions defining the bias as the difference between the true impulse response parameter and the estimated mode, rather than the mean.

Using this type of statistical diagnostics, examples for potentially weak proxies include some of the extensive list of proxies employed in Stock and Watson (2012) (see their Table 6). In addition, Känzig (2021) shows that Kilian (2008)'s oil supply proxy is potentially weak. Using a time-varying parameter version of the proxy equation (3), Amir-Ahmadi, Matthes and Wang (2022) show that the high-frequency monetary policy proxy by Gertler and Karadi (2015) is a relevant proxy

only in a small number of periods. Mumtaz and Petrova (2023) find that the tax proxies in Mertens and Ravn (2012) and Cloyne (2013) have a non-zero but small correlation with the tax shock they are designed to identify. Ramey and Zubairy (2018) find that the government spending shock proxy based on narrative information about military spending proposed in Ramey (2011) is potentially too weak for estimating impulse responses for short horizons.

A drawback of using such tests could be that, even if the proxy is found to be strong, the coverage of the standard confidence bands for impulse responses could be distorted due to pre-testing issues, as evidenced in simulations, e.g., by Andrews, Stock and Sun (2019). Instead of using a pre-test, or in cases where a pre-test suggests a weak proxy, frequentist weak-instrument robust inference procedures could be used. Montiel Olea et al. (2021) propose a method to construct confidence bands based on Anderson and Rubin (1949) which is shown to coincide with standard confidence bands if the proxy is strong. Alternatively, Angelini, Cavaliere and Fanelli (2024) show that standard asymptotic methods are valid in the presence of weak proxies for the target shocks,  $\mathbf{w}_{1t}$ , as long as strong proxies are available for the non-target shocks,  $\mathbf{w}_{2t}$ .

A Bayesian approach to dealing with weak proxies could specify a prior distribution for  $\sigma_{w_1z}$ , or transformations thereof, as in Caldara and Herbst (2019). If the researcher is convinced of the strength of the proxy before investigating the data, she can express this via a prior distribution for  $\sigma_{w_1z}$  centred away from 0. The posterior distribution of  $\sigma_{w_1z}$  can be used to compute a reliability indicator for the proxy. For this purpose the squared correlation of the proxy and the shock is sometimes used. More dogmatically, Caldara and Herbst (2019) also propose a “high relevance” prior, where exactly half of the variation in the proxy can be attributed to measurement error, i.e.,  $\text{std}(\eta_t) = 0.5 \times \text{std}(z_t)$  with probability 1 in equation (3).

## 4.5 Bootstrap Intervals for Structural Impulse Responses

The structural form of the VAR model is typically used for further structural analysis. For example, an impulse response analysis is often performed. Other important analysis tools are, for example, FEVDs and historical decompositions. In Bayesian analysis inference for impulse responses and other quantities is based in the usual way on the posterior distribution. Instead, in a frequentist setup, bootstrap approaches are often used for inference. In that case, the standard bootstrap methods for VAR analysis have to be complemented by resampling the proxies. We will therefore give more detail on the required modifications for the relevant bootstrap methods in the following. We focus on impulse responses in this context because the resampling modifications are similar for other quantities.

To account for possible heteroskedasticity, the wild bootstrap (WB) has been

used in this context in the past. More recently, the moving-block bootstrap (MBB) has replaced the WB in many instances because of its asymptotic validity and ease of implementation, see Brüggemann, Jentsch and Trenkler (2016) and Jentsch and Lunsford (2019, 2022). Its drawback is that it may require very large samples to provide reliable confidence intervals. Therefore Bruns and Lütkepohl (2023a) propose a “proxy residual-based bootstrap” (PRBB) as an alternative to the moving-block bootstrap for proxy VARs. This section draws in parts on the latter article which compares WB, MBB, and PRBB. In presenting the three bootstraps we assume that there is only one proxy that identifies a single shock.

While the bootstrap procedures differ in the way they resample the data, Bruns and Lütkepohl (2023a) describe the following common steps of the three procedures to determine bootstrap impulse responses and confidence intervals. For each bootstrap iteration  $r$  the following steps are performed:

**Step 1** A reduced-form VAR( $p$ ) model is fitted to the bootstrap sample by least squares, giving bootstrap parameter estimates  $\hat{A}_1(r), \dots, \hat{A}_p(r)$ , and residuals  $\hat{u}_t(r)$ .

**Step 2** Bootstrap estimates of the structural parameters,  $\hat{b}_1(r)$ , are determined. Any suitable method for estimating the structural parameters can be used in this step.

