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Aggregation without the aggravation? Nonparametric analysis of the representative consumer.

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Abstract

In the tradition of Afriat (1967), Diewert (1973) and Varian (1982), we provide a revealed preference characterisation of the representative consumer. Our results are simple and complement those of Gorman (1953, 1961), Samuelson (1956) and others. They can also be applied to data very readily and without the need for auxilliary parametric or statistical assumptions. We investigate the application of our characterisation by means of a balanced microdata panel survey. Our findings provide robust evidence against the existence of a representative consumer for our data.

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1 Introduction

The step from the economics of individual consumers' demands to the economics of aggregate demand is a slippery one. It has, of course, long been a core question in economics and, in particular, there has been an important literature on the problem of aggregation: the circumstances under which it is possible to treat aggregate demand as if it were the outcome of choices being made by a single, rational, optimising, normatively significant, representative consumer. These circumstances are known to be very demanding. The best known results in this area are probably those of Gorman (1953, 1961), who derived the conditions under which aggregate demand can be written a function of prices and aggregate income alone.[1] Gorman showed that such exact aggregation is possible if and only if the Engel curves of consumers are all straight lines with a common slope. Moreover, he showed that exact aggregation implies the existence of a normatively significant representative consumer.

Although the concepts of exact aggregation and the representative consumer have a long tradition in the economics literature, they became most prominent after Lucas' (1976) famous critique,

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[1] Exact linear aggregation is to be distinguished from exact nonlinear aggregation, where aggregate demand is a function of some representative level of aggregate income, which itself can be a function of the distribution of income over the individuals (see Muellbauer, 1975, 1976).

which stimulated the new research programme on the microfoundations of macroeconomics. One common feature of the first generation macromodels with solid microfoundations (with Kydland and Prescott (1982) as, perhaps, one of the most well-known examples) is that they assume a representative consumer.[2] Notwithstanding the fact that macroeconomists nowadays fully recognise the importance of heterogeneity (see, for example, Heathcote, Storesletten and Violante, 2009), we may safely argue that the representative consumer still plays a major role in many modern macroeconomic models and in macroeconomics textbooks (see, for example, Clarida, Gali and Gertler, 1999, Woodford, 2003, Uhlig, 2010, and Gourio, 2010).

In this paper we revisit the problem which Gorman addressed. We too seek necessary and sufficient conditions for exact aggregation. However, we do this from a rather different perspective, that of the revealed preference tradition of Samuelson (1938, 1948), Afriat (1967), Diewert (1973) and Varian (1982). Rather than describing the restrictions on behaviour in terms of the derivatives of certain unobservable functions (symmetry of the cross derivatives of the consumer's cost function, for example), this approach works by characterising them in terms of a finite system of inequalities involving the consumer's observed choices only.

This exercise is of a certain amount of theoretical interest, but this is not our only motivation. Our motivation is also empirical. Revealed preference methods directly analyse the raw data themselves using techniques from finite mathematics. In contrast, methods based on the derivative properties of functions require that the relevant functions are known, and since we never observe functions, these have to be estimated. The conclusions from such an exercise necessarily rest jointly on the validity of the hypothesis at stake plus a number of crucial auxiliary statistical assumptions necessary to deliver consistent estimates of the functions of interest. Revealed preference methods are, to a great extent, free of these auxiliary hypotheses, and so allow researchers to focus with much greater clarity on the economic hypothesis at the core. Furthermore, they are applicable when there are very few observations and hence when nonparametric statistical methods would be infeasible. As such, using these methods we can assess the empirical validity of exact aggregation without unnecessarily aggravating the analysis.

The cost of the revealed preference approach is that, due to its "nonparametric" nature, its empirical content is rather weak compared to methods which assume full knowledge of demand functions, cost functions, and the like.[3] In the present context this might turn out to be an advantage. This is because the microeconometric evidence, based as it is on estimated derivatives of demand functions, has been strongly anti the representative consumer. Papers which consider the question of whether or not the representative consumer exists have therefore tended to take a rather funereal tone (see especially Kirman, 1992, and Carroll, 2000). The greater empirical flexibility of the revealed preference approach, by contrast, has the potential to allow us to conclude that reports of the death of the representative consumer may in fact been "greatly exaggerated".

The remainder of this paper is structured as follows. In Section 2 we introduce the notation and some core concepts with respect to the individual consumer. Then, we distinguish between the positive and the normative representative consumer (only the latter plays a meaningful role from a welfare economics point of view). We also state the revealed preference conditions for the existence of a normative representative consumer given a socially optimal income distribution rule (following Samuelson, 1956). It will turn out that these conditions are difficult to test because they are nonlinear in nature. Nonetheless we can derive an empirically feasible test of a slightly strengthened definition of a normative representative consumer. This test requires a specific assumption on the distribution of either the marginal utility of income at the micro level or the social weights at the macro level.

Importantly, because our characterisation in Section 2 is defined for a given income distribution rule, it does not guarantee exact linear aggregation (which requires that aggregate demand depends

[2] At this point, it is worth indicating that the representative consumer actually does not only feature in the macroeconomics literature. It is also a cornerstone in some of the most important micromodels. See, for example, Dixit and Stiglitz (1977).

[3] See Beatty and Crawford (2011) for more discussion.

only on the aggregate income). However, it does provide a useful first step towards establishing the revealed preference conditions for such exact aggregation. This is discussed in Section 3, which contains the core contribution of this paper. Specifically, we here investigate the link between the conditions derived in Section 2 and the well-known Gorman aggregation conditions. Along the way we also provide a revealed preference characterisation of Gorman Polar Form preferences for an individual consumer (which is surprisingly weak from an empirical point of view) and, based upon this characterisation, we propose an easy-to-implement necessary and sufficient test for the existence of a normative representative consumer that holds for all possible income distributions across consumers. Interestingly, we can show that this test is empirically equivalent to the test developed in Section 2 (for aggregation à la Samuelson, 1956, assuming a socially optimal income distribution) under a fairly weak data requirement.

In Section 4 we bring our testable implications to a balanced microdata panel of Spanish households. Our application proceeds in two steps. Firstly, we test the conditions for exact linear aggregation for all the rational households in our sample. A main conclusion here will be that the conditions are systematically rejected, even when they are applied to small, highly homogeneous groups of households. Given this, we subsequently analyse the unobservable heterogeneity across households that causes this rejection. As we will show, our revealed preference methods (applied to panel data) are well-suited for investigating such inter-household heterogeneity. Section 5 offers some conclusions regarding the prospects for revivifying the representative consumer.

2 Samuelson revealed: a first characterization of the positive and normative representative consumer

In this section we introduce some first concepts and results that will be useful for our following discussion. We start by briefly reviewing the revealed preference conditions for rational consumption behaviour of individual consumers. Next, we make the distinction between the positive and the normative representative consumer, and we will argue that the latter concept is the only meaningful one from a welfare economics perspective. Subsequently, we derive necessary and sufficient conditions for the existence of such a normative representative consumer for a given, socially optimal income distribution rule. Essentially, this provides a revealed preference treatment of the aggregation concept originally considered by Samuelson (1956). It sets the stage for our discussion in Section 3, where we will consider the revealed preference characterisation of exact linear aggregation à la Gorman (1953, 1961), which implies a normative representative consumer independent of the income distribution.

Individual rationality. Suppose that we have a balanced microdata panel of consumers indexed by $h = 1, ..., H$ observed over a number of periods indexed $t = 1, ..., T$. Following Gorman (1953), we make the classical assumption that the law of one price holds and that prices are strictly positive K-vectors ($\mathbf{p}_t \in \mathbb{R}^K_{++}$). For each consumer h we observe non-negative quantities $\mathbf{q}_t^h \in \mathbb{R}^K_+$. We will denote these microdata by $\left\{\mathbf{p}_t, \mathbf{q}_t^h\right\}_{t\in\tau}^{h\in\eta}$, with $\eta = \{1, ..., H\}$ and $\tau = \{1, ..., T\}$ being the index sets for consumers and periods, respectively. We will use $\mathbf{Q}_t = \sum_{h\in\eta} \mathbf{q}_t^h$ to denote the aggregate demand vector in period t, so that the macrodata are $\{\mathbf{p}_t, \mathbf{Q}_t\}_{t\in\tau}$. Aggregate income is denoted by Y_t and is equal to $\mathbf{p}_t' \sum_{h\in\eta} \mathbf{q}_t^h$.

In what follows, we will assume that all the consumers are rational in the sense that observed demand results from the maximisation of a well-behaved utility function subject to an individual budget constraint. Throughout, we will assume utility functions that are nonsatiated, monotonically increasing, concave and continuous.

Definition 1 (Individual rationalisation) *A utility function u^h provides an* individual rationalisation *of the data $\left\{\mathbf{p}_t, \mathbf{q}_t^h\right\}_{t\in\tau}$ for the h'th consumer if for each observation $t \in \tau$ we have*

$u^h(\mathbf{q}_t^h) \geq u^h(\mathbf{q})$ *for all* $\mathbf{q}$ *with* $\mathbf{p}_t'\mathbf{q} \leq \mathbf{p}_t'\mathbf{q}_t^h$.