**Step 3** Bootstrap estimates of the impulse responses of interest,  $\hat{\theta}(H)^{(r)}$ , are computed and stored.

The WB, MBB and PRBB differ in the way they resample the residuals and proxies as follows:

**Wild bootstrap:** For  $t = 1, \dots, T$ , independent random variables  $\psi_t$  from a distribution with mean 0 and variance 1, e.g.,  $\psi_t \sim \mathcal{N}(0, 1)$ , are drawn and bootstrap residuals and proxy variables are generated as

$$\begin{pmatrix} u_t^{WB} \\ z_t^{WB} \end{pmatrix} = \psi_t \begin{pmatrix} \hat{u}_t \\ z_t \end{pmatrix}.$$

**Moving block bootstrap:** A block length  $\ell < T$  has to be chosen for the MBB and the dataset is divided into equal-length blocks. Then blocks of residuals and proxies are drawn with replacement. These randomly drawn blocks are joined end-to-end and the first  $T$  bootstrap residuals and proxies are retained to obtain

$$\begin{pmatrix} u_t^{MBB} \\ z_t^{MBB} \end{pmatrix}.$$

**Proxy residual-based bootstrap:** The PRBB assumes that proxies are generated as

$$z_t = D_t(\phi w_{1t} + \eta_t), \quad (20)$$

where  $D_t$  is a random 0-1 variable following a Bernoulli distribution,  $B(d)$ , with  $d$  being the share of non-zero observations of  $z_t$  in the original sample. Samples

$$\begin{pmatrix} u_t^{PRBB} \\ \eta_t^{PRBB} \\ w_{1t}^{PRBB} \end{pmatrix}, t = 1, \dots, T, \text{ are drawn from } \begin{pmatrix} \hat{u}_1 \\ \hat{\eta}_1 \\ \hat{w}_{11} \end{pmatrix}, \dots, \begin{pmatrix} \hat{u}_T \\ \hat{\eta}_T \\ \hat{w}_{1T} \end{pmatrix},$$

with replacement. A bootstrap series for the proxy,  $z_t(r)$ , is generated from (20).

Having resampled the residuals and the proxy, all three approaches then proceed to generate a bootstrap sample,  $y_t(r)$ , recursively.

In a simulation study based on a finite-order VAR process, Bruns and Lütkepohl (2023a) find that the WB can have very wide confidence intervals in realistic settings and that the MBB can have poor coverage in small sample sizes, as often encountered in applied macroeconomic studies. The PRBB on the other hand has a more accurate coverage than WB and MBB, at least in small samples. The finding that the PRBB has a shorter average length than the WB is confirmed in an illustration based on the U.S. monetary policy model by Gertler and Karadi (2015).

While the PRBB displayed good small sample properties in simulations, its drawback is that extensions to situations with multiple proxies are not immediately obvious. In fact, it would require setting up a good model for the DGP of the proxies. In contrast, extending the WB and MBB to the case of multiple proxies is straightforward. The MBB requires the choice of a block length, which in small samples might affect coverage properties. However, a rule-of-thumb is available (see Jentsch and Lunsford (2019)) for the applied user. In addition, the MBB has very good asymptotic properties. Therefore it is perhaps not surprising that it has become very popular for proxy VAR analysis lately.

## 5 Complications

So far we have mostly considered proxy VAR analysis under the assumption that a set of ideal conditions is satisfied. In practice, violations of the ideal conditions are not uncommon. Therefore researchers have also thought about the implications of such violations. In this section we review some of the related thoughts and results.

## 5.1 Heteroskedasticity

In our setup, the relation between the reduced-form errors and the structural shocks is given by  $u_t = B\mathbf{w}_t$  which implies the relation  $\Sigma_u = B\Sigma_{\mathbf{w}}B'$ , where  $\Sigma_{\mathbf{w}}$  is diagonal. In other words, the structural parameters  $B$  and  $\Sigma_{\mathbf{w}}$  are related to  $\Sigma_u$ . If the reduced-form residuals are heteroskedastic such that  $\Sigma_u$  varies over time, then the structural parameters must also change with changes in  $\Sigma_u$  which can lead to distorted estimates of the impact effects and, hence, of impulse responses and related quantities, if the time-variation is ignored.

To fix ideas, suppose during the sample there are two covariance or volatility states with covariance matrices  $\Sigma_u(1)$  and  $\Sigma_u(2)$  and that structural parameters  $B(m)$  and  $\Sigma_{\mathbf{w}}(m)$ ,  $m = 1, 2$ , are associated with the two volatility states. It is possible, however, that only the variances of the structural shocks change ( $\Sigma_{\mathbf{w}}(1) \neq \Sigma_{\mathbf{w}}(2)$ ) and the impact effects in both volatility regimes are identical such that  $B(1) = B(2)$ . In that case, the heteroskedasticity may actually contain useful identifying information for the shocks (see, e.g., Lütkepohl and Netšunajev (2017) or Kilian and Lütkepohl (2017, Chapter 14)). Thus, to assess whether this type of identifying information is available, we have to disentangle whether all the structural parameters change or only the variances of the structural shocks change or only the impact effects change with the volatility regime.

Lütkepohl and Schlaak (2022) and Bruns and Lütkepohl (2024) develop tests for time-varying impact effects of the shocks of interest when the reduced-form residuals are heteroskedastic and valid proxies for the shocks of interest are available. They use that the proxies provide over-identifying information under the null hypothesis that the impact effects of the shocks of interest are time-invariant. Thus, they combine identifying information from heteroskedasticity and from proxies. Although our discussion here has been for two volatility regimes only, this can, of course, be generalized as discussed by Lütkepohl and Schlaak (2022) and Bruns and Lütkepohl (2024). Applying the tests to the global crude oil market, Bruns and Lütkepohl (2023b) find evidence that the transmission of oil supply news shocks on macroeconomic aggregates has changed during the last decades.

## 5.2 Nonfundamental Shocks

Plagborg-Møller and Wolf (2021) start from the more general assumption of  $y_t$  being generated as

$$y_t = \sum_{i=0}^{\infty} \Theta_i \mathbf{w}_{t-i}, \quad (21)$$

where  $\mathbf{w}_t$  is an  $M$ -dimensional vector of structural shocks with  $M$  being potentially much larger than  $K$ , the number of variables in  $y_t$ . In that case, the DGP of  $y_t$

may still have a VAR representation as in (1). However, not all the  $M$  shocks  $\mathbf{w}_t$  can be linear transformations of the reduced-form residuals  $u_t$ . If, however, only  $K_1 < K$  of the shocks are of direct interest, the shocks of interest may still be linear combinations of the  $u_t$  and, hence, fundamental. If there are  $K_1$  proxies  $z_t$  that satisfy the relevance and exogeneity conditions (4) and (5), we can still use them to identify the shocks of interest.

In this context, Plagborg-Møller and Wolf (2021) point out that, in general, the impulse responses obtained from a VAR model will be distorted if the true shocks are nonfundamental and they mention that correct impulse responses in this case are obtained only if the assumed shock is a function of just the true shock and perhaps an error term which is not related to  $y_t$ . They also point out that the impulse responses of nonfundamental shocks can be estimated by local projections if suitable instruments exist (see also Stock and Watson (2018)). Plagborg-Møller and Wolf (2021) show that, if  $z_t$  is a scalar proxy, even if the shock is not fundamental, its impulse responses can be estimated properly by adding  $z_t$  as an additional variable to the VAR.

Plagborg-Møller and Wolf (2022) discuss the more general property of recoverability of nonfundamental shocks, where a shock is called recoverable if it can be obtained from past, current and future values of the  $y_t$ , while for fundamentalness it is required that it can be recovered from past and present values of  $y_t$  only. They consider FEVDs and historical decompositions and present results for such quantities when the shocks are recoverable.

### 5.3 Non-exogeneity

A necessary condition for achieving point identification is that the proxies are contemporaneously uncorrelated with the non-target shocks,  $\mathbf{w}_{2t}$ . In other words, the proxies need to be exogenous, see equation (5), which we repeat here for convenience:

$$\mathbb{E}(\mathbf{w}_{2t}z'_t) = 0.$$

Without further assumptions, the exogeneity condition cannot be assessed because the non-target shocks,  $\mathbf{w}_{2t}$ , are not observed. If the exogeneity assumption is violated, this may lead to bias in the structural analysis. Keweloh, Klein and Prüser (2025) show, for example, that fiscal elasticities may be biased if proxies for fiscal shocks are endogenous.

However, there exist a number of approaches to assess exogeneity by drawing on additional information: Schlaak et al. (2023) achieve point-identification of their model via changes in the volatility of the residuals. The exogeneity restriction (5) is then over-identifying and testable. In an application to monetary policy, the

authors compare the validity of a number of popular monetary policy proxies and find that some high-frequency proxies might be invalid.

In a Bayesian setting, Braun and Brüggemann (2023) provide tools for assessing proxy exogeneity. They impose bounds on the correlation of the proxy with the non-target shocks instead of restricting this correlation to exactly zero as in (5). They then supplement the identifying information contained in the proxy with sign restrictions, thereby achieving over-identification, and report Bayes factors as a measure of support for these over-identifying restrictions. In an application to monetary policy and imposing sign restrictions on the monetary policy rule parameters, the authors find evidence against the exogeneity of the external instrument based on narrative information proposed in Romer and Romer (2010).

Bruns and Keweloh (2024) start from the observation that the economic rationale underlying the construction of commonly used proxy variables aligns with a stronger form of exogeneity. Namely, proxies are typically constructed to not only be uncorrelated with the non-target shocks, as in (5), but to contain no information about the non-target shocks in the sense that they satisfy the mean-independence condition  $\mathbb{E}(\mathbf{w}_{2t}|z_t) = 0$ . For example, when Känzig (2021) constructs an oil supply news proxy, he argues that the proxy is a function only of the oil supply news shock and not of any non-target shocks.

Intuitively, in cases, where such “strong exogeneity” is likely the rationale behind the proxy construction, one can use the condition  $\mathbb{E}(\mathbf{w}_{2t}\tilde{z}'_t) = 0$  for a suitable “synthetic proxy”,  $\tilde{z}_t$ , obtained from the strong exogeneity condition, in addition to  $\mathbb{E}(\mathbf{w}_{2t}z'_t) = 0$ . The original proxy  $z_t$  yields  $K - 1$  moment conditions

$$\mathbb{E}[f_z(\beta, u_t)] = 0 \quad \text{with} \quad f_z(\beta, u_t) = \mathbf{u}_{2t}z_t - \beta u_{1t}z_t, \quad (22)$$