Before we focus on aggregate demand, it is useful to discuss the empirical content of individual rationalisation. A core result in the revealed preference approach to demand is that there exists a utility function that provides an individual rationalisation of the data $\{\mathbf{p}_t, \mathbf{q}_t^h\}_{t\in\tau}$ if and only if the data satisfy the *Generalised Axiom of Revealed Preference* (GARP).

Definition 2 (GARP) *The data* $\{\mathbf{p}_t, \mathbf{q}_t^h\}_{t\in\tau}$ *satisfy GARP if there exist relations* R_0^h, R^h *that meet:*
(A). if $\mathbf{p}_s'\mathbf{q}_s^h \geq \mathbf{p}_s'\mathbf{q}_t^h$ *then* $\mathbf{q}_s^h \; R_0^h \; \mathbf{q}_t^h$;
(B). if $\mathbf{q}_s^h \; R_0^h \; \mathbf{q}_u^h$, $\mathbf{q}_u^h \; R_0^h \; \mathbf{q}_v^h$, ..., $\mathbf{q}_z^h \; R_0^h \; \mathbf{q}_t^h$ *for some (possibly empty) sequence* $(u, v, ..., z)$ *then* $\mathbf{q}_s^h$ $R^h \; \mathbf{q}_t^h$;
(C). if $\mathbf{q}_s^h \; R^h \; \mathbf{q}_t^h$ *then* $\mathbf{p}_t'\mathbf{q}_t^h \leq \mathbf{p}_t'\mathbf{q}_s^h$.

In other words, the bundle of quantities $\mathbf{q}_s^h$ is directly revealed preferred over the bundle $\mathbf{q}_t^h$ (i.e. $\mathbf{q}_s^h \; R_0^h \; \mathbf{q}_t^h$) if $\mathbf{q}_s^h$ were chosen when $\mathbf{q}_t^h$ were equally attainable (i.e. $\mathbf{p}_s'\mathbf{q}_s^h \geq \mathbf{p}_s'\mathbf{q}_t^h$); see condition (A). Next, the *revealed preference* relation R^h exploits transitivity of preferences; see condition (B). Finally, condition (C) imposes that the bundle of quantities $\mathbf{q}_t^h$ cannot be more expensive than revealed preferred quantities $\mathbf{q}_s^h$.

We can now state the following result, which is usually referred to as Afriat's Theorem (Varian, 1982; based on Afriat, 1967):

Theorem 1 (Afriat's Theorem) *The following statements are equivalent:*
(1.A). There exists a nonsatiated, monotonic, concave and continuous utility function u^h *that provides an individual rationalisation of the data* $\{\mathbf{p}_t, \mathbf{q}_t^h\}_{t\in\tau}$.
(1.B). The data $\{\mathbf{p}_t, \mathbf{q}_t^h\}_{t\in\tau}$ *satisfy* GARP.
(1.C). For all $s, t \in \tau$, *there exist numbers* u_s^h, $u_t^h \in \mathbb{R}_+$ *and* $\beta_t^h \in \mathbb{R}_{++}$ *that meet the* Afriat inequalities

$$u_s^h \leq u_t^t + \beta_t^h \mathbf{p}_t' \left(\mathbf{q}_s^h - \mathbf{q}_t^h\right).$$

The equivalence between statements $(1.A)$ and $(1.B)$ captures what we mentioned above: any data set $\{\mathbf{p}_t, \mathbf{q}_t^h\}_{t\in\tau}$ can be rationalised by a well-behaved utility function if and only if these price-quantity pairs satisfy GARP. Next, the equivalent statement $(1.C)$ defines so-called Afriat inequalities, which are expressed in the unknowns u_t^h and β_t^h. These Afriat inequalities allow us to obtain an explicit construction of the utility levels and the marginal utility of income associated with each observation t: they define a utility level u_t^h and a marginal utility of income β_t^h (associated with the observed income $\mathbf{p}_t'\mathbf{q}_t^h$) for each observed $\mathbf{q}_t^h$. Importantly, as has been demonstrated by Varian (1982), and later by Blundell, Browning and Crawford (2003, 2008), the above insights can be used to formally evaluate policy reforms in terms of individual welfare.

Let us then consider rationalising the data ($\{\mathbf{p}_t, \mathbf{q}_t^h\}_{t\in\tau}^{h\in\eta}$ and $\{\mathbf{p}_t, \mathbf{Q}_t\}_{t\in\tau}$) in terms of a representative consumer. An important thing to note here is that there are actually two main personifications of this representative consumer.

The positive representative consumer. The *positive* representative consumer exists whenever aggregate demand can be modelled as the outcome of rational, maximising behaviour given prices and aggregate income. The positive representative consumer can be thought of as having classically well-behaved preferences, but those preferences need not have any normative significance.[4] As Gorman (1976) aptly put it, the positive representative consumer is

> "rather an odd chap ...he is as likely as not to be radiantly happy when those he represents are miserable and *vice versa*"
>
> Gorman (1976) reprinted in Blackorby and Shorrocks (1995).

[4] See, for example, Dow and Werlang (1988), Kirman (1992) and Jerison (1994).

The revealed preference characterisation of this "odd chap" was given by Varian (1984) and turned out to be simple: the macrodata $\{\mathbf{p}_t, \mathbf{Q}_t\}_{t\in\tau}$ must satisfy GARP. This is very easily testable and does not involve any parametric assumptions about the form of the macro-utility function.[5]

Whilst the positive representative consumer is a potentially useful character upon which one can base macro-level predictions, the trouble with him is, as Gorman (1976) was pointing out, that he is not fully "representative" in the welfare sense - none of the implied aggregate utility functions associated with his preferences can necessarily be thought of as a social welfare function. As a result the positive version of the representative consumer cannot be used for welfare analysis. We therefore say farewell to the positive representative consumer at this point and focus entirely on his more interesting and socially conscious cousin: the *normative* representative consumer.

The normative representative consumer. The normative representative consumer is a special case of the positive representative consumer. Like the positive consumer he also exists whenever aggregate demands can be modelled as the outcome of rational, maximising behaviour given prices and aggregate income. However, the normative consumer's preferences can properly be regarded as an aggregate social welfare function. This makes him a much more useful construction: you can use him both to make predictions and to make welfare statements. The normative representative consumer is modelled as solving the following problem:[6]

$$\max_{\mathbf{q}^1,\ldots,\mathbf{q}^H \in \mathbb{R}^K_+} V\left(u^1\left(\mathbf{q}^1\right), ..., u^H\left(\mathbf{q}^H\right)\right) \text{ subject to } \mathbf{p}_t' \sum_{h=1}^{H} \mathbf{q}^h = Y_t, \tag{1}$$

where Y_t is aggregate income and where $u^1, ..., u^H$ and V are well-behaved utility functions. The question we focus on concerns the conditions under which the microdata and the associated macro behaviour can be rationalised by this model. In what follows, we derive these conditions under the assumption that some income distribution rule guarantees a socially optimal distribution of the aggregate income over the individual consumers. We return to this income distribution rule concept in more detail at the end of this section.

The following defines what it means for data to be rationalised by the preferences of a normative representative consumer (when assuming a socially optimal income distribution rule).

Definition 3 (Normative representative consumer rationalisation) *The utility functions V, u^1, ... , u^H provide a normative representative consumer rationalisation of the data $\left\{\mathbf{p}_t, \mathbf{q}_t^h\right\}_{t\in\tau}^{h\in\eta}$ if $V(u^1\left(\mathbf{q}_t^1\right), \ldots, u^H\left(\mathbf{q}_t^H\right)) \geq V(u^1\left(\mathbf{q}^1\right), \ldots, u^H\left(\mathbf{q}^H\right))$ for all alternative micro-allocations $\left\{\mathbf{q}^h\right\}^{h\in\eta}$ such that $\mathbf{p}_t' \sum_{h=1}^{H} \mathbf{q}_t^h \geq \mathbf{p}_t' \sum_{h=1}^{H} \mathbf{q}^h$.*

This is simply a statement of the principle of revealed preference in the relevant context: that the normative representative consumer's utility function should associate a higher real number with the observed allocation of resources than it does for any affordable alternative allocation. The next result presents the conditions under which there exists a normative representative consumer who rationalises the data (the proofs of this and all of the following results are in the Appendix).

Theorem 2 *The following statements are equivalent:*
(2.A). There exist nonsatiated, monotonic, concave and continuous utility functions $V, u^1, ..., u^H$ that provide a normative representative consumer rationalisation of the data $\left\{\mathbf{p}_t, \mathbf{q}_t^h\right\}_{t\in\tau}^{h\in\eta}$.

[5] See, for example, Crawford and Neary (2008) for an application to country level consumption data.

[6] See, for example, Mas-Colell, Whinston and Green (1995), 4.D.1$^{\mathrm{B}}$, p.125. We note that the normative representative consumer's utility function has the same structure as a latently separable (Gorman, 1968, 1978, Blundell and Robin, 2000, and Crawford, 2006) utility function - except for the important difference that the micro-level allocations to individuals are not latent; they are observed.