which identify the  $K - 1$  non-normalised elements,  $\beta$ , of the first column of  $B$ , if  $z_t$  is valid.  $\mathbf{u}_{2t} = (u_{2t}, \dots, u_{Kt})'$  contains the last  $K - 1$  residuals. The synthetic proxy yields  $K - 1$  additional conditions

$$\mathbb{E}[f_{\tilde{z}}(\beta, u_t)] = 0 \quad \text{with} \quad f_{\tilde{z}}(\beta, u_t) = \mathbf{u}_{2t}\tilde{z}_t - \beta u_{1t}\tilde{z}_t, \quad (23)$$

which lead to a potentially over-identified system. As an example, considering the case of a scalar proxy that identifies a single shock, one could use  $\tilde{z}_t = z_t^2$ . Other measurable functions,  $h(z_t)$ , of the proxy, can be used as well. Bruns and Keweloh (2024) also discuss a generalisation which employs approximating functions to generate a set of unconditional moment conditions.

A standard Hansen’s  $J$ -test can be used to test for proxy exogeneity. It is based on the following test statistic:

$$J_T = T g_T(\hat{\beta}_T)' S^{-1} g_T(\hat{\beta}_T), \quad (24)$$

where

$$S = \lim_{T \rightarrow \infty} \mathbb{E} \left[ T g_T(\hat{\beta}_T) g_T(\hat{\beta}_T)' \right],$$

$$g_T(\beta) = \begin{bmatrix} \frac{1}{T} \sum_{t=1}^T f_z(\beta, u_t) \\ \frac{1}{T} \sum_{t=1}^T f_{\bar{z}}(\beta, u_t) \end{bmatrix},$$

$$\hat{\beta}_T = \operatorname{argmin}_{\beta \in \mathbb{R}^{K-1}} g_T(\beta)' W g_T(\beta),$$

and  $W$  is a suitable weighting matrix.

The test has power against the alternative of the proxy being contaminated by one of the non-target shocks if either the target shock or the potentially contaminating shocks are non-Gaussian or if the relation between proxy and target shock is non-linear, or if both conditions hold. Bruns and Keweloh (2024) show (i) that the resulting test statistic has good small sample properties in realistic environments, (ii) that an application to fiscal policy shocks yields empirically sensible results, and (iii) that the findings can be generalised to cases where multiple proxies identify multiple shocks as a group without the need for further identifying assumptions to disentangle such shocks.

## 6 Conclusions

In structural VAR analysis identifying the shocks of interest is often a major challenge. Therefore a number of different approaches for identifying structural shocks has been proposed and explored in the recent structural VAR literature. Using instruments or proxies in this context has become a popular device recently. Suitable instruments or proxies have to be closely related to the shocks of interest. Ideally proxies for a specific shock are uncorrelated with all other shocks and this assumption is typically made if a single shock is identified via a proxy. We have pointed out, however, that lately researchers have started to use a set of proxies to identify a set of structural shocks in which case the proxies may be related to different shocks and further identifying assumptions will usually be needed for properly identifying the shocks of interest individually.

In this survey we have addressed various issues related to structural VAR analysis with the help of proxies. We have discussed a range of possibilities how proxies have been constructed in the past. We have reviewed conditions for properly identifying the shocks via proxies also in case a set of proxies is used for identifying sets of shocks. We have considered suitable estimation and inference methods that can be used when proxies are available and we have pointed out some problems that arise if the standard assumptions are not satisfied.

We have limited our survey primarily to linear VAR analysis and, hence, we do not fully cover nonlinear models. In the linear model context, the effects of shocks have some limitations. In particular, the shape of the response of the variables is unrelated to the size or sign of the shock. Such limitations are undesirable for some situations. Therefore, nonlinear structural models have also been considered. In fact, there is a full chapter on nonlinear structural VAR analysis in Kilian and Lütkepohl (2017) (see their Chapter 18). Recently some further nonlinear studies have come up in the related literature. Examples of nonlinear studies using proxies are Bruns and Piffer (2024), Forni et al. (2025), Mumtaz and Petrova (2023), Paul (2020), Pellegrino, Castelnuovo and Caggiano (2023).

As structural VAR analysis is a standard tool for macroeconometric and financial analysis, the related methodology is still an area of intense research. Such research is likely to result in new proxies, improvements in the methodology and new insights into potential problems related to the use of proxies in the future.

## References

- Altavilla, C., Brugnolini, L., Gürkaynak, R. S., Motto, R. and Ragusa, G. (2019). Measuring euro area monetary policy, *Journal of Monetary Economics* **108**: 162–179.
- Amir-Ahmadi, P., Matthes, C. and Wang, M.-C. (2022). Understanding instruments in macroeconomics—a study of high-frequency identification, *Technical report*, Working paper.
- Anderson, T. W. and Rubin, H. (1949). Estimation of the parameters of a single equation in a complete system of stochastic equations, *The Annals of Mathematical Statistics* **20**(1): 46–63.
- Andrews, I., Stock, J. H. and Sun, L. (2019). Weak instruments in instrumental variables regression: Theory and practice, *Annual Review of Economics* **11**(1): 727–753.
- Angelini, G., Cavaliere, G. and Fanelli, L. (2024). An identification and testing strategy for proxy-SVARs with weak proxies, *Journal of Econometrics* **238**(2): 105604.
- Angelini, G. and Fanelli, L. (2019). Exogenous uncertainty and the identification of structural vector autoregressions with external instruments, *Journal of Applied Econometrics* **34**: 951–971.