(2.B). For all $s,t \in \tau$ and $h \in \eta$, there exist numbers $V_s, V_t, u_s^h, u_t^h \in \mathbb{R}_+$ and $\mu_t, b_t^h \in \mathbb{R}_{++}$ such that

$$V_s \leq V_t + \mu_t \mathbf{b}_t' \left(\mathbf{u}_s - \mathbf{u}_t\right), \tag{2.B.1}$$

$$u_s^h \leq u_t^h + \frac{1}{b_t^h} \mathbf{p}_t' \left(\mathbf{q}_s^h - \mathbf{q}_t^h\right), \tag{2.B.2}$$

with $\mathbf{u}_t = (u_t^1, \ldots, u_t^H)'$ and $\mathbf{b}_t = (b_t^1, \ldots, b_t^H)'$.

Some remarks are in order. Firstly, similar to before, this is an equivalence result, so the conditions in statement $(2.B)$ are both necessary and sufficient: if there exist solutions to the inequalities then the microdata are exactly reproducible by the model of the normative representative consumer with suitable, well-behaved utility functions; equally, if solutions to these inequalities do not exist then neither do suitable, well-behaved utility functions capable of rationalising the data. Secondly, condition $(2.B.2)$ is an Afriat inequality which applies to each consumer in the microdata, and it is equivalent to the statement that the microdata on each consumer, taken one-at-a-time, satisfies GARP. What this means is that it is a necessary condition that every consumer is rationalisable by a well-behaved, individual utility function. This, of course, is entirely natural: if the representative consumer is to be normatively significant, it is clearly necessary that those he is intended to represent are themselves rationalisable. Note that individual preferences are allowed to be arbitrarily heterogeneous across consumers and can take any form - the only restrictions are that these individual preferences must be rational and well-behaved. Thirdly, condition $(2.B.1)$ is an Afriat inequality that captures the existence of a well-behaved utility function that aggregates the consumer's utility functions. Finally, whilst the form of Theorem 2 is entirely different to the kind of results found in the exact aggregation literature, which makes use of derivative properties of functions (there are no functional forms, in particular there is nothing which indicates any kind of homotheticity, and there is nothing which relates in an obvious way to the marginal utility of income), the Afriat numbers in statement $(2.B)$ bear certain important interpretations which do relate to the standard approach. The numbers $\left\{u_t^h, 1/b_t^h\right\}_{t\in\tau}$, for example, can be interpreted as utility levels and the marginal utility of income at each observed choice for consumer h.[7] Similarly, the numbers $\{V_t, \mu_t\}_{t\in\tau}$ can be interpreted as a measure of aggregate welfare and the marginal social utility of income. Note that neither the distribution of the marginal utility of individual income or the marginal social utility of income are restricted other than via their interaction in $(2.B.1)$. This interaction is important, however, so we turn to it next.

The conditions in $(2.B)$ provide a characterisation of the necessary and sufficient empirical conditions for a normative representative consumer. They are also very general - there are no restrictions on micro-preferences other than well-behaved-ness and none at all on the type or distribution of unobservable heterogeneity. However, there is a difficulty: these conditions are not fully testable. This is because the Afriat numbers in $(2.B)$ are not unique. What this means in practice is that as soon as the investigator finds a solution to the inequalities, the search stops and a normative representative consumer is known to exist. However, if after searching for a while no solution has been found, the only option is to keep searching. Unfortunately, the set of possible Afriat numbers is infinite and it would take forever to exhaust them. Conditions like this are sometimes said to have a bias towards acceptance - simply because a falsification result would take an infinite amount

[7] To explain more in detail: given that the individual utility function u^h is concave (and assuming differentiability for ease of exposition, though this is easily relaxed), we have that $u^h\left(\mathbf{q}_s^h\right) \leq u^h\left(\mathbf{q}_t^h\right) + \boldsymbol{\nabla} u^h\left(\mathbf{q}_t^h\right)'\left(\mathbf{q}_s^h - \mathbf{q}_t^h\right)$ for all s,t. Maximising behaviour implies that the usual first order conditions are $\boldsymbol{\nabla} u^h\left(\mathbf{q}_t^h\right) \leq \left(1/b_t^h\right)\mathbf{p}_t$ (allowing for non-purchase of some goods), where $1/b_t^h$ represents the value of the Lagrange multiplier in the budget constraint. We can substitute this into the concavity condition to give $u^h\left(\mathbf{q}_s^h\right) \leq u^h\left(\mathbf{q}_t^h\right) + \left(1/b_t^h\right)\mathbf{p}_t'\left(\mathbf{q}_s^h - \mathbf{q}_t^h\right)$. This has the same form as condition $(2.B.2)$. So maximisation of the real-valued utility function means that there exist real numbers $u_t^h = u^h\left(\mathbf{q}_t^h\right)$ and $1/b_t^h$ which bear the required interpretation. See Varian (1982) for further discussion.

of time to determine while an acceptance, by definition, does not.[8]

The difficulty can be thought of as follows: in order for the observed distribution of resources to be optimal, the representative consumer needs to equalise the marginal social utility of income across consumers. Arguing loosely from the chain rule, marginal social utility can be thought of as the individual's marginal utility of income multiplied by the marginal contribution of individual utility to social utility (i.e. $\mu_t = \left(1/b_t^h\right) \nabla V\left(u_t^h\right)$). Therefore the term $\mu_t \mathbf{b}_t$ represents a tangle of unobservables which make $(2.B.1)$ nonlinear in unknowns. It is this which gives rise to the problem of infinite testability.

In order to make progress towards a computationally feasible necessary and sufficient condition it is going to be necessary to simplify the interaction between individual marginal utility and social weights. We explore this further next.

Theorem 3 *The following statements are equivalent:*
(3.A). There exist nonsatiated, monotonic, concave and continuous utility functions V, $u^1, ..., u^H$, *with common marginal utility of income, that provide a normative representative consumer rationalisation of the data* $\left\{\mathbf{p}_t, \mathbf{q}_t^h\right\}_{t\in\tau}^{h\in\eta}$.
(3.B). There exist nonsatiated, monotonic, concave and continuous utility functions V, $u^1, ..., u^H$, *for which* V *is additively separable in* $u^1, ..., u^H$, *that provide a normative representative consumer rationalisation of the data* $\left\{\mathbf{p}_t, \mathbf{q}_t^h\right\}_{t\in\tau}^{h\in\eta}$.
(3.C). For all $s, t \in \tau$ *and* $h \in \eta$, *there exist numbers* $u_s^h, u_t^h \in \mathbb{R}_+$ *and* $b_t \in \mathbb{R}_{++}$ *such that*

$$u_s^h \leq u_t^h + \frac{1}{b_t}\mathbf{p}_t'\left(\mathbf{q}_s^h - \mathbf{q}_t^h\right).$$

What this result says is that we can either tie down the social weights to be the same across consumers (i.e. have a utilitarian social welfare function; statement $(3.B)$) or we can tie down the marginal utility of income to be the same across consumers (statement $(3.A)$). Either way, what this does is simplify the inequalities in Theorem 2 to a single (and crucially) *linear* problem (statement $(3.C)$). This inequality is very straightforward to test and does not suffer from the bias-towards-acceptance problem - it is determinable in finite time.

Income distribution rule. To conclude this section, it is important to emphasise that Theorems 2 and 3 both imply the existence of an income distribution rule that distributes aggregate income optimally from a social point of view (i.e. in the sense of Samuelson, 1956, and according to the social welfare function in (1)). Formally (and within the framework of the functional-derivative based literature), an income distribution rule is a family of functions $\left(w^1\left(\mathbf{p},Y\right), w^2\left(\mathbf{p},Y\right), ..., w^H\left(\mathbf{p},Y\right)\right)$ such that $\sum_{h\in\eta} w^h\left(\mathbf{p},Y\right) = Y$ for all $\mathbf{p}$ and Y. In case there is an income distribution rule, then aggregate demand can always (and trivially) be written as a function of aggregate income through $\mathbf{Q} = \sum_{h\in\eta} \mathbf{g}^h\left(\mathbf{p}, w^h\left(\mathbf{p},Y\right)\right)$, where $\mathbf{g}^h\left(.,.\right)$ is consumer h's vector-valued demand associated with this consumer's preferences. Further, aggregate demand is the result of the representative consumer's preference relation that is represented by the social welfare function (1). Consequently, Theorems 2 and 3 imply constraints on the possible income distributions in general; this is because the aggregate demand generally depends on the income distribution rule (see Samuelson, 1956, Jerison, 1994, and Mas-Colell, Whinston and Green, 1995, for further discussion). In the next section, we consider the same question but now we will consider the existence of a normative representative consumer independent of the income distribution. This is essentially the question that Gorman (1953) originally addressed: it asks for the revealed preference conditions associated with exact linear aggregation. Interestingly, we will show that the conditions in Theorem 3 also characterise the Gorman-type normative representative consumer under a very weak data requirement.

[8] This problem is closely related to revealed preference tests for weak separability (Varian, 1983). Also these necessary and sufficient tests turn out to be based on a nonlinear system of inequalities, which is empirically less attractive. A number of alternative separability tests have been proposed, which are either necessary or sufficient. See, for example, Swofford and Whitney (1987, 1994) and Fleissig and Whitney (2003, 2008).

3 Gorman revealed: exact linear aggregation

We next investigate the conditions needed to guarantee exact linear aggregation, i.e. aggregate demand only depends on aggregate income and is not affected by how the income is actually distributed across consumers. From the functional derivative-based literature, we know that this independence result applies *if and only if* consumers have preferences of the Gorman Polar Form and linear Engel curves with common slopes. As demonstrated by Gorman (1953, 1961), this implies that aggregate demand can be written in the simple form $\mathbf{Q} = \mathbf{g}(\mathbf{p}, Y)$, where $\mathbf{g}(.,.)$ is the vector-valued demand equation that results from the maximisation of the normative representative consumer's preferences given aggregate income Y and taking as given market prices $\mathbf{p}$. Clearly, this requires that *any* income distribution, such that $\sum_{h \in \eta} y^h = Y$, gives rise to the same aggregate demands $\mathbf{Q}$; this is guaranteed by the conditions as they have been stated in Theorem 3 (or Theorem 2).

Our following discussion, however, will show a close link between the result in Theorem 3 and Gorman-type aggregation. We proceed in four steps. Firstly, we derive a revealed preference characterisation of individual preferences of the Gorman Polar Form. Secondly, we show the remarkable and important result that if observed prices are nonproportional, then GARP is equivalent to having preferences of the Gorman Polar Form. In practice, this data requirement is very weak: it is met for the application that we present in the next section and, indeed, we are not aware of observational (non-experimentally generated) data on consumer behaviour which exhibits price-proportionality. Next, we provide the revealed preference counterpart to Gorman's aggregation results and show that, in the revealed preference sense, aggregate demand is independent of the income distribution if and only if all consumers have preferences of the Gorman Polar Form with common marginal utilities of income. In other words, all consumers are associated with parallel linear Engel curves. Finally, we propose an easy-to-apply linear test for a normative representative consumer, which holds for any possible income distribution, by combining the above steps. Interestingly, as we will discuss, the linear condition that is tested is empirically equivalent to the condition (3.C) in Theorem 3.

Gorman Polar Form preferences. We begin by defining what it means for the data of an individual consumer to be rationalisable with the Gorman Polar Form. The Gorman Polar Form is usually defined in terms of an indirect utility function w^h. Let y^h represent the income of consumer h. The indirect utility function w^h is connected with the utility function u^h in the following way:

$$w^h\left(\mathbf{p}, y^h\right) = \max_{\mathbf{q}^h}\{u^h\left(\mathbf{q}^h\right) | \mathbf{p}'\mathbf{q}^h \leq y^h\}.$$

We can now state the next definition.

Definition 4 (Gorman Polar Form Rationalisation) *The data* $\left\{\mathbf{p}_t, \mathbf{q}_t^h\right\}_{t \in \mathcal{T}}$ *are rationalisable by the Gorman Polar Form if they are rationalisable by a utility function* u^h *(in the sense of Definition 1) such that the indirect utility function* $w^h(\mathbf{p}, y^h) = \frac{y^h - a^h(\mathbf{p})}{b^h(\mathbf{p})}$*, with* $a^h(\mathbf{p}) \in \mathbb{R}$ *and* $b^h(\mathbf{p}) \in \mathbb{R}_{++}$ *for all* $\mathbf{p}$ *and the functions* a^h *and* b^h *linearly homogeneous of degree 1.*

In this definition, the price index $a^h(\mathbf{p})$ is often interpreted as subsistence expenditure - although this interpretation is not always valid (see Pollak, 1971, p 403, fn 4) - while the price index $b^h(\mathbf{p})$ is interpreted as the inverse of the marginal utility of income.

Before moving on it is worth pointing out the well-established fact that the Gorman Polar Form does not necessarily give rise to well-behaved preferences in all parts of the quantity-space: in general, well-behaved preferences only apply to a limited range of possible income values; and therefore Gorman Polar Form preferences are defined in terms of boundary conditions for the possible income levels.[9] To keep the exposition simple, our following analysis only considers income values that lie within this range; and, thus, we will not explicitly consider the income boundary values. More specifically, we restrict attention to the cases in which preferences are both representable by the Gorman Polar Form *and* rational. This is because (as pointed out above) the existence of the normative representative consumer requires consumers to be utility-maximisers. We can now give the characterisation.[10]

Theorem 4 *The following statements are equivalent:*
(4.A). The data $\left\{\mathbf{p}_t, \mathbf{q}_t^h\right\}_{t\in\tau}$ *are rationalisable by the Gorman Polar Form.*
(4.B). For all $s,t \in \tau$*, there exist numbers* w_s^h, $w_t^h \in \mathbb{R}_+$, $a_t^h \in \mathbb{R}$ *and* $b_t^h \in \mathbb{R}_{++}$ *such that*

$$w_s^h \leq w_t^h + \frac{1}{b_t^h}\mathbf{p}_t'\left(\mathbf{q}_s^h - \mathbf{q}_t^h\right), \tag{4.B.1}$$

$$w_t^h = \frac{\left(\mathbf{p}_t'\mathbf{q}_t^h\right) - a_t^h}{b_t^h}, \tag{4.B.2}$$

$$a_t^h = \delta a_s^h \text{ and } b_t^h = \delta b_s^h \text{ if } \mathbf{p}_t = \delta \mathbf{p}_s \text{ for } \delta > 0. \tag{4.B.3}$$

As before the Afriat numbers in this result have certain structural interpretations. Condition $(4.B.1)$, for example, is again an Afriat inequality, which has a directly similar interpretation as before. In this inequality, we can interpret each number w_t^h as an indirect utility value (the function value $w^h(\mathbf{p}, y^h)$ in Definition 4, which equals the utility value $u^h\left(\mathbf{q}^h\right)$ under rational consumer behaviour). Condition $(4.B.2)$ then states the Gorman Polar Form restriction, with the numbers a_t^h and b_t^h corresponding to the price indices $a^h(\mathbf{p})$ and $b^h(\mathbf{p})$ in Definition 4 evaluated at $\mathbf{p}_t$. Condition $(4.B.3)$, finally, imposes linear homogeneity of these price indices.

Two further notes are in order. First, the Gorman Polar Form characterisation in Theorem 4 is nonlinear in a_t^h and b_t^h. However, in our proof of Theorem 4 we show that it can be equivalently expressed in linear form. In turn, this makes it easily testable.

The second remark combines the results in Theorems 1 and 4. In particular, it follows that, under the weak data requirement of nonproportional prices, Gorman Polar Form preferences provide *no additional restrictions* over and above the standard Afriat inequalities (or, equivalently, GARP).[11] In other words, Gorman Polar Form preferences and rational preferences are nonparametrically (in the revealed preference sense) *equivalent*: for data in which proportional prices movements are not observed their empirical implications are identical. This result is formally stated as follows:

Corollary 1 *The following statements are equivalent when prices* $\mathbf{p}_t \neq \delta\mathbf{p}_s$ $(\delta > 0)$ *for all* $s,t \in \tau$*:*
(A). The data $\left\{\mathbf{p}_t, \mathbf{q}_t^h\right\}_{t\in\tau}$ *are rationalisable by the Gorman Polar Form.*
(B). The data $\left\{\mathbf{p}_t, \mathbf{q}_t^h\right\}_{t\in\tau}$ *satisfy GARP.*

This is an important result. It implies that if the data satisfy GARP and observed prices are nonproportional, then we can *always* construct an indirect utility function which exactly rationalises the data with the Gorman Polar Form. This is perhaps surprising as the Gorman Polar Form is usually thought of as a very demanding restriction. However, it seems that this is only the case

[9] See, for example, Pollak (1971) and Blackorby, Boyce and Russell (1978) for a more detailed discussion on the local nature of Gorman Polar Form preferences. As we explain in the Appendix, similar boundary conditions are needed to prove Theorem 4.

[10] An alternative revealed preference characterisation of the Gorman Polar Form can be found in work in progress by Brown and Shannon. In a certain sense, the work of these authors is complementary to ours as Brown and Shannon characterise Gorman Polar Form preferences in terms of so-called 'dual' Afriat numbers (which have an interpretation in terms of of indirect utility functions; see Brown and Shannon, 2000), whereas our analysis starts from the original 'primal' Afriat numbers (to be interpreted in terms of direct utility functions). We thank Don Brown for revealing this to us in a private conversation.