- Angelini, G., Fanelli, L. and Neri, L. (2024). Invalid proxies and volatility changes, *arXiv preprint arXiv:2403.08753* .
- Arias, J. E., Rubio-Ramírez, J. F. and Waggoner, D. F. (2021). Inference in Bayesian Proxy-SVARs, *Journal of Econometrics* **225**: 88–106.
- Arias, J. E., Rubio-Ramírez, J. F. and Waggoner, D. F. (2025). Uniform priors for impulse responses, *Econometrica* **93**(2): 695–718.
- Bahaj, S. (2020). Sovereign spreads in the euro area: Cross border transmission and macroeconomic implications, *Journal of Monetary Economics* **110**: 116–135.
- Baleyte, J., Bazot, G., Monnet, E. and Morys, M. (2024). High temperature shocks are supply shocks. Evidence from one century of monthly data, *Technical report*, CEPR Discussion Paper 19682, CEPR Press.
- Bañbura, M., Giannone, D. and Reichlin, L. (2010). Large Bayesian vector autoregressions, *Journal of Applied Econometrics* **25**: 71–92.
- Barakchian, S. M. and Crowen, C. (2013). Monetary policy matters: Evidence from new shocks data, *Journal of Monetary Economics* **60**: 950–966.
- Bassett, W. F., Chosak, M. B., Driscoll, J. and Zakrajšek, E. (2014). Changes in bank lending standards and the macroeconomy, *Journal of Monetary Economics* **62**(C): 23–40.
- Bastianin, A. and Manera, M. (2018). How does stock market volatility react to oil price shocks?, *Macroeconomic Dynamics* **22**: 666–682.
- Baumeister, C. (2025). Discussion of “Local projections or VARs? A Primer for Macroeconomists” by José Luis Montiel Olea, Mikkel Plagborg-Møller, Eric Qian, and Christian K. Wolf, *NBER Macro Annual Conference* .
- Baumeister, C. and Hamilton, J. D. (2015). Sign restrictions, structural vector autoregressions, and useful prior information, *Econometrica* **83**(5): 1963–1999.
- Bertsche, D. and Braun, R. (2022). Identification of structural vector autoregressions by stochastic volatility, *Journal of Business & Economic Statistics* **40**(1): 328–341.
- Bilal, A. and Känzig, D. R. (2024). The macroeconomic impact of climate change: Global vs. local temperature, *Technical report*, National Bureau of Economic Research.

- Bloom, N. (2009). The impact of uncertainty shocks, *Econometrica* **77**: 623–685.
- Boer, L. and Lütkepohl, H. (2021). Qualitative versus quantitative external information for proxy vector autoregressive analysis, *Journal of Economic Dynamics and Control* **127**: 104118.
- Braun, R. and Brüggemann, R. (2023). Identification of SVAR models by combining sign restrictions with external instruments, *Journal of Business & Economic Statistics* **41**(4): 1077–1089.
- Braun, R., Miranda-Agrippino, S. and Saha, T. (2025). Measuring monetary policy in the UK: The UK monetary policy event-study database, *Journal of Monetary Economics* **149**: 103645.
- Breitung, J. and Brüggemann, R. (2023). Projection estimators for structural impulse responses, *Oxford Bulletin of Economics and Statistics* **85**: 1320–1340.
- Brüggemann, R., Jentsch, C. and Trenkler, C. (2016). Inference in VARs with conditional heteroskedasticity of unknown form, *Journal of Econometrics* **191**: 69–85.
- Bruns, M. and Keweloh, S. A. (2024). Testing for strong exogeneity in proxy-VARs, *Journal of Econometrics* **245**(1-2): 105876.
- Bruns, M. and Lütkepohl, H. (2022). Comparison of local projection estimators for proxy vector autoregressions, *Journal of Economic Dynamics & Control* **134**: 104277.
- Bruns, M. and Lütkepohl, H. (2023a). An alternative bootstrap for proxy vector autoregressions, *Computational Economics* **62**: 1857–1882.
- Bruns, M. and Lütkepohl, H. (2023b). Have the effects of shocks to oil price expectations changed? Evidence from heteroskedastic proxy vector autoregressions, *Economics Letters* **233**: 111416.
- Bruns, M. and Lütkepohl, H. (2024). Heteroskedastic proxy vector autoregressions: An identification-robust test for time-varying impulse responses in the presence of multiple proxies, *Journal of Economic Dynamics and Control* **161**: 104837.
- Bruns, M. and Lütkepohl, H. (2025). Comparing external and internal instruments for vector autoregressions, *Journal of Economic Dynamics and Control* **177**: 105131.

- Bruns, M., Lütkepohl, H. and McNeil, J. (2025). Avoiding unintentionally correlated shocks in proxy vector autoregressive analysis, *Journal of Business and Economic Statistics* . forthcoming.
- Bruns, M. and Piffer, M. (2024). Tractable Bayesian estimation of smooth transition vector autoregressive models, *The Econometrics Journal* **27**(3): 343–361.
- Caldara, D. and Herbst, E. (2019). Monetary policy, real activity, and credit spreads: Evidence from Bayesian proxy SVARs, *American Economic Journal: Macroeconomics* **11**: 157–192.
- Caldara, D. and Kamps, C. (2017). The analytics of SVARs: A unified framework to measure fiscal multipliers, *The Review of Economic Studies* **84**(3): 1015–1040.
- Carriero, A., Marcellino, M. and Tornese, T. (2024). Blended identification in structural VARs, *Journal of Monetary Economics* **146**: 103581.
- Carriero, A., Mumtaz, H., Theodoridis, K. and Theophilopoulou, A. (2015). The impact of uncertainty shocks under measurement error: A proxy SVAR approach, *Journal of Money, Credit and Banking* **47**(6): 1223–1238.
- Cesa-Bianchi, A., Thwaites, G. and Viccondoa, A. (2020). Monetary policy transmission in the United Kingdom: A high frequency identification approach, *European Economic Review* **123**: 103375.
- Cevik, S. and Gwon, G. (2024). This is going to hurt: Weather anomalies, supply chain pressures and inflation, *International Economics* **180**: 100560.
- Ciccarelli, M., Kuik, F. and Martínez Hernández, C. (2024). The asymmetric effects of temperature shocks on inflation in the largest euro area countries, *European Economic Review* **168**: 104805.
- Clemens, M., Michelsen, C. and Rieth, M. (2025). An estimation and decomposition of the government investment multiplier, *Discussion Paper 2106*, DIW Berlin.
- Cloyne, J. (2013). Discretionary tax changes and the macroeconomy: New narrative evidence from the United Kingdom, *American Economic Review* **103**(4): 1507–1528.
- Degasperi, R. (2023). Identification of expectational shocks in the oil market using OPEC announcements, *Technical report*, University of Warwick, Department of Economics.

- Dolado, J. J. and Lütkepohl, H. (1996). Making Wald tests work for cointegrated VAR systems, *Econometric Reviews* **15**: 369–386.
- Drautzburg, T. (2020). A narrative approach to a fiscal DSGE model, *Quantitative Economics* **11**(2): 801–837.
- Dufour, J.-M., Pelletier, D. and Renault, E. (2006). Short run and long run causality in time series: Inference, *Journal of Econometrics* **132**: 337–362.
- Eickmeier, S., Quast, J. and Schüler, Y. (2024). Macroeconomic and financial effects of natural disasters, *Technical Report 45/2024*, Deutsche Bundesbank.
- Fernald, J. (2014). A quarterly, utilization-adjusted series on total factor productivity, *Working paper*, Federal Reserve Bank of San Francisco.
- Fisher, J. D. M. and Peters, R. (2010). Using stock returns to identify government spending shocks, *Economic Journal* **120**: 414–436.
- Forni, M., Franconi, A., Gambetti, L. and Sala, L. (2025). Asymmetric transmission of oil supply news, *Quantitative Economics* **16**: 947–979.
- Forni, M. and Gambetti, L. (2014). Sufficient information in structural VARs, *Journal of Monetary Economics* **66**: 124–136.
- Fusari, F., Marlow, J. and Volpicella, A. (2024). Estimation and inference of the forecast error variance decomposition for set-identified SVARs, *Technical report*, School of Economics, University of Surrey.
- Gertler, M. and Karadi, P. (2015). Monetary policy surprises, credit costs, and economic activity, *American Economic Journal: Macroeconomics* **7**: 44–76.
- Giacomini, R., Kitagawa, T. and Read, M. (2022). Robust Bayesian inference in proxy SVARs, *Journal of Econometrics* **228**(1): 107–126.
- Gilchrist, S. and Zakrajšek, E. (2012). Credit spreads and business cycle fluctuations, *American Economic Review* **102**(4): 1692–1720.  
**URL:** <https://www.aeaweb.org/articles?id=10.1257/aer.102.4.1692>
- Gonçalves, S., Herrera, A. M., Kilian, L. and Pesavento, E. (2024). Nonparametric local projections, *Technical report*, Federal Reserve Bank of Dallas.
- Gregory, A. W., McNeil, J. and Smith, G. W. (2024). US fiscal policy shocks: Proxy-SVAR overidentification via GMM, *Journal of Applied Econometrics* **39**: 607–619.