[11] Specifically, under nonproportional prices condition $(4.B.3)$ becomes redundant. Then, one can easily verify that, for any given solution for the Afriat inequalities $(4.B.1)$, there also exists a solution for condition $(4.B.2)$.

when proportional prices are observed in the data. In such a case, the Gorman Polar Form is *extremely* demanding as we can directly observe points on an Engel curve and this Engel curve must be perfectly straight. However, we are not aware of *any* observational (non-experimentally generated) consumer panel data in which proportional prices changes are ever observed. Thus, it turns out that, empirically, the Gorman Polar Form is without additional empirical content from a revealed preference point of view.[12]

Exact linear aggregation. We can now use these insights to provide the revealed preference counterparts of Gorman's conditions for exact linear aggregation. As stressed above, exact linear aggregation implies a normative representative consumer for any income distribution and thus does not restrict attention to a particular income distribution rule. Gorman proved that such exact aggregation holds if and only if consumers' preferences are of the Gorman Polar Form with common slopes for the (linear) Engel curves. In revealed preference terms, we get the following characterisation.

Theorem 5 *The following statements are equivalent for the data* $\left\{\mathbf{p}_t, \mathbf{q}_t^h\right\}_{t\in\tau}^{h\in\eta}$*:*
(5.A). Aggregate demand is independent of the income distribution.
(5.B). For all $s,t \in \tau$ *and* $h \in \eta$*, there exist numbers* w_s^h*,* $w_t^h \in \mathbb{R}_+$*,* $a_t^h \in \mathbb{R}$ *and* $b_t \in \mathbb{R}_{++}$ *such that*

$$w_s^h \leq w_t^h + \frac{1}{b_t}\mathbf{p}_t'\left(\mathbf{q}_s^h - \mathbf{q}_t^h\right), \tag{5.B.1}$$

$$w_t^h = \frac{\left(\mathbf{p}_t'\mathbf{q}_t^h\right) - a_t^h}{b_t}, \tag{5.B.2}$$

$$a_t^h = \delta a_s^h \text{ and } b_t = \delta b_s \text{ if } \mathbf{p}_t = \delta \mathbf{p}_s \text{ for } \delta > 0. \tag{5.B.3}$$

As compared to Theorem 4, the key requirement is that the Afriat number b_t is common across consumers who face the same prices (i.e. $b_t^h = b_t$ for all h). In terms of Definition 4, this effectively imposes Gorman Polar Form preferences with a common $b(\mathbf{p})$ index for all consumers. The idea is that the marginal utility of income must be independent of income variations across consumers but can vary with prices. Without these restrictions on the individual preferences (and, by implication, on the preferences of the normative representative consumer), one typically has to assume some income distribution rule (as discussed in Section 2). We note, finally, that our characterisation in Theorem 5 can be linearised in a directly similar way as our earlier characterisation in Theorem 4. As such, it implies an easy-to-apply test for a normative representative consumer that is independent of the income distribution.

Interestingly, the characterisation in Theorem 5 also generalises several special cases that generate the same independence of the income distribution. Two important examples are Varian's (1983) revealed preference characterisation of identical homothetic preferences (where $a^h(\mathbf{p}) = 0$ in Definition 4) and Brown and Calsamiglia's (2007) revealed preference characterisation of quasi-linear preferences (where $a^h(\mathbf{p}) = -p^i\phi(\mathbf{p})$ and $b^h(\mathbf{p}) = p^i$, with p^i the price of the numeraire and ϕ a homogeneous of degree one function).

As a final result, we connect the characterisations in Theorems 2 and 5. Similar to Corollary 1, we find that if observed prices are nonproportional, then a necessary and sufficient condition for a Gorman-type normative representative consumer is that each consumer satisfies the standard Afriat inequalities with a common marginal utility of income. This is formally stated in the following result:

Corollary 2 *The following statements are equivalent when prices* $\mathbf{p}_t \neq \delta\mathbf{p}_s$ $(\delta > 0)$ *for all* $s,t \in \tau$*:*
(A). Aggregate demand is independent of the income distribution.

[12] At this point it is worth recalling that we focus on preferences taking the Gorman Polar Form for income values within bounded ranges, which here means that the equivalence in Corollary 1 has a local nature by construction.

(B). For all $s,t \in \tau$ and $h \in \eta$, there exist numbers w_s^h, $w_t^h \in \mathbb{R}_{++}$ and $b_t \in \mathbb{R}_{++}$ such that

$$w_s^h \leq w_t^h + \frac{1}{b_t}\mathbf{p}_t'\left(\mathbf{q}_s^h - \mathbf{q}_t^h\right).$$

Thus, we get exactly condition $(3.C)$ for aggregate demand to be independent of the income distribution. This means that, under nonproportional prices, the condition in Theorem 5 conveniently reduces to the condition in Theorem 3. In other words, under the weak data requirement of nonproportional prices, the characterisation of a normative representative consumer in Theorem 3 holds for all income distributions across consumers and no longer relies on the existence of an income distribution rule. On the other hand, if prices are proportional, then the condition in Corollary 2 (or condition $(3.C)$ in Theorem 3) is not empirically equivalent to the one in Theorem 5. In that case, it still (but only) defines a necessary (and not sufficient) test for exact linear aggregation: if the condition is violated we can (only) conclude that there certainly does not exist a normative representative consumer that is independent of the income distribution.

4 An application

In the previous section we established the revealed preference conditions for exact linear aggregation. Such exact aggregation effectively implies the existence of a normative representative consumer that is independent of the income distribution, which is the most commonly used concept of representative consumer in the economics literature. Our characterisation provides the revealed preference counterparts of Gorman's conditions that have played a crucial role in the functional-derivative based literature. Importantly, the characterisation can be linearised in unknowns, which makes it easily testable.

We will next illustrate our revealed preference based aggregation results by means of an empirical application. Here, it is worth to recall from our discussion in the Introduction that revealed preference methods are intrinsically "nonparametric": in contrast to the more standard functional-derivative based methods, they do not need auxiliary parametric or statistical assumptions. As such, our empirical revealed preference analysis avoids unnecessary aggravation and should thus lead to robust conclusions on the existence of the normative representative consumer (for the data at hand).

One preliminary remark is in order. It pertains to the fact that prices in our application are nonproprortional (see below). As explained at the end of Section 3, nonproportional prices make that our conditions for exact linear aggregation are empirically equivalent to those for a normative representative consumer under a socially optimal income distribution rule. As a result, while we will interpret our following results in terms of aggregation à la Gorman (1953, 1961), it is important to keep in mind that the same results may actually also be given a specific meaning related to aggregation à la Samuelson (1956).

Rationalisability by the Gorman Polar Form. The data we use are drawn from the Spanish Continuous Family Expenditure Survey (ECPF). This is one of the few surveys with detailed expenditure information for a panel of households. The ECPF is a quarterly budget survey of Spanish households which interviews about 3200 households every quarter.[13] We focus on a subsample of couples (with or without children), in which the husband is in full-time employment in a non-agricultural activity while the wife is out of the labour force. This choice is driven by the fact that we want to minimise the impact of possible nonseparabilities between consumption and leisure. To keep things simple, we restrict attention to a balanced panel of households.[14] Given the construction of the ECPF, households can be interviewed for up to eight consecutive quarters.

[13] See Crawford (2010) for more detailed information about the ECPF.

[14] Thus, our empirical analysis uses households/families as consumers. As Samuelson (1947, p.224) pointed out, "Attention should also be called to the fact that even the classical economist does not literally have the individual

However, our sample would be rather small if we would focus on those households observed for a full eight period. Therefore, we have drawn a balanced panel of 342 households which are observed 5 times in order to balance the desire for a reasonable number of observations both across households and time. In what follows, we focus on a set of 15 nondurable commodity groups.[15] We note that the 5 observed price vectors are nonproportional.

Our following analysis proceeds in two steps. First we check, individual household by individual household, whether observed behaviour is rationalisable by the Gorman Polar Form, albeit with heterogeneous $b^h(\mathbf{p})$ indices for the different households h. In the second step we then pool the data across households to investigate the conditions for exact linear aggregation. As discussed above, such aggregation essentially requires a common $b(\mathbf{p})$ index for the different households (i.e. $b^h(\mathbf{p}) = b(\mathbf{p})$ for all h).

The first requirement for exact linear aggregation is that each individual household acts as of it were maximising its own well-behaved utility function. We first check this condition by testing the Afriat inequalities for each household individually (i.e. without pooling the data). That is, we use linear programming to check, for each household h and for all observations $s, t \in \tau$, whether there exist numbers $u_s^h, u_t^h \in \mathbb{R}_{++}$ and $b_t^h \in \mathbb{R}_{++}$ such that

$$u_s^h \leq u_t^h + \frac{1}{b_t^h}\mathbf{p}_t'\left(\mathbf{q}_s^h - \mathbf{q}_t^h\right).$$

In light of Corollary 1 and the fact that our data does not exhibit proportional price movements, it is important to note that if a household's consumption choices satisfy these inequalities it means more than just the fact that they are rationalisable by well-behaved preferences. It also implies that they can be rationalised by preferences of the Gorman Polar Form. In this interpretation, b_t^h equals household h's price index $b^h(\mathbf{p})$ when prices are $\mathbf{p_t}$. The results of this procedure are reported in Table 1.