- Gürkaynak, R. S., Sack, B. P. and Swanson, E. T. (2007). Market-based measures of monetary policy expectations, *Journal of Business & Economic Statistics* **25**: 201–212.
- Hamilton, J. D. (2018). Why you should never use the Hodrick-Prescott filter, *Review of Economics and Statistics* **100**(5): 831–843.
- Hou, C. (2024). Large Bayesian SVARs with linear restrictions, *Journal of Econometrics* **244**(1): 105850.
- Inoue, A. and Kilian, L. (2026). When is the use of Gaussian-inverse Wishart-Haar priors appropriate?, *Journal of Political Economy* **134**(2): 000–000.
- Jarociński, M. and Karadi, P. (2020). Deconstructing monetary policy surprises - the role of information shocks, *American Economic Journal: Macroeconomics* **12**: 1–43.
- Jentsch, C. and Lunsford, K. G. (2019). The dynamic effects of personal and corporate income tax changes in the United States: Comment, *American Economic Review* **109**: 2655–2678.
- Jentsch, C. and Lunsford, K. G. (2022). Asymptotically valid bootstrap inference for proxy SVARs, *Journal of Business & Economic Statistics* **40**: 1876–1891.
- Jermann, U. and Quadrini, V. (2012). Macroeconomic effects of financial shocks, *American Economic Review* **102**: 238–271.
- Jordà, O. (2005). Estimation and inference of impulse responses by local projection, *American Economic Review* **95**: 161–182.
- Känzig, D. R. (2021). The macroeconomic effects of oil supply news: Evidence from OPEC announcements, *American Economic Review* **111**(4): 1092–1125.
- Keweloh, S. A., Klein, M. and Prüser, J. (2025). Estimating fiscal multipliers by combining statistical identification with potentially endogenous proxies, *Technical report*, TU Dortmund University.
- Kilian, L. (2008). Exogenous oil supply shocks: How big are they and how much do they matter for the U.S. economy?, *Review of Economics and Statistics* **90**: 216–240.
- Kilian, L. (2024). How to construct monthly VAR proxies based on daily surprises in futures markets, *Journal of Economic Dynamics and Control* **168**: 104966.

- Kilian, L. and Lütkepohl, H. (2017). *Structural Vector Autoregressive Analysis*, Cambridge University Press, Cambridge.
- Kilian, L., Plante, M. D. and Richter, A. W. (2025). Macroeconomic responses to uncertainty shocks: The perils of recursive orderings, *Journal of Applied Econometrics* **40**: 395–410.
- Kim, H. S., Matthes, C. and Phan, T. (2025). Severe weather and the macroeconomy, *American Economic Journal: Macroeconomics* **17**(2): 315–341.
- Kubota, H. and Shintani, M. (2025). Macroeconomic effects of monetary policy in Japan: an analysis using interest rate futures surprises, *Empirical Economics* **68**(2): 783–801.
- Kuttner, K. N. (2001). Monetary policy surprises and interest rates: Evidence from the Fed funds futures market, *Journal of Monetary Economics* **47**: 523–544.
- Lakdawala, A. (2019). Decomposing the effects of monetary policy using an external instruments SVAR, *Journal of Applied Econometrics* **34**: 934–950.
- Lewis, D. J. and Mertens, K. (2026). Weak instrument bias in impulse response estimators, *Technical report*, Institute for Fiscal Studies.
- Li, D., Plagborg-Møller, M. and Wolf, C. K. (2024). Local projections vs. VARs: Lessons from thousands of DGPs, *Journal of Econometrics* **244**: 105722.
- Liao, W., Sheng, X., Gupta, R. and Karmakar, S. (2024). Extreme weather shocks and state-level inflation of the United States, *Economics Letters* **238**: 111714.
- Lucidi, F. S., Pisa, M. M. and Tancioni, M. (2024). The effects of temperature shocks on energy prices and inflation in the euro area, *European Economic Review* **166**: 104771.
- Ludvigson, S. C., Ma, S. and Ng, S. (2021). Uncertainty and business cycles: exogenous impulse or endogenous response?, *American Economic Journal: Macroeconomics* **13**(4): 369–410.
- Lunsford, K. (2015). Identifying structural VARs with a proxy variable and a test for a weak proxy, *Technical report*, Federal Reserve Bank of Cleveland.
- Lusompa, A. (2023). Local projections, autocorrelation, and efficiency, *Quantitative Economics* **14**: 1199–1220.
- Lütkepohl, H. (2005). *New Introduction to Multiple Time Series Analysis*, Springer-Verlag, Berlin.

- Lütkepohl, H. and Netšunajev, A. (2017). Structural vector autoregressions with heteroskedasticity: A review of different volatility models, *Econometrics and Statistics* **1**: 2–18.
- Lütkepohl, H. and Schlaak, T. (2022). Heteroscedastic proxy vector autoregressions, *Journal of Business and Economic Statistics* **40**: 1268–1281.
- Martínez-Hernández, C. (2020). Disentangling the effects of multidimensional monetary policy on inflation and inflation expectations in the euro area, *School of Business & Economics Discussion Paper 2020/18*, Freie Universität Berlin.
- Mertens, K. and Ravn, M. O. (2012). Empirical evidence on the aggregate effects of anticipated and unanticipated U.S. tax policy shocks, *American Economic Journal: Economic Policy* **4**: 145–181.
- Mertens, K. and Ravn, M. O. (2013). The dynamic effects of personal and corporate income tax changes in the United States, *American Economic Review* **103**: 1212–1247.
- Miranda-Agrippino, S. and Ricco, G. (2021). The transmission of monetary policy shocks, *American Economic Journal: Macroeconomics* **13**: 74–107.
- Miranda-Agrippino, S. and Ricco, G. (2023). Identification with external instruments in structural VARs, *Journal of Monetary Economics* **135**: 1–19.
- Montiel Olea, J. L. and Pflueger, C. (2013). A robust test for weak instruments, *Journal of Business & Economic Statistics* **31**(3): 358–369.
- Montiel Olea, J. L. and Plagborg-Møller (2021). Local projection inference is simpler and more robust than you think, *Econometrica* **89**: 1789–1823.
- Montiel Olea, J. L., Plagborg-Møller, M., Qian, E. and Wolf, C. (2025). Local projections or VARs? A primer for macroeconomists, *Technical report*, NBER Working Paper.
- Montiel Olea, J. L., Plagborg-Møller, M., Qian, E. and Wolf, C. K. (2024). Double robustness of local projections and some unpleasant varithmetic, *Technical report*, National Bureau of Economic Research.
- Montiel Olea, J. L., Stock, J. H. and Watson, M. W. (2021). Inference in structural vector autoregressions identified with an external instrument, *Journal of Econometrics* **225**: 74–87.