TABLE 1: Tests of Gorman Polar Form rationalisation

	Pass	Fail
N	326	16
Proportion	0.953	0.047

It turns out that the behaviour of 95% of the households in our data is exactly rationalisable by preferences of the Gorman Polar Form. A little under 5% of the data (16 households) are not rationalisable by well-behaved preference at all. Since it is a necessary condition for normative aggregation that individual households act as if they are utility maximisers, we therefore set these 16 households to one side.[16]

Exact linear aggregation. We now ask whether the aggregate (macro) behaviour of the remaining 95% of the original sample satisfy exact linear aggregation (or, equivalently, can be described by a normative representative consumer that is independent of the income distribution rule). To

in mind, so much as the family; of course, some hardy souls will pursue the will-o'-the-wisp of sovereignty within the family so as to reduce even these collective indifference curves to an individualistic basis". See Cherchye, De Rock and Vermeulen (2007, 2011) for a revealed preference characterisation of Chiappori's (1988) model of collective household consumption that starts from an individualistic basis.

[15] The commodity groups are (i) food and non-alcoholic drinks at home; (ii) alcohol; (iii) tobacco; (iv) energy at home (heating by electricity); (v) services at home (heating: not electricity, water, furniture repair); (vi) nondurables at home (cleaning products); (vii) non-durable medicines; (viii) medical services; (ix) transportation; (x) petrol; (xi) leisure (cinema, theatre, clubs for sports); (xii) personal services; (xiii) personal nondurables (toothpaste, soap); (xiv) restaurants and bars and (xv) traveling (holiday).

[16] An alternative would perhaps be to impose rationalisability on them. Blundell, Browning and Crawford (2008) describe a way to do this. In this case, and in view of the very small number of such households, we opted for simplicity.

do this we simply check the condition for exact linear aggregation that is given in Theorem 5, again using a linear programming approach. Specifically, we need to check for the data *pooled across households* whether there exist numbers $u_t^h \in \mathbb{R}_+$ and $b_t \in \mathbb{R}_{++}$ such that, for all observations $s, t \in \tau$ and all households $h \in \eta$,

$$u_s^h \leq u_t^h + \frac{1}{b_t}\mathbf{p}_t'\left(\mathbf{q}_s^h - \mathbf{q}_t^h\right),$$

where the parameter previously denoted by b_t^h is now common across all households for the prices $\mathbf{p}_t$ (which corresponds to $b^h(\mathbf{p}) = b(\mathbf{p})$ for all h). We find that this condition is rejected - the data cannot be rationalised by a common b_t parameter. Despite the fact that these households all satisfy the necessary condition (Gorman Polar Form preferences) perfectly, and despite the very flexible nature of revealed preference tests, it seems that the data still cannot bear the weight of the theory required for exact linear aggregation.

To the extent that variation in the b_t parameter might be driven by observables, stratification might be a flexible way in which to allow for this - the idea being that a representative consumer might be valid when applied to sub-groups of demographically similar households, even though when applied to the data *in toto* it is rejected. To investigate this further we allocated the data to smaller homogeneous groups on the basis of observables such as their age profiles, schooling level, household size and number of children. This resulted in 52 groups - some of them very small indeed. Table 2 reports the group frequency for different group sizes (measured as number of households). For obvious reasons, we conduct aggregation tests only for the 34 groups which contain more than one household. Although the number of different households for a given stratum could be as small as just two, *even then* we could not find any group that satisfies the condition for aggregation. This despite the fact that the strength of revealed preference tests in general (weakly) increases with the number of observations, so that reducing the number of households involved in a test, by considering only those with similar observables, should make it easier to rationalise behaviour with a representative consumer. We take this to mean that, even in very small groups of demographically similar households, the distribution of income plays an essential role in understanding group (macro) behaviour.

TABLE 2: Household sub-groups

Group size	Frequency
1	18
2	6
3	4
4	6
5	1
6	2
7	3
8	2
9	1
10	2
11	1
14	1
20	1
22	1
24	1
35	1
51	1

Inter-household heterogeneity. This is the first time that Gorman's aggregation conditions have been tested in a revealed preference framework. As such, the result is very robust. It is worth, therefore, considering for a moment the source of this rejection. All of the households used in our test satisfied the conditions for rational preferences of the Gorman Polar Form, i.e. their behaviour is consistent with the indirect utility function

$$w^h\left(\mathbf{p}, y^h\right) = \frac{y^h - a^h\left(\mathbf{p}\right)}{b^h\left(\mathbf{p}\right)},$$

in which the price indices are allowed to be heterogeneous. Regarded as an affine function of income, heterogeneity in the "intercept" of the indirect utility function is not relevant to aggregation considerations, but heterogeneity in the "slope" index $1/b^h\left(\mathbf{p}\right)$ is.[17] The *prima facie* reason that the data fail to satisfy the conditions for aggregation must, therefore, be heterogeneity in the slope of the indirect utility function with respect to income. Our revealed preference characterisation allows us to investigate this a little further - in a way that naturally exploits the specific panel structure of our data set. We introduce a time-varying idiosyncratic heterogeneity parameter ϵ_t^h to the slope index for each household. Then, we consider the problem of how much heterogeneity around a common slope index is required in order to rationalise the data, and we therefore look at the values ε_t^h which solve the following minimisation problem (for $s, t \in \tau$ and $h \in \eta$):

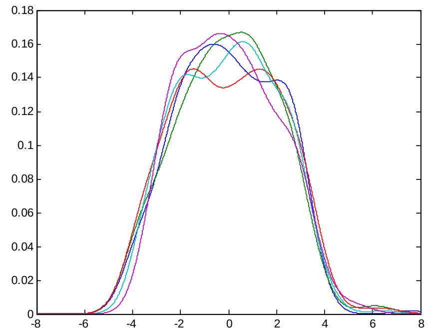
$$\begin{aligned} & \min_{u_t^h \in \mathbb{R}_+, \varepsilon_t^h \in \mathbb{R}, b_t \in \mathbb{R}_{++}} \sum_{t \in \tau, h \in \eta} \left(\epsilon_t^h\right)^2 \\ & s.t. \\ u_s^h \quad \leq & \quad u_t^h + \left(\frac{1}{b_t} + \epsilon_t^h\right) \mathbf{p}_t' \left(\mathbf{q}_s^h - \mathbf{q}_t^h\right) \\ 0 \quad < & \quad \frac{1}{b_t} + \epsilon_t^h. \end{aligned} \tag{2}$$

Clearly, only when the stated condition for exact linear aggregation is satisfied, we have that all the parameters ε_t^h equal zero in this problem. We already know from the previous results that this is not the case for our dataset. Thus, our minimisation problem evaluates how close we can come to this in a least-squares sense. The larger the solution values for ε_t^h, the further we are from exact aggregation. Each parameter ε_t^h captures the (minimum) household-specific deviation from a common $1/b\left(\mathbf{p}\right)$ index to obtain consistency with our conditions for exact linear aggregation. As such, the distribution of the parameters ε_t^h over all households h effectively captures the minimal heterogeneity underlying the observed violation of the conditions for exact linear aggregation.

Our empirical procedure delivers the following parameters which solve (2): $\left\{b_t, \varepsilon_t^h\right\}_{t \in \tau}^{h \in \eta}$. Figure 1 shows a nonparametric estimate of the density of the distribution of the heterogeneity term ε_t^h. The five curves show the distribution for each of the five periods. The heterogeneity distribution is stable across time and approximately symmetric as to be expected given our quadratic loss function, but it fails a Kolmogorov-Smirnov test for normality (we standardise the variable and test against a null that the distribution is a standard normal). From Figure 1, we conclude that we need considerable heterogeneity across households to rationalise the observed consumption behaviour. Recall too that we have found only the minimal heterogeneity required for rationalisation. It therefore seems that variation in the marginal utility of income is economically significant: certainly significant enough to kill off the representative consumer - perhaps once and for all.

[17] Indeed, it is the requirement of a common "slope" index (i.e. $b\left(\mathbf{p}\right) = b^h\left(\mathbf{p}\right)$ for all h) that drives the empirical restrictions associated with exact linear aggregation. In revealed preference terms, this is particularly apparent from Theorem 5 and Corollary 2. Here, we can draw a parallel with typical panel data regression models, which can allow for cross-sectional heterogeneity in the intercept term (corresponding to $a^h\left(\mathbf{p}\right)$ in our setting) while, for identification purposes, they need to assume slope coefficients to be the same cross-sectionally.

Figure 1: Estimates of the density of heterogeneity terms ε_t^h (by period)



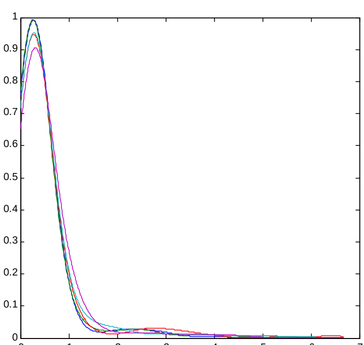
As an interesting side product, our minimum-distance procedure also provides solutions for b_t which is the value of the common component of the $b^h(\mathbf{p}_t)$ price index in each period. This is reported in Table 3. As is clear from the table, the index $b(\mathbf{p}_t)$ is roughly increasing over the five time periods. This reflects the increase of most nominal commodity prices over the time frame considered.[18]

Table 3: Annual values of $b(\mathbf{p}_t)$

Period	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$
$b(\mathbf{p}_t)$	0.2451	0.2402	0.2534	0.2594	0.2887

Finally, given a solution for $1/b_t + \varepsilon_t^h$, which is interpretable as a solution for the slope index $1/b^h(\mathbf{p}_t)$, we can also describe the distribution of the implied idiosyncratic $b^h(\mathbf{p}_t)$ price index. This is shown in Figure 2, which brings together the results in Figure 1 and Table 3. In line with these earlier results, whilst the distribution of $b^h(\mathbf{p}_t)$ appears to be fairly stable over time, the heterogeneity term swamps the variability in the common component of the index.

Figure 2: Estimates of the density of $b^h(\mathbf{p}_t) = \left(\frac{1}{b_t} + \varepsilon_t^h\right)^{-1}$ (by period)



5 Conclusion

The concept of the normative representative consumer has since long played a central role in many areas in economics. Although the conditions for its existence have been argued to be demanding, it

[18] Following this interpretation, when considering the raw price data, we found that it is the decrease of a single commodity price that drives the drop in the index $b(\mathbf{p}_t)$ between periods 1 and 2.

is fair to say that existing evidence is solely based on Gorman's well-known exact linear aggregation results within a functional-derivative based framework. To test Gorman's conditions for exact linear aggregation (which boil down to consumers having preferences of the Gorman Polar Form with an equal marginal utility of income), one needs to make many additional assumptions to bring these conditions to the data. In this paper, we revisited the exact aggregation problem by bringing in tools from the revealed preference literature. These tools are based solely on the data at hand and do not need any additional parametric or statistical assumptions. As such, they allow for robustly analysing the empirical validity of exact aggregation.

In addition to a few interesting and rather important side results (like a revealed preference characterisation of Gorman Polar Form preferences for an individual consumer), we proposed a revealed preference test for the existence of a consumer that can normatively represent a set of consumers regardless of the income distribution (i.e. exact linear aggregation). Interestingly, the test is linear and thus easy to apply in practice. Our analysis also clarified the relationship between the empirical restrictions associated with Samuelson-type aggregation and Gorman-type aggregation. Most notably, we made explicit the conditions under which the two notions of aggregation become empirically equivalent.

We showed the practical usefulness of our revealed preference characterisation by means of an empirical application to a Spanish balanced microdata panel. We could not find any evidence for the existence of a normative representative consumer that is independent of the income distribution: our data systematically reject the necessary and sufficient conditions for exact linear aggregation even for groups with only *two* households. Given this negative result, we have subsequently analysed the unobservable heterogeneity across households that drives the rejection. Specifically, we have used our revealed preference characterisation to reconstruct heterogeneous household-specific price indices which make exact aggregation impossible. Overall it seemed that the inter-household heterogeneity in the slope index of the indirect utility function is economically significant. This implies that there is also important variation in the marginal propensity to consumer across households. We take our results to mean that the distribution of income plays an essential role in understanding group (macro) behaviour. As a consequence, our results may provide further ammunition to those who wish to wipe out the normative representative consumer once and for all.

Appendix

Proof of Theorem 2.

$(2.A) \Rightarrow (2.B)$: First consider the implications of optimising behaviour and the first order conditions from the consumer's problem. Continuity ensures that suitable subgradients exist such that $\boldsymbol{\nabla}V\left(\mathbf{q}_t^h\right) \leq \mu_t \mathbf{p}_t$ where $\boldsymbol{\nabla}V\left(\mathbf{q}_t^h\right) = \nabla V\left(u_t^h\right)\boldsymbol{\nabla}u^h\left(\mathbf{q}_t^h\right)$. Define $\mu_t b_t^h = \nabla V\left(u_t^h\right)$. Then $\boldsymbol{\nabla}u^h\left(\mathbf{q}_t^h\right) \leq \left(b_t^h\right)^{-1}\mathbf{p}_t$. Now consider the concavity conditions for this structure

$$\begin{aligned} V\left(\mathbf{u}_s\right) &\leq V\left(\mathbf{u}_t\right) + \boldsymbol{\nabla}V\left(\mathbf{u}_t\right)'\left(\mathbf{u}_s - \mathbf{u}_t\right) \\ u^h\left(\mathbf{q}_s^h\right) &\leq u^h\left(\mathbf{q}_t^h\right) + \boldsymbol{\nabla}u^h\left(\mathbf{q}_t^h\right)'\left(\mathbf{q}_s^h - \mathbf{q}_t^h\right) \end{aligned}$$

Substituting in $\boldsymbol{\nabla}u^h\left(\mathbf{q}_t^h\right) \leq \left(b_t^h\right)^{-1}\mathbf{p}_t$ and $\mu_t b_t^h = \nabla V\left(u_t^h\right)$ preserves the inequalities and gives

$$\begin{aligned} V\left(\mathbf{u}_s\right) &\leq V\left(\mathbf{u}_t\right) + \mu_t \mathbf{b}_t'\left(\mathbf{u}_s - \mathbf{u}_t\right) \\ u^h\left(\mathbf{q}_s^h\right) &\leq u^h\left(\mathbf{q}_t^h\right) + \frac{1}{b_t^h}\mathbf{p}_t'\left(\mathbf{q}_s^h - \mathbf{q}_t^h\right) \end{aligned}$$

which are conditions $(2.B.1)$ and $(2.B.2)$.

$(2.B) \Rightarrow (2.A)$: Suppose we have numbers $\{V_t, \mu_t > 0\}_{t\in\tau}$ and $H-$vectors $\{\mathbf{u}_t, \mathbf{b}_t > 0\}_{t\in\tau}$ such that conditions $(2.B.1)$ and $(2.B.2)$ hold. Consider some arbitrary $\left\{\mathbf{q}^h\right\}^{h\in\eta}$ such that $\mathbf{p}_t'\sum\mathbf{q}_t^h \geq \mathbf{p}_t'\sum_h \mathbf{q}^h$ for some observation t. We need to show that there exists utility functions, with the stated properties such that $V\left(u^1\left(\mathbf{q}_t^1\right),...,u^H\left(\mathbf{q}_t^H\right)\right) \geq V\left(u^1\left(\mathbf{q}^1\right),...,u^H\left(\mathbf{q}^H\right)\right)$. Using $(2.B.2)$ we can construct T upper bounds on $u^h\left(\mathbf{q}^h\right)$ and if we take the minimum of these then we have, as in Varian (1982), a piecewise linear, nonsatiated, monotonic, concave and continuous utility function

$$u^h\left(\mathbf{q}^h\right) = \min_s \left\{u_s^h + \frac{1}{b_s^h}\mathbf{p}_s'\left(\mathbf{q}^h - \mathbf{q}_s^h\right)\right\}_{s\in\tau} \leq u_t^h + \frac{1}{b_t^h}\mathbf{p}_t'\left(\mathbf{q}^h - \mathbf{q}_t^h\right).$$

Summing this inequality over h after multiplying it with the strict positive number b_t^h gives

$$\mathbf{b}_t'\mathbf{u}_t - \mathbf{p}_t'\sum\mathbf{q}_t^h \geq \mathbf{b}_t'\mathbf{u} - \mathbf{p}_t'\sum\mathbf{q}^h$$

where $\mathbf{u}_t = \left(u_t^1,...,u_t^H\right)'$, $\mathbf{u} = \left(u^1,...,u^H\right)'$, $u^h = u^h\left(\mathbf{q}^h\right)$ and $\mathbf{b}_t = \left(b_t^1,...b_t^H\right)'$. Since $\mathbf{p}_t'\sum\mathbf{q}_t^h \geq \mathbf{p}_t'\sum\mathbf{q}^h$ we must have that $\mathbf{b}_t'\mathbf{u}_t \geq \mathbf{b}_t'\mathbf{u}$. Using $(2.B.1)$ we can then similarly construct the following macro-utility function

$$V\left(\mathbf{u}\right) = \min_s\left\{V_s + \mu_s\mathbf{b}_s'\left(\mathbf{u} - \mathbf{u}_s\right)\right\}_{s\in\tau} \leq V_t + \mu_t\mathbf{b}_t'\left(\mathbf{u} - \mathbf{u}_t\right).$$

Since $\mu_t\mathbf{b}_t'\left(\mathbf{u} - \mathbf{u}_t\right) \leq 0$ we obtain $V\left(\mathbf{u}\right) \leq V_t$ as required.$\blacksquare$

Proof of Theorem 3.