- Mumtaz, H. and Petrova, K. (2023). Changing impact of shocks: A time-varying proxy SVAR approach, *Journal of Money, Credit and Banking* **55**(2-3): 635–654.
- Mumtaz, H., Pinter, G. and Theodoridis, K. (2018). What do VARs tell us about the impact of a credit supply shock?, *International Economic Review* **59**: 625–646.
- Paul, P. (2020). The time-varying effect of monetary policy on asset prices, *Review of Economics and Statistics* **102**(4): 690–704.
- Pellegrino, G., Castelnuovo, E. and Caggiano, G. (2023). Uncertainty and monetary policy during the great recession, *International Economic Review* **64**: 577–606.
- Piffer, M. and Podstawski, M. (2018). Identifying uncertainty shocks using the price of gold, *The Economic Journal* **128**(1549): 3266–3284.
- Plagborg-Møller, M. and Wolf, C. K. (2021). Local projections and VARs estimate the same impulse responses, *Econometrica* **89**: 955 – 980.
- Plagborg-Møller, M. and Wolf, C. K. (2022). Instrumental variable identification of dynamic variance decompositions, *Journal of Political Economy* **130**(8): 2164–2202.
- Ramey, V. A. (2011). Identifying government spending shocks: It’s all in the timing, *The Quarterly Journal of Economics* **126**(1): 1–50.
- Ramey, V. A. and Zubairy, S. (2018). Government spending multipliers in good times and in bad: Evidence from US historical data, *Journal of Political Economy* **126**(2): 850–901.
- Ricco, G., Savini, E. and Tuteja, A. (2024). Monetary policy, information and country risk shocks in the euro area, *Discussion Paper DP19679*, CEPR.
- Romer, C. D. and Romer, D. H. (2009a). Do tax cuts starve the beast? The effect of tax changes on government spending, *Brookings Papers on Economic Activity* **Spring 2009**: 139–200.
- Romer, C. D. and Romer, D. H. (2009b). A narrative analysis of postwar tax changes, *Technical report*, University of California, Berkeley.
- Romer, C. D. and Romer, D. H. (2010). The macroeconomic effects of tax changes: Estimates based on a new measure of fiscal shocks, *American Economic Review* **100**: 763–801.

- Schlaak, T., Rieth, M. and Podstawski, M. (2023). Monetary policy, external instruments, and heteroskedasticity, *Quantitative Economics* **14**(1): 161–200.
- Smets, F. and Wouters, R. (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach, *American Economic Review* **97**: 586–606.
- Staiger, D. and Stock, J. (1997). Instrumental variables regression with weak instruments, *Econometrica* **65**(3): 557–586.
- Stock, J. H. and Watson, M. W. (2012). Disentangling the channels of the 2007-09 recession, *Brookings Papers on Economic Activity* pp. 81–135.
- Stock, J. H. and Watson, M. W. (2018). Identification and estimation of dynamic causal effects in macroeconomics using external instruments, *Economic Journal* **128**: 917–948.
- Stock, J. H., Wright, J. H. and Yogo, M. (2002). A survey of weak instruments and weak identification in generalized method of moments, *Journal of Business & Economic Statistics* **20**(4): 518–529.
- Stock, J. and Yogo, M. (2005). Asymptotic distributions of instrumental variables statistics with many instruments, *Identification and Inference for Econometric Models: Essays in Honor of Thomas Rothenberg*, Cambridge University Press Cambridge, chapter 6, pp. 109–120.
- Swanson, E. T. (2021). Measuring the effects of federal reserve forward guidance and asset purchases on financial markets, *Journal of Monetary Economics* **118**: 32–53.
- Toda, H. Y. and Yamamoto, T. (1995). Statistical inference in vector autoregressions with possibly integrated processes, *Journal of Econometrics* **66**: 225–250.
- Usman, S., Fernández, G. G.-T. and Parker, M. (2025). Going nuts: The regional impact of extreme climate events over the medium term, *European Economic Review* **178**: 105081.
- Wright, J. H. (2012). What does monetary policy do to long-term interest rates at the zero lower bound?, *Economic Journal* **122**: F447–F466.