$(3.A) \Leftrightarrow (3.C)$. The condition $(3.C)$ is simply $(2.B.2)$ from Theorem 1 with the common marginal utility of income requirement added. Condition $(2.B.1)$ is redundant according to the following argument. Sum $(3.C)$ over h

$$\sum_h u_s^h \leq \sum_h u_t^h + \frac{1}{b_t}\sum_h \mathbf{p}_t'\left(\mathbf{q}_s^h - \mathbf{q}_t^h\right)$$

Define $V_t = \sum u_t^h$ and $\mu_t = \frac{1}{b_t}$ then

$$V_s - V_t = \mu_t b_t\left(\mathbf{1}'\mathbf{u}_s - \mathbf{1}'\mathbf{u}_t\right)$$

since $\mu_t b_t = 1$. Hence there exist numbers such that

$$V_s \leq V_t + \mu_t b_t \left(\mathbf{1}'\mathbf{u}_s - \mathbf{1}'\mathbf{u}_t\right)$$

which is $(2.B.1)$ when $b_t^h = b_t$. Thus the conditions are equivalent to those in Theorem 2 with the extra restriction that $b_t^h = b_t$.
$(3.B) \Leftrightarrow (3.C)$ Analogous to the proof of Theorem 2. However given the additive separability of V we have $\nabla V\left(u_t^i\right) = \nabla V\left(u_t^j\right)$, i.e. this derivative is constant for all i, j. So define $\mu_t b_t = \nabla V\left(u_t^h\right)$ and note the lack of the h superscript on b_t. The rest of the proof follows that for Theorem 2 to give condition $(2.B.2)$. Summing $(2.B.2)$ across h and defining $V_t = \mathbf{1}'\mathbf{u}_t$ gives

$$V_s \leq V_t + \frac{1}{b_t}\mathbf{p}_t'\left(\sum \mathbf{q}_s^h - \sum \mathbf{q}_t^h\right)$$

$$V_s = V_t + \mathbf{1}'\left(\mathbf{u}_s - \mathbf{u}_t\right)$$

which satisfies condition (2.B.1) where we interpret $\mu_t \mathbf{b}_t = \mathbf{1}$.■

Proof of Theorem 4.

As a preliminary step, we provide an equivalent linear formulation of the conditions in $(4.B)$. Let $\alpha_t = -a_t^h/b_t^h$ and $\beta_t^h = 1/b_t^h$. Then we get the following linear reformulations of the conditions $(4.B.1) - (4.B.3)$:

$$w_s^h \leq w_t^h + \beta_t^h \mathbf{p}_t'\left(\mathbf{q}_s^h - \mathbf{q}_t^h\right), \tag{4.B.1'}$$

$$w_t^h = \alpha_t + \beta_t^h\left(\mathbf{p}_t'\mathbf{q}_t^h\right), \tag{4.B.2'}$$

$$\alpha_t^h = \alpha_s^h \text{ and } \beta_t^h = \beta_s^h/\delta \text{ if } \mathbf{p}_t = \delta\mathbf{p}_s \text{ for } \delta > 0. \tag{4.B.3'}$$

.
$(4.A) \Rightarrow (4.B)$: Condition (4.B.1') readily follows Theorem 1 for a utility function u^h that rationalises the data $\left\{\mathbf{p}_t, \mathbf{q}_t^h\right\}_{t\in\tau}$. Then, we can use $w_t^h = \max_{\mathbf{q}}\{u^h\left(\mathbf{q}\right)|\mathbf{p}_t'\mathbf{q} \leq \mathbf{p}_t'\mathbf{q}_t^h\}$ (using $\mathbf{p}_t'\mathbf{q}_t^h = y_t^h$). Given this, Definition 4 directly implies (4.B.2') and (4.B.3') when using $\alpha_t = -a^h(\mathbf{p}_t)/b^h(\mathbf{p}_t)$ and $\beta_t^h = 1/b^h(\mathbf{p}_t)$.

$(4.B) \Rightarrow (4.A)$: Consider

$$u^h\left(\mathbf{q}\right) = \min_t\ [w_t^h + \beta_t^h\mathbf{p}_t'\left(\mathbf{q} - \mathbf{q}_t^h\right)].$$

Varian (1982) has shown that this utility function rationalises the data $\left\{\mathbf{p}_t, \mathbf{q}_t^h\right\}_{t\in\tau}$. Using (4.B.2'), we have

$$u^h\left(\mathbf{q}\right) = \min_t[\alpha_t^h + \beta_t^h\mathbf{p}_t'\mathbf{q}]. \tag{3}$$

Let us then verify whether the function u^h meets Definition 4. Consider some arbitrary prices $\mathbf{p}_0$ and income y_0^h. As a preliminary step, we recall that

$$w^h(\mathbf{p}_0, y_0^h) = \max_{\mathbf{q}}\{u^h\left(\mathbf{q}\right)|\mathbf{p}_0'\mathbf{q} \leq y_0^h\}.$$

Thus, using (3), we get

$$w^h(\mathbf{p}_0, y_0^h) = \max_{\mathbf{q}}\left\{\min_t[\alpha_t^h + \beta_t^h\mathbf{p}_t'\mathbf{q}]|\mathbf{p}_0'\mathbf{q} \leq y_0^h\right\}.$$

Dropping the min operator, we can equivalently state

$$w^h(\mathbf{p}_0, y_0^h) = \max_{w,\mathbf{q}}\left\{w|w \leq \alpha_t^h + \beta_t^h\mathbf{p}_t'\mathbf{q}\ \ (t \in \tau)\,,\ \mathbf{p}_0'\mathbf{q} \leq y_0^h\right\},$$

which obtains the linear program

$$
\begin{aligned}
w^h(\mathbf{p}_0, y_0^h) = &\max_{w \in \mathbb{R}, \mathbf{q} \in \mathbb{R}_+^N} w \\
&\text{s.t.} \\
w - \beta_t^h &\mathbf{p}_t' \mathbf{q} \leq \alpha_t^h \ \ (t \in \tau), \\
&\mathbf{p}_0' \mathbf{q} \leq y_0^h.
\end{aligned} \tag{4}
$$

The dual linear program is given as

$$
\begin{aligned}
w^h(\mathbf{p}_0, y_0^h) = &\min_{\theta_t \in \mathbb{R}_+ \ (t \in \tau), \lambda \in \mathbb{R}_+} \sum\nolimits_{t=1}^{T} \alpha_t^h \theta_t + \lambda y_0^h \\
&\text{s.t.} \\
&\sum\nolimits_{t=1}^{T} \theta_t = 1, \\
-&\sum\nolimits_{t=1}^{T} \theta_t \beta_t^h \mathbf{p}_t + \lambda \mathbf{p}_0 \geq 0.
\end{aligned} \tag{5}
$$

Let θ_t^* $(t \in \tau)$ and λ^* define the optimum of program (5). In general, these optimal values are independent of y_0^h when y_0^h respects boundary conditions that limit the domain of y_0^h. In practice, the boundary values for y_0^h can be determined by standard methodology for sensitivity analysis of linear programming. (Technically, these bounds will correspond to the range of y_0^h (as the objective coefficient of λ) for which the optimal basic feasible solution of the linear program (5) remains constant.) These boundary conditions parallel the usual conditions that apply to indirect utility functions representing Gorman Polar Form preferences; see our discussion following Definition 4 in the main text.

Thus, because the solution of the problem (5) is independent of y_0^h (under the stated boundary conditions), we conclude that the function w^h in (5) meets the requirement in Definition 4 for

$$\lambda^* = 1/b^h(\mathbf{p}_0) \text{ and } -a^h(\mathbf{p}_0)/b^h(\mathbf{p}_0) = \sum\nolimits_t \theta_t^* \alpha_t.$$

Specifically, for w^* the optimal value of linear program (5) (or, equivalently, (4)), we get

$$
\begin{aligned}
w^h(\mathbf{p}_0, y_0^h) &= w^* \\
&= \lambda^* y_0^h + \sum\nolimits_{t=1}^{T} \theta_t^* \alpha_t^h \\
&= \frac{y_0^h - a^h(\mathbf{p}_0)}{b^h(\mathbf{p}_0)}.
\end{aligned}
$$

Inspection of problems (4) and (5) reveals that the price indices a^h and b^h are linearly homogenous of degree 1 (if again the same income boundary conditions hold).■

Proof of Corollary 1.

As a first step, we note that the conditions $(4.B.2)$ and $(4.B.3)$ in Theorem 4 are void if $\mathbf{p}_t \neq \delta \mathbf{p}_s$ $(\delta > 0)$ for all s, t. As such, rationalisability by Gorman Polar Form only requires consistency with the condition $(4.B.1)$. The equivalence between the statements (A) and (B) in Corollary **1** then follows directly from the equivalence between statements $(1.B)$ and $(1.C)$ in Theorem 1.■

Proof of Theorem 5.

This follows from Theorem 4 (i.e. each household is rationalisable by the Gorman Polar Form) and the result of Gorman (i.e. the marginal utility of income is household independent, which is captured by the common b_t (i.e. $b_t^h = b_t$ for all h)). ■

Proof of Corollary 2.

The result follows from combining Corollary 1 with Theorem 5.■

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